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# INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

## QUANTIZATION OF ROBERTSON-WALKER GEOMETRY COUPLED TO FERMIONIC MATTER

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# QUANTIZATION OF ROBERTSON-WALKER GEOMETRY COUPLED TO FERMIONIC MATTER *

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ABSTRACT

A Robertson-Walker universe coupled to a spin $\frac{1}{2}$ Dirac field is quantized following Dirac's formalism for constrained Hamiltonian systems. It is found that in nearly all cases it can be asserted that the universe avoids the collapse.

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## I. INTRODUCTION

The first attempts to develop a quantum theory of gravity are almost as old as quantum field theory itself [1,2]. With the development by Dirac [3] of a consistent treatment for constrained Hamiltonian systems, the way was paved for the canonical formulation of a quantum theory of gravity. Today, the use of the canonical formalism in the reduction of the Einstein gravitational action to Hamiltonian form is well known [4-6]. However, the passage from the classical to the quantum theory using the substitution of dynamical variables by operators and Poisson brackets (P.B.) by commutators is complicated by the problem of operator ordering [7,8] so that one is left with the choice of either abandoning the canonical approach or to study simplified models. Along the lines of the latter, one can consider a system in which all but a finite number of degrees of freedom have been frozen out by only allowing the geometry to remain within a certain specified class [9,10], i.e. by considering special solutions to Einstein's equations. The resulting "quantum cosmology" is more manageable than the full theory because it is essentially an ordinary quantum mechanical model. In this way attention can be focused on the problems which are peculiar to the gravitational field rather than those which are common to all quantum field theories. In particular, the phenomenon of gravitational collapse and the influence of quantum effects upon it can be sensibly discussed in this context. Indeed, since singularities were proven to be a quite general feature of classical general relativity [11], it is expected that quantum gravity will give the final word.

In this paper we consider the quantization of the Robertson-Walker geometry coupled to an anticommuting spin $\frac{1}{2}$ matter field. Nelson and Teitelboim [12] have thoroughly discussed the constrained Hamiltonian system of gravity plus a Dirac field. The difference between their approach to the classical problem and ours is their use of commuting instead of anticommuting spinors and our particular choice of geometry,which makes the discussion of the constraints a lot simpler in our case. The quantum cosmological problem with fermions has also been discussed by Isham and Nelson [13]. They do not discuss the constraints a la Dirac and this leads them to conclude that no fermions can be supported in a Robertson-Walker universe with $k \neq 0$; for $k = 0$ the fermion mass is quantized and its quanta are of the order of the mass of the known universe. In our treatment of the problem we carry out

Dirac's programme first and once the Dirac brackets have been defined and the second class constraints put strongly to zero, the transition is made to quantum mechanics. The only first class constraint $\mathcal{H} \approx 0$ becomes the "Schrödinger equation" $\mathcal{H}\psi = 0$. In this way we find that for $k \neq 0$ fermions are not only consistent with a R-W background, but the probability density of the universe, $R^{1/2}|\psi|^2$, vanishes as $R \to 0$ - signalling a possible avoidance of the singualrity. In the case $k = 0$ fermions can be defined as well and we find that $R^{1/2}|\psi|^2 \to 0$ as $R \to 0$, but depending on the fermion number, the wave function might be non-normalizable.

## II. THE ACTION

We are interested in the Robertson-Walker geometry described by the metric

$$ds^2 = -N^2 dt^2 + R^2 \Delta_{ij}\, dx^i dx^j$$

$$\Delta_{ij} = \delta_{ij} + \frac{k x^i x^j}{1-kr^2} \quad , \quad r^2 = x_i x_i \tag{2.1}$$

i,j,k, etc. = 1,2,3.

Both $N$ and $R$ are only functions of the time parameter $x^0$ and $\Delta_{ij}$ is the metric of the three space of constant curvature $k$ (in our units $k = +1, -1, 0$). The source term required by Einstein's equations is provided by a massive Dirac field with Lagrangian

$$\mathcal{L}_M = -\frac{i}{2} e^{\mu a}\left(\bar{\psi}\gamma_a \nabla_\mu \psi - h.c.\right) - im\bar{\psi}\psi \tag{2.2}$$

where $\nabla_\mu = \partial_\mu - iB_\mu$ and $B_\mu = \frac{1}{4} e_{\beta a} e^{\beta}{}_{b;\mu}\ \sigma^{ab} \equiv \frac{1}{4}\omega_{ab\mu}\ \sigma^{ab}$

$\mu, \nu, \rho, \ldots$ etc. = 0 - 3 are world indices

a, b, c, ...etc. = 0 - 3 are tangent space indices.

The tetrads $e^{\mu a}$ are in this case

$$e^{\mu a} = \begin{bmatrix} N^{-1} & \\ & R^{-1}\Sigma^{i\underline{k}} \end{bmatrix} \quad , \quad e_{\mu a} = \begin{bmatrix} N & \\ & R\,\Sigma_{i\underline{k}} \end{bmatrix}$$

$$e^{\mu a} e_{\mu b} = \delta^a{}_b \quad , \quad e_\mu{}^a e_{\nu a} = g_{\mu\nu} \tag{2.3}$$



and, in particular,

$$\Sigma_{i\underline{k}}\ \Sigma_j{}^{\underline{k}} = \Delta_{ij}$$

$$\Sigma_{i\underline{k}}\ \Sigma^i{}_{\underline{j}} = \delta_{\underline{k}\underline{j}}$$

$$\Delta_{ik}\ \Sigma^k{}_{\underline{\ell}} = \Sigma_{i\underline{\ell}}$$

$$\Delta_{ik}\ \Delta^{ij} = \delta_k{}^j$$

A bar under a latin index indicates that it refers to tangent space. (For notation and $\gamma$-matrix conventions see the appendix.)

