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NONLINEAR DYNAMICS OF BOILING WATER REACTORS*

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Recent stability tests[1] in Boiling Water Reactors (BWRs) have indicated that these reactors can exhibit the special nonlinear behavior of following a closed trajectory called limit cycle. The existence of a limit cycle corresponds to an oscillation of fixed amplitude and period. During these tests,[1] such oscillations had their amplitudes limited to about ± 15% of the operating power. Since limit cycles are fairly insensitive to parameter variations, it is possible to operate a BWR under conditions that sustain a limit cycle (of fixed amplitude and period) over a finite range of reactor parameters.

This paper presents an investigation of the qualitative dynamic behavior of a BWR in this nonlinear regime. For this purpose, the following spatially averaged quantities are selected as dependent variables: the excess neutron population $n(t)$, the excess population of delayed neutron precursors $c(t)$, the excess fuel temperature $T(t)$, the relative excess coolant density $\gamma(t)$, and the excess reactivity $\rho(t)$. Here, "excess" signifies departure from the initial, steady-state values at $t=0$ when the reactor is just critical; thus, $n(t)$, $c(t)$, $T(t)$, $\gamma(t)$ and $\rho(t)$ are all zero at $t=0$. (The definitions of $n(t)$ and $c(t)$ involve normalization to the initial steady-state neutron population N_0, e.g., $n(t) \equiv [N(t)-N_0]/N_0$, and $\gamma(t)$ depends on the ratio between vapor and liquid water in the reactor.)

For $t>0$, the dependent variables satisfy the following equations:

$$dn(t)/dt = n(t)[\rho(t)-\beta]/\Lambda + \lambda c(t) + \rho(t)/\Lambda, \quad (1)$$

$$dc(t)/dt = n(t)\beta/\Lambda - \lambda c(t), \quad (2)$$

$$dT(t)/dt = Q[n(t) + H(t)\Delta] - a_3 T(t) \quad (3)$$

$$d^2\gamma(t)/dt^2 + a_2\, d\gamma(t)/dt + a_1\gamma(t) = k\, T(t), \quad (4)$$

$$\rho(t) = \Gamma_1\gamma(t) + \Gamma_2\, T(t), \quad (5)$$

where H(t) denotes the customary Heaviside functional. The nonlinear reactivity feedback in Eq. (1) arises from the Doppler coefficient of reactivity $\Gamma_2 = -2.61 \times 10^{-5} K^{-1}$ and the coolant-moderator density reactivity coefficient $\Gamma_1 = 0.15$. These coefficients, together with the constants $a_1 = 6.8166 s^{-2}$, $a_2 = 2.2494 s^{-1}$, $a_3 = 0.2325 s^{-1}$, and $Q = 25.044$ K/s, are obtained from a detailed study[2-4] of the mass, energy, and momentum conservation equations for the BWR's thermal-hydraulic loop. The constants $\lambda = 0.08 s^{-1}$, $\Lambda = 4 \times 10^{-5} s$, and $\beta = 0.0056$ are determined by the nuclear properties and geometry of the reactor.

When the excess neutron population is suddenly increased at $t=0^+$ by $\Delta = -0.1$, the BWR modeled by Eqs. (1)-(5) becomes a dynamical system whose behavior will now be investigated as a function of the parameter k. This parameter controls the gain of the feedback loop, and consequently controls the BWR's local stability. When k is increased (e.g., by control rod motion or turbine trip) above a critical value $k_o = 0.01318\ K^{-1}s^{-2}$, a limit cycle appears, just as has been experimentally observed in Ref. 1. This limit cycle, of period T_0, is shown in Fig. 1 (in the n(t) versus T(t) phase-space). At $k = k_1 = 1.47\ k_o$, this limit cycle becomes unstable, to be replaced by a new stable limit cycle of period $T_1 = 2T_0$. Increasing k causes the second limit cycle to lose its stability at $k = k_2 = 1.584\ k_o$, to be replaced by a new limit cycle of period $T_2 = 4T_0$. As k is further increased, this process repeats itself at successive critical values k_j, where a limit cycle of period $2^j T_0$ loses its stability but is replaced by a stable limit cycle of period $2^{j+1} T_0$. The sequence k_j converges[5] to

k_∞ = 1.618 beyond which the state variables exhibit aperiodic behavior after having undergone an infinite number of period doublings. This behavior is shown in Fig. 2. This figure also shows that the dynamic behavior of the BWR model remains bounded at all times.

In summary, this work has highlighted the qualitative nonlinear behavior of a lumped-parameter representation of a BWR when the reactor's neutron population is increased beyond that at steady-state criticality. The appearance of limit cycle oscillations such as those predicted in this work has already been observed experimentally.[1] This work also shows that a continued reactivity increase will destabilize these limit cycles, leading to aperiodic reactor behavior. Finally, this work shows that the reactor's dynamical evolution remains bounded at all times. An increased understanding of the reactor behavior in the limit cycle and aperiodic regions may open the possibility to practical operation of reactors beyond their current operating conditions.

