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CHARGED PION ELECTROPRODUCTION, A SELECTIVE PROBE OF NUCLEAR SPIN ISOSPIN RESPONSES

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Abstract : We study the reaction of pion electroproduction on nuclei in the quasi-elastic region. We show that detection of the pion in the direction of the virtual photon permits the separation of the spin longitudinal and transverse responses through a Rosenbluth plot. Emphasis is also put on consistency between medium effects and gauge invariance.

Spin-isospin modes of nuclear excitation are among the most vividly discussed subjects of present day nuclear physics. The recent discovery of both the M1 and Gamow-Teller giant resonances (see e.g. a review in ref. [1]) has focussed the attention on the existence of a strong repulsion in this channel at zero momentum transfer wich moreover through coupling of nucleon-hole to Δ isobar-hole excitations stands as an attractive candidate to the explanation of the controversial problem of the "missing strength" (Lorentz-Lorenz effect). The origin of this repulsion though not really elucidated is certainly connected to the short range behaviour of the nucleon-nucleon force. With increasing momenta one expects manifestations of the attractive components of the force arising from π and ρ meson exchanges which are privileged vectors of spin-isospin excitations. Pion condensation would be an extreme consequence, precursors of which have been unsuccessfully searched for in the recent years [2] . This failure can be presumably attributable to the weakness of the p-h interaction, the repulsive component balancing the attractive pion exchange at the momenta relevant to critical phenomena. It should be stressed however that one does not precisely know how the repulsion manifested at $|\underset{\sim}{t}| = 0$ evolves with increasing transfer and that the exact behaviour of the interaction deserves further exploration.

It has been emphasized [3] that a "remnant" of the precursor effect could survive thanks to the large difference of ranges between π and ρ meson exchanges, viz. the long range pion attraction manifests itself at much lower momenta than that produced by the rho. A strong contrast has thus been predicted between the two nuclear responses corresponding to the modes which have the projection of the spin along ("longitudinal" or pionic mode) or perpendicular ("transverse" or rho-mesonic) to the transferred momentum. The knowledge of the gross features of the two responses in the continuum (quasi-elastic region) and the evolution of their more or less collective chararcter with the energy and momentum transfer would be of precious help to pin down the spin-isospin interaction.

Information is already available on the transverse ($m_s=\pm 1$) mode from electron scattering where a Rosenbluth plot permits to disentangle the magnetic response which is mainly excited by the spin isovector operator. The longitudinal ($m_s=0$) or pionic mode has been much more elusive and has up to now never been separated, though some specific transitions where it dominates have been explored to test the proximity of critical behaviour [2]. It is the purpose of this letter to show that under special kinematical conditions, charged pion electroproduction would permit a separation of the two responses by the same experiment in the very same way as electron scattering separates the transverse from the charge responses.

The elementary electroproduction reaction on the nucleon belongs to the class of photopion reactions which are well described from threshold to the 3-3 resonance region by a pseudovector Lagrangian supplemented by a Δ excitation term [4,5]. These reactions on nuclei have long been signaled as probes of spin-isospin modes [6] and more recently emphasis has been put on their sensitivity to the pionic mode [7,8] . Indeed one can write the following simplified transition operator for photoproduction of $\pi^{\pm}(\underset{\sim}{q},q_0)$ by a photon of energy-momentum $(\underset{\sim}{k},k_0)$ and polarization vector $\underset{\sim}{\varepsilon}$, in a non-relativistic approximation ignoring terms proportionnal to the nucleon velocity :