Since the 3-space is homogeneous and isotropic, we expect the spatial dependence of $\psi$ to be trivial, therefore we assume, following Ref.13, that $\psi = \psi(t)$. The Lagrangian (2.2) then takes the form

$$\mathcal{L}_M = \frac{i}{2N}\left(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi\right) - im\bar{\psi}\psi + \frac{i}{4R}\bar{\psi}\gamma^0\gamma_5\psi \tag{2.4}$$

where $s = \Sigma^i{}_{\underline{k}}\ (\partial_i \Sigma^j{}_{\underline{\ell}})\ \Sigma_{j\underline{m}}\ \epsilon^{\underline{k}\underline{\ell}\underline{m}}$. The quantity $s$ can be evaluated knowing the Lie brackets of the triad fields

$$[\Sigma_{\underline{a}}, \Sigma_{\underline{b}}]_{\mathcal{L}} = C_{\underline{a}\underline{b}}{}^{\underline{c}}\ \Sigma_{\underline{c}} \tag{2.5}$$

where the Lie bracket $[\Sigma_{\underline{a}}, \Sigma_{\underline{b}}]_{\mathcal{L}}$ is the operator

$$\Sigma^i{}_{\underline{a}}\ \partial_i \Sigma_{j\underline{b}} - \Sigma^i{}_{\underline{b}}\ \partial_i \Sigma_{j\underline{a}}\ . \tag{2.6}$$

Putting (2.5) and (2.6) together we find [14]

$$s = \frac{1}{2} C_{\underline{k}\underline{\ell}\underline{m}}\ \epsilon^{\underline{k}\underline{\ell}\underline{m}} = \begin{cases} 3 & \text{for } k = +1 \\ 0 & \text{for } k = 0, -1\ . \end{cases} \tag{2.7}$$

The full (gravity + matter) action is

$$S = \int \sqrt{-g}\ [\mathbf{R} + \mathcal{L}_M]\ dx^0 d^3x \tag{2.8}$$

where we have absorbed a factor $16\pi$ in the normalization of the spinor field and we work in the units in which $c = \hbar = G = 1$. For the metric in question the scalar curvature $\mathbf{R}$ is found to be [15]

$$\mathbf{R} = 6\left[-\frac{\ddot{R}}{N^2 R} - \frac{\dot{R}^2}{N^2 R^2} + \frac{\dot{R}\dot{N}}{N^3 R} - \frac{k}{R^2}\right] \tag{2.9}$$

and

$$\sqrt{-g} = N R^3 (1 - k r^2)^{-1/2} .$$

Since the only relevant co-ordinate of the problem is $t = x^0$, the integration over three space can be carried out, yielding an action in terms of the dynamical variables $R(t)$, $N(t)$, $\psi(t)$ and $\bar{\psi}(t)$:

$$S = c_k \int \left\{ 6\left[-\frac{R^2 \ddot{R}}{N} - \frac{R\dot{R}^2}{N} + \frac{R^2 \dot{R}\dot{N}}{N^2} - kNR\right] + L_M \right\} dt \tag{2.10}$$

where $c_k = \int (1 - kr^2)^{-1/2} \, d^3x$ is an irrelevant (possibly infinite) constant which does not affect the variation of $S$, and

$$L_M = \frac{i}{2} R^3 (\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi) - imNR^3\bar{\psi}\psi + \frac{is}{4} N R^2 \bar{\psi}\gamma^0\gamma_5\psi \tag{2.11}$$

In (2.10) one can eliminate the $\ddot{R}$ and $\dot{N}$ terms by partial integration; the result is

$$S = c_k \int \left\{ 6\left[\frac{R\dot{R}^2}{N} - kNR\right] + L_M \right\} dt \tag{2.12}$$

Although the constant $c_k$ becomes infinite for $k \leq 0$ (open space), it just accounts for the fact that the proper volume of three space is infinite. In fact, it can be checked that the equations obtained below by variations of (2.12) convey the same information as the ten Einstein equations [16]. So, we will take as our starting point the action

$$S = \int \left[\frac{R\dot{R}^2}{N} - kNR + \frac{i}{2} R^3 (\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi) - imNR^3\bar{\psi}\psi + \frac{is}{4} R^2 N \bar{\psi}\gamma^0\gamma_5\psi\right] dt \tag{2.13}$$

where we have again absorbed a numerical factor into the normalization of $\psi$.



