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PERTURBATION THEORY FROM STOCHASTIC QUANTIZATION

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Abstract

By using a diagrammatical method it is shown that in scalar theories the stochastic quantization method of Parisi and Wu gives the usual perturbation series in Feynman diagrams. It is further explained how to apply the diagrammatical method to gauge theories, discussing the origin of ghost effects.

1. Introduction

Parisi and Wu [1] introduced a stochastic quantization method which is based on the Langevin equation of non-equilibrium statistical mechanics. They applied their method to the quantization of scalar and gauge field theories with the main aim of constructing a new perturbation theory for gauge theories without introducing gauge fixing. Therefore for non-Abelian gauge theories their method is free of the Gribov ambiguity [2] and no ghost fields are required.

This talk is organized as follows: First we recall the basic features of the stochastic quantization method and how to obtain "stochastic diagrams" from an iterative solution of the Langevin equation. For the case of a self-interacting scalar field we show how to perform the time integrations of stochastic diagrams by introducing time ordering of the vertices and explain the diagrammatical proof [3] of the equivalence of sums of stochastic diagrams and Feynman diagrams.

The last part of this talk will be devoted to gauge theories: We study the Langevin equation for the gauge independent degrees of freedom of the gauge field. The conventional ghost effects are seen to result from a new type of interaction which appears now in the transformed Langevin equation.

2. Langevin Equation and Stochastic Diagrams

The starting point of the discussion of stochastic diagrams is the introduction of a fictitious time for the fields $\phi(x) \to \phi(x,t)$ whose time evolution is supposed to be given by the Langevin equation

$$\frac{\partial\phi(x,t)}{\partial t} = -\frac{\delta S}{\delta\phi(x,t)} + \eta(x,t) \qquad (2.1a)$$

$$S = \int d^D x \,\{\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{3!}g\phi^3\} . \qquad (2.1b)$$

We have written down (2.1) for a real self-interacting scalar field. S is the Euclidean action in D space-time dimensions. The fictitious time t should not be confused with the physical time contained in x. η is a random source with Gaussian distribution (white noise)

$$\langle\eta(x,t)\rangle_\eta = 0$$

$$\langle\eta(x,t)\eta(x',t')\rangle_\eta = 2\delta(x-x')\delta(t-t') \qquad \text{etc.} \tag{2.2}$$

Explicitly we can write the random average over η

$$\langle\ldots\rangle_\eta = \frac{\int[d\eta](\ldots)\ \exp(-\frac{1}{4}\int d^Dx dt\,\eta^2(x,t))}{\int[d\eta]\ \exp(-\frac{1}{4}\int d^Dx dt\,\eta^2(x,t))}\,. \tag{2.3}$$

Now the crucial point is that in the limit $t\to\infty$ equilibrium is reached and the random average of correlation functions of $\phi(x,t)$ tends to the corresponding Green functions of the quantum field theory with action (2.1b)

$$\lim_{t\to\infty}\langle\phi(x_1,t)\ \ldots\ \phi(x_L,t)\rangle_\eta = \langle\phi(x_1)\ \ldots\ \phi(x_L)\rangle. \tag{2.4}$$

This equivalence has been shown by various authors by various methods [1,4,5] and will now be discussed by a diagrammatical technique [3].

It should be noted that for gauge theories the situation is more complicated [4,6,7], part of which will be discussed later on.

The Langevin equation (2.1a) can be transformed into an integral equation in momentum space

$$\phi(k,t) = \int_0^t d\tau\ G(k,t-\tau)\{\eta(k,\tau) - \frac{1}{2!}\,g\int\frac{d^Dp}{(2\pi)^D}\,\phi(p,\tau)\ \phi(k-p,\tau)\} \tag{2.5a}$$

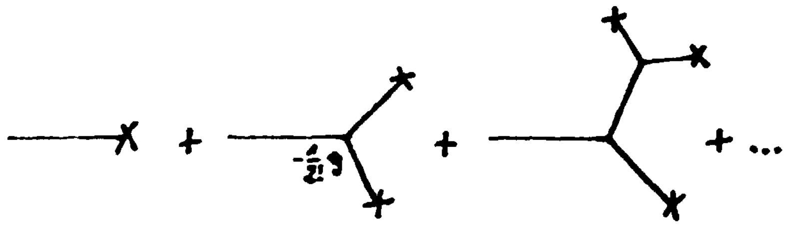
where

$$G(k,t-\tau) = e^{-(t-\tau)(k^2+m^2)}\ \theta(t-\tau) \tag{2.5b}$$

is the retarded Green function corresponding to the Langevin equation (2.1a) and where the boundary condition $\phi(x,0) = 0$ has been used. However, any trace of a specific boundary condition should die out with $t \to \infty$ due to the equilibrium property of the system. In momentum space (2.2) reads