$$T_{\gamma\pi^{\pm}} = \mp i e_r f_r \sqrt{2}\,\tau^{\mp}\Big\{ \underset{\sim}{\sigma}\cdot\underset{\sim}{\varepsilon}\Big[F_A(k^2) \mp \frac{k_0}{2M}F_1^S(k^2)\Big] - \frac{\underset{\sim}{\sigma}\cdot\underset{\sim}{t}\,(2\underset{\sim}{t}-\underset{\sim}{k})\cdot\underset{\sim}{\varepsilon}}{t^2+m_\pi^2}\,F_\pi(k^2)$$
$$\pm\frac{\underset{\sim}{\sigma}\cdot\underset{\sim}{q}\;\underset{\sim}{k}\cdot\underset{\sim}{\varepsilon}}{2Mk_0}F_1^S(k^2) + \frac{\underset{\sim}{\sigma}\cdot[\underset{\sim}{q}\times(\underset{\sim}{k}\times\underset{\sim}{\varepsilon})]}{2Mk_0}\Big[\frac{2k_0G_M^{\Delta}(k^2)}{9(\omega_R-k_0-i\frac{\Gamma}{2})} \mp G_M^S(k^2)\Big] \quad (1)$$
$$+ i\,\frac{\underset{\sim}{q}\cdot(\underset{\sim}{k}\times\underset{\sim}{\varepsilon})}{2Mk_0}\Big[\frac{4k_0G_M^{\Delta}(k^2)}{9(\omega_R-k_0-i\frac{\Gamma}{2})} + G_M^V(k^2)\Big]\Big\}$$

In the formula, $e_r f_r = 4\pi(0.08/137)^{1/2}$, M and m_π are the nucleon and pion masses, ω_R($\approx 2,15 m_\pi$) and Γ the position and width of the Δ resonance, $t = k-q = (\underset{\sim}{t},\omega)$, the energy-momentum transfer (the static limit strictly corresponds to $\omega = 0$ but we have to release this condition and to use a slightly different amplitude than [1] in our study of the quasi-elastic region). The quantities $F_A, F_1^{V(S)}$, $G_M^{V(S)}$, are the axial,Dirac and magnetic isovector (isoscalar) form factors of the nucleon whereas F_π and G_M^{Δ} refer to the electromagnetic form factors of the pion and the Δ . They are normalized so that $F_A = F_1^{V(S)} = F_\pi = 1$ at $k^2 = 0$ and $G_M^S(0) = 0.88$, $G_M^V(0) = G_M^{\Delta}(0) = 4.71$ (note that we have assumed that the $\gamma N\Delta$ coupling follows the scaling law of Chew et al. [4]). This amplitude [1] is applicable to the case of virtual photons (electroproduction) as testified by the presence of longitudinal contributions in $\underset{\sim}{k}\cdot\underset{\sim}{\varepsilon}$ (scalar terms have been eliminated).

The cross-section for electroproduction is expressible in terms of that for photoproduction, the situation of real photon being easily deducible. The formulae for the inverse reactions (radiative capture and pair production) are obtained readily through detailed balance and appropriate modifications of the kinematics. One can write the standard decomposition for the reaction produced by an electron of 4-momentum $(\varepsilon_1, \underset{\sim}{k}_1)$ scattered in the solid angle Ω_2 with 4-momentum $(\varepsilon_2, \underset{\sim}{k}_2)$:

$$\frac{d\sigma}{d\Omega_\pi dq_0 d\Omega_2 d\epsilon_2} = \Gamma \left[\frac{d\sigma_T(\gamma,\pi)}{d\Omega_\pi dq_0} + \epsilon \frac{d\sigma_L(\gamma,\pi)}{d\Omega_\pi dq_0} + \sqrt{\frac{\epsilon(1+\epsilon)}{2}} \frac{d\sigma_I(\gamma,\pi)}{d\Omega_\pi dq_0} \right] \qquad (2)$$

with the flux factor $\Gamma = \frac{e_r^2}{(2\pi)^3} \frac{k_2}{k_1} \frac{(k_0 - k^2/2M)}{k^2(1-\epsilon)}$, $\epsilon = \left[1 + 2\frac{\underset{\sim}{k}^2}{k^2} tg^2\left(\frac{\hat{k}_1 \cdot \hat{k}_2}{2}\right)\right]^{-1}$ being the value of the photon polarization. The three terms of eq.(2) correspond to photoproduction by transverse and longitudinal photons and an interference term. In the case of the reaction on a nucleus they are expressible in terms of the response functions :