## III. THE CLASSICAL EQUATIONS

The Euler-Lagrange equations obtained from (2.13) are as follows:

$$\frac{R\dot{R}^2}{N^2} + kR + imR^3\bar{\psi}\psi - \frac{is}{4} R^2 \bar{\psi}\gamma^0\gamma_5\psi = 0 \tag{3.1}$$

$$\frac{d}{dt}\left(\frac{2R\dot{R}}{N}\right) - \frac{\dot{R}^2}{N} + kN - \frac{3i}{2}(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi)R^2 + 3imNR^2\bar{\psi}\psi - \frac{is}{2} NR\bar{\psi}\gamma^0\gamma_5\psi = 0 \tag{3.2}$$

$$iR^3\gamma^0\dot{\psi} + \left[\frac{3i}{2} R^2\dot{R}\gamma^0 - imNR^3 + \frac{is}{4} NR^2\gamma^0\gamma_5\right]\psi = 0 \tag{3.3}$$

$$i\dot{\bar{\psi}}\gamma^0 R^3 + \bar{\psi}\left[\frac{3i}{2} R^2\dot{R}\gamma^0 + imNR^3 - \frac{is}{4} NR^2\gamma^0\gamma_5\right] = 0 . \tag{3.4}$$

On the other hand, the Einstein equations $G_{\mu\nu} = T_{\mu\nu}$ read

$$G_{00}: \quad -2\left(\frac{R\dot{R}^2}{N^2} + kR\right)\frac{N^2}{R^3} = \frac{iN}{2}(\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi) + imN^2\bar{\psi}\psi - \frac{is}{4R} N^2 \bar{\psi}\gamma^0\gamma_5\psi, \tag{3.5}$$

$$G_{0i}: \quad 0 = \frac{i}{2}\bar{\psi}\gamma_{\underline{k}}\gamma_5\psi \, e_{j\underline{m}} e^{j}{}_{\underline{n};i} \, \epsilon^{\underline{kmn}} + iR\Sigma_{i\underline{k}}(\bar{\psi}\gamma^{\underline{k}}\dot{\psi} - \dot{\bar{\psi}}\gamma^{\underline{k}}\psi), \tag{3.6}$$

$$G_{ij}: \quad (2R\ddot{R} + \dot{R}^2 + k)\Delta_{ij} = \left(\frac{i}{2N}[\bar{\psi}\gamma^0\dot{\psi} - \dot{\bar{\psi}}\gamma^0\psi] - im\bar{\psi}\psi\right)R^2\Delta_{ij} + \frac{i}{2} R\Omega_{ij}\bar{\psi}\gamma^0\gamma_5\psi, \tag{3.7}$$

where $\Omega_{ij} \equiv (\frac{1}{2}\Delta_{ij}\Sigma^{k}{}_{\underline{\ell}}\,\omega_{\underline{mnk}} - \Sigma_{i\underline{\ell}}\,\omega_{\underline{mnj}})\,\epsilon^{\underline{\ell mn}}$ .

Finally, the equations for $\psi$, $\bar{\psi}$ are the same as those obtained from the variations of (2.4) or (2.9). Combining (3.3) and (3.4) one gets the equation

$$\bar{\psi}\gamma_{\underline{k}}\dot{\psi} - \dot{\bar{\psi}}\gamma_{\underline{k}}\psi + \frac{s}{2}\frac{N}{R}\bar{\psi}\gamma_{\underline{k}}\gamma_5\psi = 0 , \tag{3.8}$$

which is the form Eq.(3.6) takes upon integration over $d^3x$. Using (3.8), Eq.(3.5) reduces to (3.1) and (3.7) to (3.2), thus showing the equivalence of the two treatments. Our interest is now to proceed to quantize this system in the Hamiltonian form, so we will not attempt to solve the classical equations (3.1)-(3.4) in contrast with other studies (see e.g. Henneaux [15]).

Before going on to the next section, we would like to point out that our treatment does not give rise to two separate constraints $\bar{\psi}\gamma_i\gamma_5\psi = 0 = \bar{\psi}\gamma_i\dot{\psi} - \dot{\bar{\psi}}\gamma_i\psi$ as in Ref.13, but only to the combined form (3.8). Furthermore, we disagree with Isham and Nelson's conclusion [13] that $\bar{\psi}\gamma_i\gamma_5\psi = 0$ implies $\psi \equiv 0$. Indeed, it is easy to check that for commuting spinors this only means that $\psi$ is of the form

$$\psi = e^{i\Omega}\begin{bmatrix} a \\ -b e^{i(\alpha-\beta)} \\ b e^{i\beta} \\ a e^{i\alpha} \end{bmatrix}$$

where the five parameters $a$, $b$, $\alpha$ $\beta$ and $\Omega$ are real, corresponding to the eight variables of $\psi$ restricted by three relations. For anticommuting spinors there is more freedom due to their nilpotent character and one can choose for instance

$$\psi_\alpha = e^{is_\alpha}\lambda_\alpha \quad , \quad \alpha = 1\ldots 4 ,$$

where $s_\alpha$ are ordinary real numbers and $\lambda_\alpha$ are real Grassmann numbers. If we set, for example, $\lambda_1 = \lambda_2$ and $\lambda_3 = \lambda_4$, the constraints are identically satisfied.