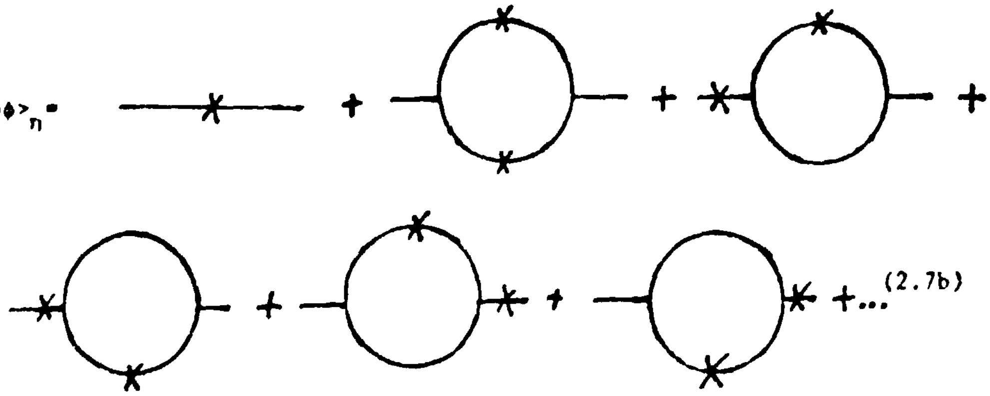
$$\langle \eta(k,t)\ \eta(k',t')\rangle_\eta = 2(2\pi)^D\ \delta(k+k')\ \delta(t-t')\ . \qquad (2.6)$$

Solving (2.5) by iteration one arrives at a power series expansion of ϕ in the coupling constants, which can be written diagrammatically

$\phi =$  (2.7a)

where we denote G by a line and η by a cross; integration over the momenta and the fictitious times at all the vertices and crosses is included.

Let us now consider a L point function $\langle \phi(x_1,t)\ \dots\ \phi(x_L,t)\rangle_\eta$ and substitute for ϕ its diagrammatical expansion (2.7a). When the random average over the η's is taken all crosses are joined together in all possible ways due to the Wick-decomposition property of the white noise. In this way diagrams are obtained which we call stochastic diagrams, as e.g.

$\langle \phi\phi\rangle_\eta =$  (2.7b)

Let us give some of the characteristics of a stochastic diagram:

- It has the form of an ordinary Feynman diagram of the theory described by the action S, apart from crosses on the lines where two η's have been joined together.
- To every Feynman diagram there exists a number of stochastic diagrams of the same topology. (Actually the sum of all stochastic diagrams to a given Feynman diagram yields just this Feynman diagram; see later.)
- Cutting a stochastic diagram of a L-point function at all crosses gives L connected trees.
- From every vertex there is "a way out just using G's". More precisely at every vertex there is just one G-function leading to another vertex with a higher fictitious time.
- Due to the δ functions in (2.6) the momentum integration in stochastic diagrams is just like the usual one of Feynman diagrams.

The time integrations on both sides of a cross can be performed similarly and lead to the propagator

$$D(k,\tau,\tau') = 2\int_0^{\min(\tau,\tau')} d\tau'' G(k,\tau-\tau'')G(k,\tau'-\tau'') = \frac{1}{k^2+m^2}\left(e^{-|\tau-\tau'|(k^2+m^2)} - e^{-(\tau+\tau')(k^2+m^2)}\right) \tag{2.8}$$

where τ, τ' are the times of the neighbour vertices. This means that in a stochastic diagram each line represents the Green function G and each crossed line the propagator D respectively.

3. Integrations over the Fictitious Times

In this section we will show how to perform the time integrationsof stochastic diagrams by introducing time ordering of the vertices. Let us consider a stochastic diagram belonging to some Feynman diagram, each vertex carrying a time τ_i which has to be integrated over. Due to the absolute values of time differences in the D propagators it is however convenient to introduce fixed time orderings of the vertices, as e.g.

$$\tau_1 < \tau_2 < \ldots < \tau_N < t\ .$$

For convenience of notation we substitute $p^2+m^2 \to p^2$ for all momenta. Thus masses are trivially contained in our discussion.