$$\frac{d\sigma_T(\gamma,\pi^\pm)}{d\Omega_\pi dq_0} = K \Big\{ \frac{1}{2} \frac{q^2}{\underset{\sim}{t}^2} \sin^2\theta \, |A_1 - B_1|^2 R_{SL} + \Big[|B_2 - \frac{\underset{\sim}{q} \cdot \underset{\sim}{k}}{\underset{\sim}{k}^2} B_1|^2 +$$
$$+ \frac{q^2 \sin^2\theta}{2\underset{\sim}{t}^2} \left(\frac{\underset{\sim}{t}^2}{\underset{\sim}{k}^2} |B_1|^2 - |B_1 - B_2|^2 \right) \Big] R_{ST} + \frac{1}{2} \underset{\sim}{q}^2 \underset{\sim}{k}^2 \sin^2\theta \, |C|^2 R_C \qquad (3a)$$
$$+ \frac{\epsilon}{2} \frac{q^2}{\underset{\sim}{t}^2} \sin^2\theta \cos 2\phi \Big[|A_1 - B_1|^2 R_{SL} + \left(\frac{\underset{\sim}{t}^2}{\underset{\sim}{k}^2} |B_1|^2 - |B_1 - B_2|^2 \right) R_{ST} - \underset{\sim}{k}^2 \underset{\sim}{t}^2 |C|^2 R_C \Big] \Big\}$$

$$\frac{d\sigma_L(\gamma,\pi^\pm)}{d\Omega_\pi dq_0} = K \frac{k^2}{k_0^2} \Big\{ |(A_1 + A_3) \hat{\underset{\sim}{k}} \cdot \hat{\underset{\sim}{t}} + A_2|^2 R_{SL} + \frac{\underset{\sim}{q}^2 \sin^2\theta}{\underset{\sim}{t}^2} |B_2 + A_3|^2 R_{ST} \Big\} \qquad (3b)$$

$$\frac{d\sigma_I(\gamma,\pi^\pm)}{d\Omega_\pi dq_0} = K \sqrt{\frac{k^2}{k_0^2}} \frac{|\underset{\sim}{q}|}{|\underset{\sim}{t}|} \sin\theta \cos\phi \Big\{ 2 Re(A_1 - B_1) \left[(A_1^* + A_3^*) \hat{\underset{\sim}{k}} \cdot \hat{\underset{\sim}{t}} + A_2^* \right] R_{SL}$$
$$- 2 Re(B_2 + A_3) \left[\frac{|\underset{\sim}{t}|}{|\underset{\sim}{k}|} B_1^* + (B_2^* - B_1^*) \hat{\underset{\sim}{k}} \cdot \hat{\underset{\sim}{t}} \right] R_{ST} \Big\} \qquad (3c)$$

where $\hat{\underline{k}}$ and $(\hat{\underline{k}}_1 x \hat{\underline{k}}_2) x \hat{\underline{k}}$ have been chosen as the z and x axes and one has introduced the common factor K and the coefficients :

$$A_1^{(\pm)} = F_A - \frac{2\underline{t}^2}{t^2+m_\pi^2} F_\pi \mp \frac{k_0}{2M} F_1^S \qquad A_2^{(\pm)} = |\underline{t}||\underline{k}| \left(\frac{F_\pi}{t^2+m_\pi^2} \mp \frac{F_1^S}{2Mk_0} \right)$$

$$A_3^{(\pm)} = \pm \frac{\underline{k}^2}{2Mk_0} F_1^S \qquad B_1^{(\pm)} = \frac{\underline{k}^2}{2Mk_0} \left[\frac{2k_0 G_M^\Delta}{9(\omega_k - k_0 - i\Gamma/2)} \mp G_M^S \right] \tag{4}$$

$$B_2^{(\pm)} = F_A \mp \frac{k_0}{2M} F_1^S \qquad C = \frac{1}{2Mk_0} \left[\frac{4k_0 G_M^\Delta}{9(\omega_k - k_0 - i\Gamma/2)} + G_M^V \right]$$

The nuclear responses R_{SL}, R_{ST} and R_C corresponding to the spin longitudinal, spin transverse and charge isospin modes are defined according to :

$$R(\underline{t},\omega) = \sum_n \left| \langle n | \sum_{j=1}^{A} O_j \tau_j^{\mp} e^{i\underline{t}\cdot\underline{r}_j} | 0 \rangle \right|^2 \delta(E_n - \omega) \tag{5}$$

where the operator O_j is $\underline{\sigma}_j \cdot \hat{\underline{t}}$, $1/\sqrt{2}(\underline{\sigma}_j \times \hat{\underline{t}})$ and 1 for the indices SL,ST and C respectively.