## IV. THE CONSTRAINTS

Among the dynamical variables $R$, $N$, $\psi$ and $\bar{\psi}$ (or $\psi^+$), we immediately note that there exist some constraints that prevent us from expressing "velocities" in terms of momenta and co-ordinates [3]. Indeed, from the definitions of the canonical momenta we find the primary constraints:

$$\frac{\partial L}{\partial \dot{N}} \equiv \pi_N \approx 0 \tag{4.1}$$

$$\phi_{\bar{\psi}} \equiv \frac{\overrightarrow{\partial}}{\partial\dot{\bar{\psi}}} L + \frac{i}{2} R^3\gamma^0\psi \equiv \pi_{\bar{\psi}} + \frac{i}{2} R^3\gamma^0\psi \approx 0 \tag{4.2}$$



$$\phi_\psi \equiv L\frac{\overleftarrow{\partial}}{\partial\dot{\psi}} - \frac{i}{2}R^3\bar{\psi}\gamma^0 \equiv \pi_\psi - \frac{i}{2}R^3\bar{\psi}\gamma^0 \approx 0 \tag{4.3}$$

Here we have assumed the spinor field $\psi$ to be an anticommuting object, so that we must distinguish between left and right derivatives [18]. The canonical Hamiltonian is

$$H_c = N\left\{ \frac{\pi_R^2}{4R} + kR + imR^3\bar{\psi}\psi - \frac{is}{4}R^2\bar{\psi}\gamma^0\gamma_5\psi \right\}, \tag{4.4}$$

where

$$\pi_R = \frac{2R\dot{R}}{N} . \tag{4.5}$$

The time evolution of any function of the canonical variables will be given by its Poisson bracket with $\tilde{H}$, where

$$\tilde{H} = H_c + u_N\pi_N + u_{\bar{\psi}}\phi_{\bar{\psi}} + \phi_\psi u_\psi , \tag{4.6}$$

in which $u_N$, $u_\psi$, $u_{\bar{\psi}}$ are arbitrary Lagrange multipliers. The preservation in time of the constraints (4.1)-(4.3) leads to the consistency requirement

$$\dot{\pi}_N = [\pi_N, \tilde{H}] = \frac{1}{N}H_c \equiv \chi_0 \approx 0 . \tag{4.7}$$

The equations for $\dot{\phi}_\psi$ and $\dot{\phi}_{\bar{\psi}}$ do not give new constraint relations; they are just equations for the functions $u_\psi$, $u_{\bar{\psi}}$. After this point, no new constraints are found by the requirement that they be preserved in time. From (4.7) we observe that the Hamiltonian is a linear combination of constraints, as is the case for any theory which is invariant under reparametrizations in time. We also note that $N$ is indeed a Lagrange multiplier which is not restricted by the equations of motion; we need not worry about its dynamics and the term $u_N\pi_N$ can be dropped from the Hamiltonian.

In order to distinguish between constraints which reflect invariances of the system (first class) and those that correspond to redundancies of variables (second class), we calculate the P.B. of the constraints:

$$[\chi_0, \phi_\psi] = \bar{\Psi}\left[\tfrac{3i}{4} R\pi_R + \tfrac{is}{4} R^2\gamma^0\gamma_5 - imR^3\right] \tag{4.8}$$

$$[\chi_0, \phi_{\bar{\Psi}}] = -\left[\tfrac{3i}{4} R\pi_R + \tfrac{is}{4} R^2\gamma^0\gamma_5 - imR^3\right]\psi \tag{4.9}$$

$$\{\phi_{\bar{\Psi}}, \phi_\psi\} = -iR^3\gamma^0 \tag{4.10}$$

(our notation is $[R,\pi_R] = 1$, $\{\bar{\Psi}_\alpha, \pi_{\bar{\Psi}_\beta}\} = \{\pi_{\bar{\Psi}_\beta}, \bar{\Psi}_\alpha\} = \{\psi_\alpha, \pi_{\psi_\beta}\} = \{\pi_{\psi_\beta}, \psi_\alpha\} = \delta_{\alpha\beta}$).

From (4.8) and (4.9) it would seem as if all our constraints were second class, however the linear combination

$$\chi_0' = \chi_0 - i\bar{\Psi}\tilde{M}\gamma^0\phi_{\bar{\Psi}} + i\phi_\psi\gamma^0 M\psi, \tag{4.11}$$

where

$$M = \frac{3i}{4R^2}\pi_R\gamma^0 - \frac{is}{4R}\gamma^0\gamma_5 + im \,, \quad \tilde{M} = \gamma^0 M^\dagger\gamma^0, \tag{4.12}$$

commutes with both $\phi_\psi$ and $\phi_{\bar{\psi}}$, and is therefore first class. The two constraints $\phi_\psi \approx 0 \approx \phi_{\bar{\psi}}$ instead, are second class and linearly independent. The second class constraints come from the fact that the Dirac Lagrangian is first order in the velocities and therefore the set $\psi$, $\bar{\psi}$, $\pi_\psi$, $\pi_{\bar{\psi}}$ is artificially large; only half of these variables are really independent. This redundancy has to be eliminated from the theory before quantization. This can be done consistently if one uses, instead of the Poisson brackets, the Dirac brackets

$$\begin{aligned} &[A,B]^* = [A,B] - [A,\varphi_i]\, C^{-1ij}\, [\varphi_j, B], \\ &\{\varphi_i\} = \text{all second class constraints,} \end{aligned} \tag{4.13}$$

where $C^{-1ij}$ is the inverse of $C_{ij} = [\varphi_i, \varphi_j]$. In our case we find

$$C_{ij} = \{\phi_\psi, \phi_{\bar{\Psi}}\} = -iR^3\begin{bmatrix} 0 & \gamma^0 \\ \gamma^{0T} & 0 \end{bmatrix} \tag{4.14}$$



and

$$C^{-1ij} = \frac{i}{R^3}\begin{bmatrix} 0 & \gamma^{0T} \\ \gamma^0 & 0 \end{bmatrix}$$