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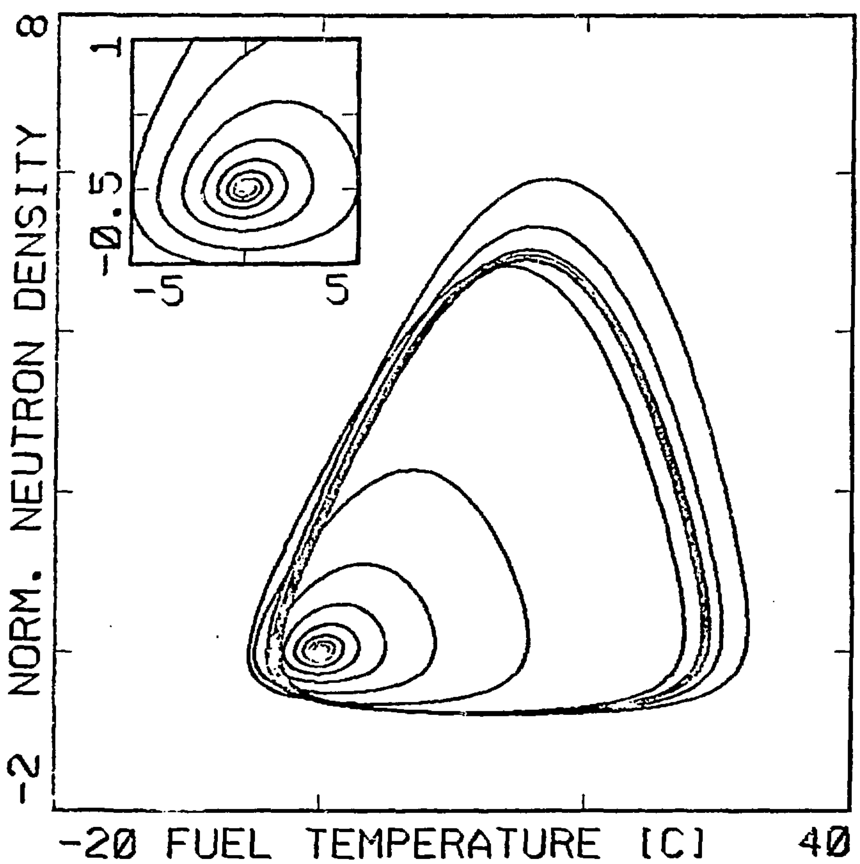

Fig. 1. Phase-space diagram n(t) versus T(t); for $k > k_0$, trajectories diverge from the unstable critical point but remain bounded, creating a limit cycle.

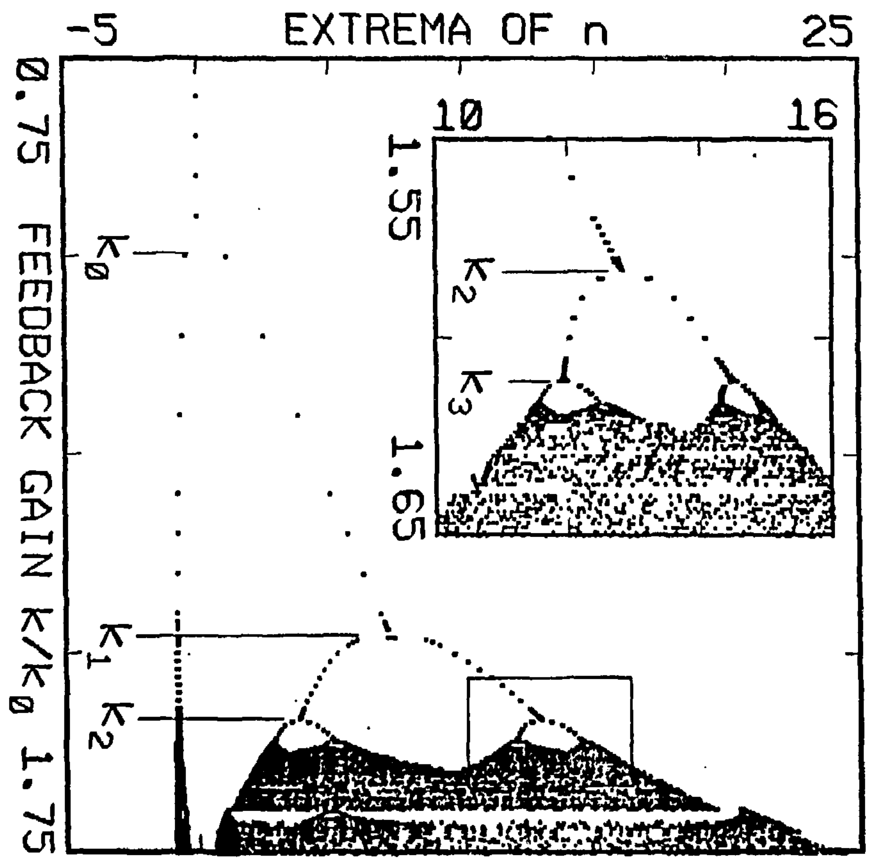

Fig. 2. Bifurcation diagram for n(t) as a function of k.