The expression (3a) is readily applicable to photoproduction by real photons partially polarized with probabilities $\frac{1 \pm \varepsilon}{2}$ in the directions of the x and y axes. It appears relatively deceiving as concerns the possibility of disentangling the long searched longitudinal response R_{SL}. Working at $\theta = 0$ or π would merely display the transverse response which is already known from electron scattering. Polarized beams would not help much, though an experiment at $\theta = 0$ or π with $\epsilon = 1$ would eliminate the charge response R_C. In any case R_{ST} would always dominate over R_{SL}. The situation is much more promising for electroproduction. Indeed at forward angle ($\theta = 0$), it is clear that the longitudinal (photon) cross section (3b) determines R_{SL} (the backward kinematics are also useful a priori but the contribution of R_{SL} is small due to destructive interference between F_A and F_π). This proves our liminary assertion that forward kinematics permit the separation of R_{SL} and R_{ST} by exploiting the linearity in ϵ of the Rosenbluth plot (cf. eq.(2)) :

$$\left.\frac{d\sigma}{d\Omega_\pi dq_0 d\Omega_2 d\epsilon_2}\right|_{\theta=0} = \Gamma K\left(|B_2 - \frac{|q|}{|\underset{\sim}{k}|}B_1|^2 R_{ST} + \epsilon \frac{k^2}{k_0^2}|A_1+A_2+A_3|^2 R_{SL}\right) \tag{6}$$

It is interesting to notice that such a separation has already been performed experimentally in the case of the proton [9] where the aim was a determination of the pion electromagnetic form factor $F_\pi(k^2)$ contained in the A coefficients.

For a numerical illustration of the power of the method we perform the calculation in the simplest nuclear model, viz. that of the infinite Fermi gas with a Fermi momentum k_F and an effective mass $M^* = 0.8M$. The Fermi motion broadens the nucleon response and gives rise to the familiar form of the quasi-elastic peak. For non interacting particles the three responses are equal : $R_{SL} = R_{ST} = R_C$. In the presence of interaction the responses are reshaped differently. We will make use of the results of ref.3 where the calculations were made in the ring approximation to the RPA. The particle-hole interactions in the T and

L channels were described by a short range Landau-Migdal parameter g' and the exchange of π and ρ mesons :

$$V_L(t) = \frac{f_\pi^2}{m_\pi^2}\left(g' - \frac{t^2}{t^2+m_\pi^2}\right) v_\pi^2(t^2) \qquad (7)$$

$$V_T(t) = \frac{f_\pi^2}{m_\pi^2} g' v_\pi^2(t^2) - \frac{f_\rho^2}{m_\rho^2}\frac{t^2}{t^2+m_\rho^2} v_\rho^2(t^2)$$

where $f_{\pi(\rho)}$, $v_{\pi(\rho)}$, $m_{\pi(\rho)}$ are the coupling constant, form factor and mass of the π (ρ) meson. The response functions which were related to the polarizability $\Pi_0(t)$(including Δ -hole as well as nucleon-hole excitations) by $R = -\frac{3\pi Z}{k_F^3}\,\mathrm{Im}\,\Pi_0$ become :

$$\tilde{R}_L(t) = -\frac{3\pi Z}{k_F^3}\,\mathrm{Im}\,\frac{\Pi_0(t)}{1-V_L(t)\Pi_0(t)} \qquad (8)$$

and the analogous expression for $\tilde{R}_T(t)$(see fig.1).

The kinematical region is chosen for invariant hadronic masses of 1125 MeV so as to cover the energy range of the quasi-elastic peak and the interesting momentum domain $|\underset{\sim}{t}| = 0$ to $3m_\pi$. The outgoing pions have thus a typical energy of 100 MeV on top of the peak. The effects produced by the distortion have been estimated by a eikonal approximation. The optical potential is built from the pion self-energy consistently with the p-wave Lagrangian used to compute $\Pi_0(t)$ with however the addition of a two nucleon absorption term V_A(complete consistency would imply the introduction of 2p-2h components in the polarizability). The pion momentum becomes complex according to the solution of the dispersion equation :

$$\underset{\sim}{q}^2+\frac{f_r^2}{m_\pi^2}\Pi_0(\underset{\sim}{q},q_0)\mathrm{Re}\,\underset{\sim}{q}^2/\left(1-\frac{f_r^2}{m_\pi^2}g'\Pi_0(\underset{\sim}{q},q_0)\right)+2q_0V_A+m_\pi^2-q_0^2=0 \qquad (9)$$