Then, the elimination of the redundant variables can be carried out by calculating all Dirac brackets among the dynamical variables and then setting $\phi_\psi = 0 = \phi_{\bar{\psi}}$. Thus we find

$$[R, \pi_R]^* = 1 \tag{4.15}$$

$$\{\psi_\alpha, \psi^\dagger_\beta\}^* = -\frac{i}{R^3}\delta_{\alpha\beta} \tag{4.16}$$

$$[R,\psi]^* = [R,\psi^\dagger]^* = 0 \tag{4.17}$$

$$[\pi_R, \psi]^* = \frac{3}{2R}\psi \tag{4.18}$$

$$[\pi_R, \psi^\dagger]^* = \frac{-3}{2R}\psi^\dagger . \tag{4.19}$$

The Hamiltonian is now, after setting $\phi_\psi = \phi_{\bar{\psi}} = 0$

$$H = N\left[\frac{i}{4R}\pi_R^2 + kR + i\psi^\dagger(mR^3\gamma^0 + \tfrac{s}{4}R^2\gamma_5)\psi\right] \approx 0 \tag{4.20}$$

and it is obviously first class. In view of (4.17), we can rescale $\psi$ and $\psi^+$ so that their anticommutator is normalized. Let $\chi \equiv R^{3/2}\psi$ and $\chi^+ = R^{3/2}\psi^+$, then we find

$$\{\chi_\alpha, \chi^\dagger_\beta\}^* = -i\delta_{\alpha\beta} \tag{4.21}$$

$$[R,\chi]^* = [R,\chi^\dagger]^* = 0 \tag{4.22}$$

$$[\pi_R, \chi]^* = 0 \tag{4.23}$$

$$[\pi_R, \chi^\dagger]^* = -\frac{3}{R}\chi^\dagger . \tag{4.24}$$

Now we are ready to make the transition to quantum mechanics: the truly independent dynamical variables are $R$, $\pi_R$, $\chi$ and $\chi^+$, their Dirac brackets are explicitly given and the only first class constraint, $H \approx 0$, is expressed in terms of those as

$$H = H_G + H_F \tag{4.25}$$

where

$$H_G = \frac{\pi_R^2}{4R} + kR \tag{4.26}$$

and

$$H_F = i\chi^+\left(m\gamma^0 + \frac{s}{4R}\gamma_5\right)\chi\ , \tag{4.27}$$

represent the purely gravitational and fermionic parts, respectively.

## V. QUANTIZATION

The transition from classical to quantum mechanics is carried out with the prescription

$$[A,B]^*_\mp \longrightarrow -i(\hat{A}\hat{B} \mp \hat{B}\hat{A}) \equiv -i[\hat{A},\hat{B}]_\mp\ . \tag{5.1}$$

Thus, the Dirac brackets (4.15), (4.21)-(4.24) become

$$[\hat{R},\hat{\pi}_R] = i \tag{5.2}$$

$$\{\hat{\chi}_\alpha, \hat{\chi}_\beta\} = \delta_{\alpha\beta} \tag{5.3}$$

$$[\hat{R},\hat{\chi}_\alpha] = [\hat{R},\hat{\chi}^+_\alpha] = 0 \tag{5.4}$$

$$[\hat{\pi}_R, \hat{\chi}_\alpha] = 0 \tag{5.5}$$

$$[\hat{\pi}_R, \hat{\chi}^+_\alpha] = -3\hat{R}^{-1}\hat{\chi}^+_\alpha\ . \tag{5.6}$$



One can choose the Schrödinger representation for $\hat{R}$ and $\hat{\pi}_R$:

$$\hat{R} = R\ , \quad \hat{\pi}_R = -i\frac{\partial}{\partial R}\ . \tag{5.7}$$

On the other hand, it is clear from (5.3) that $\hat{\chi}^+_\alpha$, $\hat{\chi}_\alpha$ are, respectively, creation and annihilation operators of a fermion in the state $\alpha$. The states on which $\hat{\chi}$ and $\hat{\chi}^+$ act form a 16-dimensional space whose basis, in the occupation number representation, is

$$\begin{aligned}
&|\Omega\rangle \equiv \text{the vacuum}\ , \quad \hat{\chi}_\alpha|\Omega\rangle = 0 && (\text{unique})\\
&|\alpha\rangle \equiv \hat{\chi}^+_\alpha|\Omega\rangle && (4 \text{ of them})\\
&|\alpha,\beta\rangle \equiv \hat{\chi}^+_\alpha\hat{\chi}^+_\beta|\Omega\rangle\ , \quad \alpha<\beta && (6 \text{ of them})\\
&|\alpha,\beta,\gamma\rangle \equiv \hat{\chi}^+_\alpha\hat{\chi}^+_\beta\hat{\chi}^+_\gamma|\Omega\rangle\ , \quad \alpha<\beta<\gamma && (4 \text{ of them})\\
&|\upsilon\rangle \equiv \hat{\chi}^+_1\hat{\chi}^+_2\hat{\chi}^+_3\hat{\chi}^+_4|\Omega\rangle\ , \quad \hat{\chi}^+_\alpha|\upsilon\rangle = 0\ . && (\text{unique})
\end{aligned} \tag{5.8}$$