The pion wave is then $e^{(i\mathrm{Re}|\underset{\sim}{q}|-\mathrm{Im}|\underset{\sim}{q}|)z}$ so that we have replaced $|\underset{\sim}{q}|$ by its real part in eq.(6) and have evaluated an average damping factor over the nuclear volume from the attenuated wave.

However one should not forget that consideration of such medium effects should be made consistently with the requirements of gauge invariance [7] . The latter dictates that polarization bubbles corresponding to distortion imply the presence of counter terms proportionnal to $\Pi_e(q)=\Pi_0(q)/\left(1-\frac{f_r^2}{m_\pi^2}g'\Pi_0(q)\right)$ in the amplitude [1] . For instance, fig.2 shows graphs associated with the photoproduction of both real and virtual pions of momenta q and t respectively according to the amplitude

$$\delta T_{\gamma\pi^\pm}=\mp i e_r f_r\sqrt{2}\,\tau^\mp\,\frac{f_r^2}{m_\pi^2}\Pi_e(q)\,\frac{\underset{\sim}{\sigma}\cdot\underset{\sim}{t}}{t^2+m_\pi^2}\Big[\big(\underset{\sim}{t}\cdot\underset{\sim}{\epsilon}-\hat{\underset{\sim}{k}}\cdot\underset{\sim}{\epsilon}\,\hat{\underset{\sim}{k}}\cdot\underset{\sim}{t}\big)\,B_1^{(\pm)}$$

$$+\,\underset{\sim}{q}\cdot\underset{\sim}{\epsilon}\;B_2^{(\pm)}+\frac{\underset{\sim}{k}\cdot\underset{\sim}{\epsilon}\;\underset{\sim}{q}\cdot\underset{\sim}{t}}{\underset{\sim}{k}^2}\,A_3^{(\pm)}\Big] \qquad (10)$$

In the same way one should add contributions from the photoproduction of both real pions and virtual rho mesons, the leading term of which reads $\mp i e_r f_r\sqrt{2}\,\tau^\mp\,\frac{f_\rho^2}{m_\rho^2}\Pi_e(q)\,(\underset{\sim}{\sigma}\times\underset{\sim}{t})\cdot(\underset{\sim}{q}\times\underset{\sim}{\epsilon})/(t^2+m_\rho^2)$. It is important to remind that at frequencies like q_0, the polarizability is dominated by its Δ-hole component. Both these gauge terms and the above mentioned renormalization of the response functions can be considered as π and ρ exchange current effects as is exemplified by the lowest order graphs of fig.3.

The results of the calculations for the case of a ^{40}Ca nucleus are summarized on fig.4 where the longitudinal and transverse cross sections integrated over the quasi-elastic peak are plotted as a function of k^2 for the free gas and the interacting situation at several values of g' (the latter case including gauge and distortion effects). It turns out that there is considerable cancellation between gauge terms and distortion effects, which points towards the importance of a treatment consistent with gauge invariance. The contrast emphasized in ref.3 is clearly displayed. The common quenching of the two cross-sections at low transfers (hence low k^2) leaves place at increasing momenta to an opposite behaviour arising from the manifestation of pion exchange in the longitudinal response. Despite the crudeness of the models we have used in this work, we expect that the contrast will survive in a more refined treatment.

Measurements of the electroproduction reaction in the specific kinematics that we have emphasized would thus be welcome. The separability of the two spin responses is a nearly unique property which is shared up to now only with the recent possibility of measuring transfers of polarization in (p,p') reactions [10] . An advantage over the latter would be the good understanding of the elementary process. However such a coïncidence experiment is probably more appropriate to high duty cycle electron accelerators than to existing machines.