The final step in setting up the quantum theory is to substitute the first class constraint $H \approx 0$ by the condition

$$\hat{H}(\hat{\pi}_R, \hat{R}, \hat{\chi}, \hat{\chi}^+)\Psi = 0 \tag{5.9}$$

on the quantum states.

a) <u>The fermionic part</u>

We observe that since $\hat{R}$ commutes with both $\hat{\chi}$ and $\hat{\chi}^+$, one can choose the wave function $\Psi$ to be a simultaneous eigenstate of $\hat{R}$ and $\hat{H}_F$. In a straightforward way one finds the eigenvalues of $\hat{H}_F$ to be [19]

| eigenvalue | degeneracy | |
|---|---|---|
| $2\lambda$ | 1 | |
| $\lambda$ | 4 | |
| 0 | 6 | |
| $-\lambda$ | 4 | |
| $-2\lambda$ | 1 | (5.10) |

where

$$\lambda = \sqrt{m^2 + \frac{S^2}{16R^2}} .$$

b) The bosonic part

It is immediately obvious that in order to give meaning to (5.9), one must first resolve the ordering ambiguity for the kinetic term $\pi_R^2/R$ of $H_G$. Superficially it might seem like one ordering would be as good as any other, however at least in this simple case it can be shown that this is not so. We first observe that the problem originates because the kinetic term in the Lagrangian is of the form $R\dot{R}^2$ which becomes $\pi_R^2/R$ in the Hamiltonian. If one makes the canonical (point) transformation $R = X^{2/3}$, the kinetic term in L is $4\dot{X}^2/9$ and in H it becomes $\frac{9}{16}p_x^{\ 2}$. (One can verify that the Poisson brackets are unaffected by this transformation.) Then, in the Schrödinger representation we can take

$$\hat{p}_x = -i\frac{\partial}{\partial x}$$
$$= -\frac{2}{3}i R^{-1/2}\frac{\partial}{\partial R} = \frac{2}{3}\hat{R}^{-1/2}\hat{\pi}_R . \qquad (5.11)$$

One thus arrives at the natural prescription

$$\frac{\pi_R^2}{4R} = \frac{9}{16}p_x^2 \longrightarrow \frac{9}{16}\hat{p}_x^2 = \frac{1}{4}\hat{R}^{-1/2}\hat{\pi}_R\hat{R}^{-1/2}\hat{\pi}_R \qquad (5.12)$$

(classically) (quantum-mechanically)

It might be objected that the result (5.12) is not manifestly hermitian: assuming $\hat{\pi}_R^{\ +} = \hat{\pi}_R$ and $\hat{R}^+ = \hat{R}$, the right-hand side of (5.12) is not equal to its adjoint. On the other hand, the kinetic term in the x-respresentation is self adjoint. What this indicates is that in making the transformation $x \to R$ one has to define the inner product in the Hilbert space consistently so that the result of (5.12) be hermitian. In fact, $p_x^{\ 2}$ is hermitian under the inner product

$$\langle\Psi_1,\Psi_2\rangle = \int_0^\infty \Psi_1^*(x)\,\Psi_2(x)\,dx . \qquad (5.13)$$



We want the inner product to be independent of the co-ordinate choice so that our predictions make sense, i.e. the inner product must be an invariant

$$\langle\Psi_1,\Psi_2\rangle_x = \langle\Psi_1,\Psi_2\rangle_R \qquad (5.14)$$

which in turns means

$$\langle\Psi_1,\Psi_2\rangle_R = \frac{3}{2}\int_0^\infty \Psi_1^*(R)\,\Psi_2(R)\,R^{1/2}\,dR \qquad (5.15)$$

One can now check that the operator $R^{-1/2}\frac{\partial}{\partial R}(R^{-1/2}\frac{\partial}{\partial R})$ is indeed hermitian under this inner product.

It must be stressed that the preceding is not a purely academic argument since the scalar density which is to be interpreted as the probability density of finding the universe with a radius in the neighbourhood of $R$ is

$$P(R) = R^{1/2}|\Psi(R)|^2 \qquad (5.16)$$

and not just $|\Psi(R)|^2$ as it might seem naively. This will be of crucial importance in the discussion of singularity at $R = 0$.

Collecting the above results, one arrives at the "Schrödinger equation" for the quantum system:

$$\left[-\frac{1}{R}\frac{d^2}{dR^2} + \frac{1}{2R^2}\frac{d}{dR} + 4kR + 4n\lambda(R)\right]\Psi_n(R) = 0 \qquad (5.17)$$

where $\lambda(R) = \sqrt{m^2 + \frac{S^2}{16R^2}}$ and $n = \pm 2, \pm 1, 0$.