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Figure captions

Fig.1 The renormalization of the longitudinal response where each nucleon line may be replaced by a Δ line. A similar figure can be drawn for R_{ST} with $\vec{\sigma}.\hat{t}$ and V_L replaced by $(\vec{\sigma}\, x\hat{t})$ and V_T.

Fig.2 An exemple of a gauge consistency graph. The hatched circle represents a photoproduction amplitude without internal pion line (the Δ-hole bubbles are by far more important than N-hole ones). A similar figure can be drawn with ρ meson coupled to the transverse response.

Fig.3 An exemple of exchange current contributions contained in the renormalization of the spin responses (3a) or in the gauge consistency terms (3b).

Fig.4 The longitudinal and transverse (curves labelled L and T) virtual photon cross-sections (per proton) for ^{40}Ca at a final hadron state energy of 1125 MeV without (dashed lines) and with particle-hole interaction (full lines for g'=0.5 and 0.7) as a function of the squared photon four-momentum k^2.

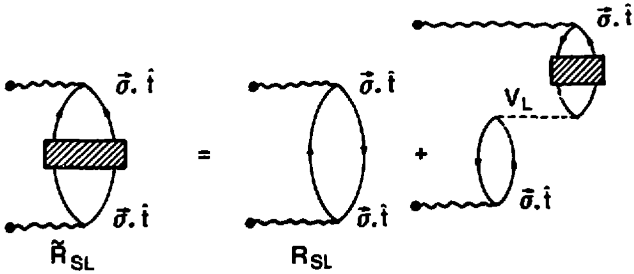

Fig. 1

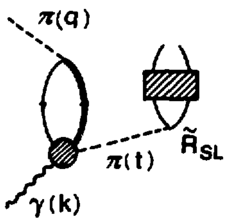

Fig. 2

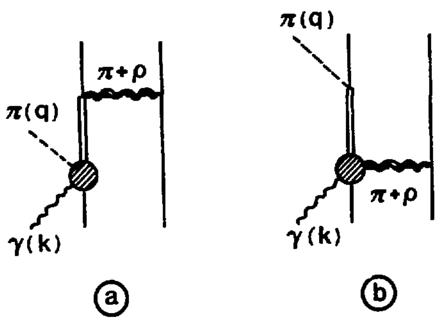

Fig. 3

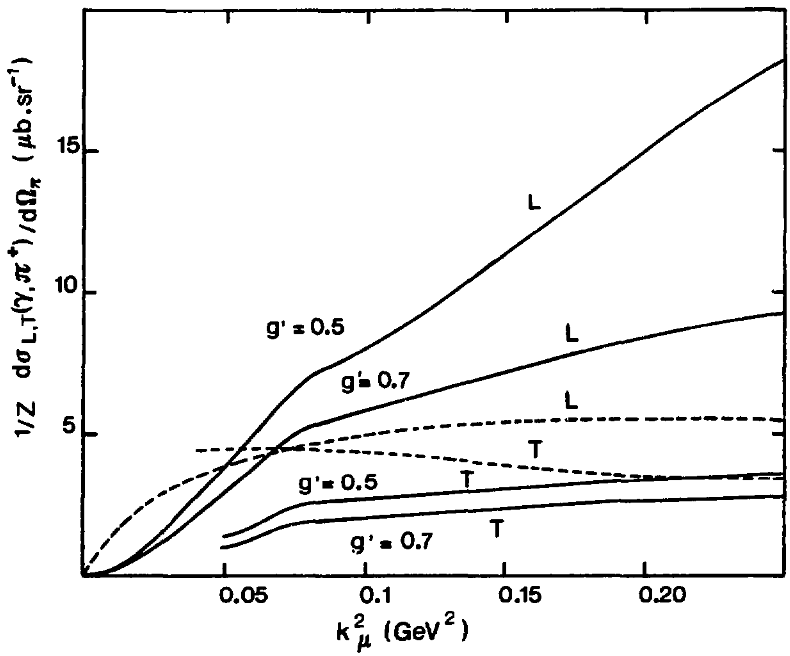
1/Z dσ L,T(γ,π+)/dΩπ (μb.sr-1)
15
10
5
0.05
0.1
0.15
0.20
k2μ (GeV2)
L
L
L
T
T
T
g' = 0.5
g' = 0.7
g' = 0.5
g' = 0.7

Fig. 4