## VI. THE SOLUTIONS

In looking for solutions of (5.17) we will distinguish the three cases $k = 0, -1, +1$. In some cases only approximate forms for $R \to \infty$ and $R \to 0$ will be given, since this will suffice to discuss the problem of singularities.

a) k = 0

In this case $s = 0$ and $\lambda = m$, then equation (5.17) becomes

$$\left[-\frac{1}{R}\frac{d^2}{dR^2}+\frac{1}{2R}\frac{d}{dR}+4mn\right]\Psi_n(R)=0 \quad . \qquad (6.1)$$

This is a modified Bessel equation and the two linearly independent solutions are [20]

$$\Psi_n^{(1,2)}(R)=(\text{const})\times R^{3/4}H^{(1,2)}_{1/2}\left(\tfrac{4}{3}\sqrt{-mn}\,R^{3/2}\right) , \qquad (6.2)$$

where $H^{(1,2)}$ are Bessel functions of the third kind. The asymptotic behaviour of the two solutions is

$$\Psi_n^{(1,2)}(R\to\infty)\sim(\text{const})\times e^{\pm i\frac{4}{3}\sqrt{-mn}\,R^{3/2}} . \qquad (6.3)$$

So, for $n \leqslant 0$, $\psi^{(1,2)}$ are not normalizable and for $n > 0$, one of the two solutions is convergent and normalizable while the other blows up at large $R$. In the region $R\to 0$

$$\Psi_n^{(1,2)}(R\to 0)\sim(\text{const})\times R^{3/4(1\pm 1)} . \qquad (6.4)$$

b) k = -1

Now (5.17) becomes

$$\left[\frac{d^2}{dR^2}-\frac{1}{2R}\frac{d}{dR}+4R^2-4mnR\right]\Psi_n(R)=0 \qquad (6.5)$$

The equation is not exactly solvable now, but we can study its behaviour for $R\to\infty$ and $R\to 0$. In the $R\to\infty$ region we find that for any $n$ the equation is

$$R^2\Psi_\infty''-\frac{1}{2}R\Psi_\infty'+4R^4\Psi_\infty=0 \qquad (6.6)$$

which gives asymptotically unnormalizable waves



$$\Psi_n^{(1,2)}(R\to\infty)\sim(\text{Const})\times R^{-1/4}e^{\pm iR^2} . \qquad (6.7)$$

In the limit $R\to 0$ the equation becomes

$$\Psi_0''-\frac{1}{2R}\Psi_0'=0 \qquad (6.8)$$

so that the behaviour of $\Psi(R)$ is again (6.4):

$$\Psi(R\to 0)\sim R^{3/4(1\pm 1)} . \qquad (6.9)$$

c) k = +1

i) In this case the equation takes a simple form only for $m = 0$

$$\left[\frac{d^2}{dR^2}-\frac{1}{2R}\frac{d}{dR}-4R^2-\frac{3}{\sqrt{2}}n\right]\Psi_n(R)=0 \qquad (6.10)$$

Its solutions can be expressed in terms of Whittaker's functions [20]:

$$\Psi_n^{(1)}(R)=N_1 e^{-R} R^{15/8} M\left(\tfrac{1}{8}[7-\tfrac{3}{\sqrt{2}}n], 7/4, 2R\right), \qquad (6.11a)$$

$$\Psi_n^{(2)}(R)=N_2 e^{-R} R^{15/8} U\left(\tfrac{1}{8}[7-\tfrac{3}{\sqrt{2}}n], 7/4, 2R\right), \qquad (6.11b)$$

where $N_1$, $N_2$ are some constants. For $R\to\infty$ we find, up to constants,

$$\Psi_n^{(1,2)}(R\to\infty)\sim\left(e^R R^{\frac{3}{\sqrt{2}}n}\right)^{\pm 1} , \qquad (6.12)$$

while for $R\to 0$

$$\Psi_n^{(1,2)}(R\to 0)\sim(\text{const})\times R^{3/8(3\pm 2)}\longrightarrow 0 . \qquad (6.13)$$

ii) The case $m\neq 0$ cannot be solved in closed form. For $R\to\infty$ the equation reads

$$R^2 \Psi''_\infty - \frac{1}{2} R \Psi'_\infty - 4R^2 \Psi_\infty = 0 , \qquad (6.14)$$

whose solutions are again expressible in terms of Bessel functions. Thus we find

$$\Psi^{(1,2)}_n (R \to \infty) \sim R^{-1/4} e^{\pm R^2} . \qquad (6.15)$$

For $R \to 0$, the equation reduces again to (6.8), therefore

$$\Psi^{(1,2)}_n (R \to 0) \sim R^{3/4 (1 \pm 1)} . \qquad (6.16)$$

## VII. DISCUSSION OF THE SOLUTIONS

a) We observe that in all cases $R^{1/2}|\Psi(R)|^2 \to 0$ as $R \to 0$ which can be interpreted to indicate that the probability of finding the universe in a neighbourhood of the collapsed state $R = 0$ is vanishingly small. This probabilistic interpretation however breaks down when the wave function $\Psi(R)$ is not normalizable as it happens for $k = 0$, $n \lesssim 0$ and for $k = -1$, any $n$; so one can genuinely state that the collapse is avoided in the remaining cases: $k = 0$, $n > 0$; $k = +1$. One can relax this criterion to include solutions which are not square integrable but bounded as $R \to \infty$, as it is done in scattering problems in quantum mechanics. In this case one can also accept the solutions for $k = -1$ as genuine non-collapsing cases.

b) Note that the result $R^{1/2}|\Psi|^2 \to 0$ as $R \to 0$ is due entirely to the fact that in all cases the two linearly independent solutions of the Schrödinger operator for $R \to 0$ are of the form $R^p$ with $p \gtrsim 0$. This is a property of $\hat{H}_G$ alone, so one might be tempted to say that even in the absence of matter ($\hat{H}_F = 0$), there is no collapse. This is true in the cases $k = \pm 1$ since our results in those cases do not depend on $n$. For $k = 0$ however, this would not be true since if we set $n = 0$ the wave function is not normalizable ($R^{1/2}|\psi|^2$ blows up at infinity) while the problem is well defined when $n > 0$.



c) In the only case in which the solutions are sensitive to the presence of matter (i.e. $k = 0$) we see that if $n > 0$ the wave function is more divergent at infinity than in the matter free case, while for $n < 0$ the wave function damps off rapidly. If $R^{1/2}|\Psi(R)|^2$ is to be interpreted as a probability density, this might be related to the fact that the universe would tend to an infinitely expanded state or it would remain bounded, depending on the state of matter.

d) The system we have discussed above can be considered as a very simplified version of a cosmological model, presumably applicable at the very early stages of our universe ($t \lesssim 10^{-44}$ sec, after the big bang). In that case, the behaviour of the wave function as $R \to \infty$ ($t \to \infty$) would be irrelevant and one should put a cut-off at a sufficiently large $R$, since the model cannot be expected to describe that region accurately. On the other hand, if the model is a good reflection of what happens for small R, the statement $P(R) \xrightarrow[R \to 0]{} 0$ seems more compelling and one should perhaps believe that quantum gravity effects can prevent our universe from collapsing.

## VIII. SUMMARY

We have discussed the quantization of the Robertson-Walker geometry interacting with a homogeneous distribution of fermionic matter. The method of Dirac [3] was used as the only guide to formulate the problem. The quantum problem was set up following the standard prescriptions. The problem of ordering is uniquely and unambiguously resolved using the freedom to make canonical transformations in the classical theory before quantizing. In this simple case it is not necessary to give up hermiticity, as in more complicated situations[7,8]. Finally, we find that the solutions to the quantum problem always predict a vanishing of the probability density at $R = 0$, which can be interpreted in most of the cases as a genuine avoidance of the collapse, without resorting to ad hoc potential barriers [9] at $R = 0$, or other artifacts.


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## APPENDIX

### Conventions

We adopt the flat (tangent space) metric

$$\eta_{ab} = \mathrm{diag}\,(-,+,+,+) \quad . \tag{A.1}$$

We define the Dirac matrices to satisfy

$$\{\gamma^a, \gamma^b\} = 2\,\eta^{ab} \rightarrow (\gamma^0)^2 = -(\gamma^i)^2 = -1 \quad . \tag{A.2}$$

We also choose them to be either hermitean or anti-hermitean

$$\gamma^{0\dagger} = -\gamma^0 \quad , \quad \gamma^{i\dagger} = \gamma^i \quad , \tag{A.3}$$

so that $\gamma^{a\dagger} = \gamma^0 \gamma^a \gamma^0$. Finally, the $\gamma_5$ matrix is taken as

$$\gamma_5 = \gamma_0 \gamma_1 \gamma_2 \gamma_3 = -\gamma^0 \gamma^1 \gamma^2 \gamma^3 \tag{A.4}$$

so that

$$(\gamma_5)^2 = -1, \quad \gamma_5^\dagger = -\gamma_5 \tag{A.5}$$

and

$$\{\gamma^a, \gamma_5\} = 0 \quad .$$

An explicit representation of the $\gamma$-matrices with these properties is

$$\gamma^0 = i\begin{bmatrix} I & \\ & -I \end{bmatrix}, \quad \gamma^k = i\begin{bmatrix} & \sigma_k \\ -\sigma_k & \end{bmatrix}, \quad \gamma_5 = -i\begin{bmatrix} & I \\ I & \end{bmatrix} , \tag{A.6}$$

where $\sigma_k$ are the Pauli matrices. The matrices $\sigma^{ab}$ are defined as

$$\sigma^{ab} = -\frac{i}{2}[\gamma^a, \gamma^b] \; . \tag{A.7}$$

Then we find

$$\sigma^{ab\dagger} = -\gamma^0 \sigma^{ab} \gamma^0 \tag{A.8}$$

$$B_i^\dagger = -\gamma^0 B_i \gamma^0 \tag{A.9}$$

and

$$\{\gamma^a, \sigma^{bc}\} = -2i\,\epsilon^{abcd}\,\gamma_d \gamma_5 \tag{A.10}$$

With these definitions one can check that $i\bar{\psi}\psi$, $i\bar{\psi}\gamma^0\gamma_5\psi$, etc. are hemitean (real).



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