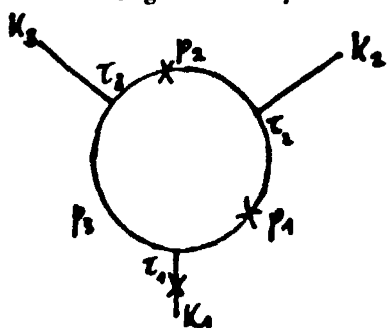
Given now a fixed time ordering of vertices we find the following rules for performing the time integrations and taking the limit $t \to \infty$ (for details see [3]):

a) beginning with the lowest vertex time and going succesively to higher ones one has to write in the denominator and multiply

- sum of $(\text{momenta})^2$ around τ_1
- sum of $(\text{momenta})^2$ around τ_1, τ_2 but not those between τ_1 and τ_2
- sum of $(\text{momenta})^2$ around τ_1, τ_2, τ_3 but not those between τ_1 and τ_2, τ_1 and τ_3, τ_2 and τ_3
- etc.
- sum of external $(\text{momenta})^2$.

b) For every cross in the stochastic diagram one has to divide by the corresponding $(\text{momentum})^2$.

Let us give an explicit example:



let e.g. $\tau_2 < \tau_1 < \tau_3$

one has as the contribution for $t \to \infty$

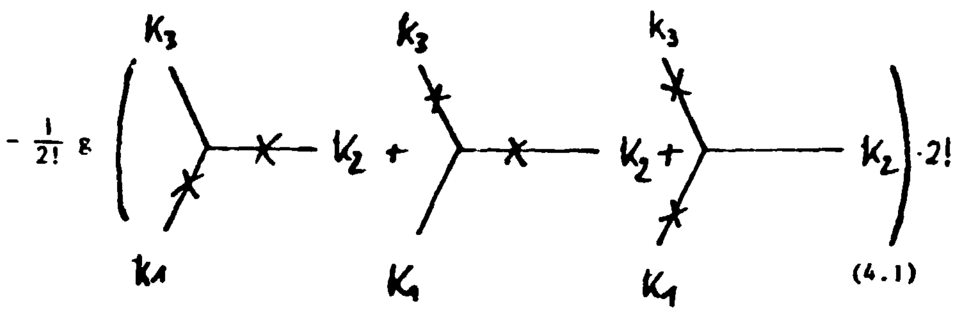
$$\frac{1}{p_1^2+p_2^2+k_2^2}\ \frac{1}{p_2^2+p_3^2+k_1^2+k_2^2}\ \frac{1}{k_1^2+k_2^2+k_3^2} \cdot \frac{1}{p_1^2\ p_2^2\ k_1^2}\ . \tag{3.1}$$

c) To obtain the full contribution of a stochastic diagram one has to sum over all time orderings.

4. Equivalence Proof

By induction on the number of vertices of an arbitrarily given Feynman diagram one can show [3] that the sum over all stochastic diagrams with the same topology gives for $t \to \infty$ exactly the Feynman diagram in the case of the real scalar field theory (2.1b).

Just as an example let us consider the 3 point vertex function in lowest order. According to the simplest application of the time integration rules a, b, c we get



(4.1)

$$= -g \frac{1}{k_1^2+k_2^2+k_3^2}\left(\frac{1}{k_1^2k_2^2}+\frac{1}{k_2^2k_3^2}+\frac{1}{k_3^2k_1^2}\right) = \frac{-g}{k_1^2\,k_2^2\,k_3^2}$$

where 2! is the number of possibilities of joining the η's together to obtain the desired diagram. We see that we have exactly obtained the corresponding Feynman diagram. Because of the expansion (2.7) all Green functions we consider are non-truncated.

5. Gauge Theories

The Langevin equation for the gauge field reads

$$\frac{\partial A_\mu^a(x,t)}{\partial t} = -\frac{\delta S[A]}{\delta A_\mu^a(x,t)} + \eta_\mu^a(x,t) \qquad (5.1)$$

where S is the pure Yang-Mills action

$$S = \frac{1}{4} \int d^D x \, F^a_{\mu\nu} F^a_{\mu\nu} \,. \tag{5.2}$$

The stochastic Green function and propagator are given in momentum space by [1]

$$G_{\mu\nu}(k,t-\tau) = \{(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})\, e^{-k^2(t-\tau)} - \frac{k_\mu k_\nu}{k^2}\}\, \theta(t-\tau) \tag{5.3}$$

and

$$D_{\mu\nu}(k,t-t') = \frac{1}{k^2}(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})(e^{-k^2|t-t'|} - e^{-k^2|t+t'|}) + \frac{2k_\mu k_\nu}{k^2} \min(t,t') \,. \tag{5.4}$$

The linear t-dependence in the propagator shows that with respect to the fictitious time t the gauge fields A^a_μ perform a random walk in the gauge parameter space; only gauge invariant quantities converge when performing the limit $t \to \infty$. On the other hand it is just the random walk which produces the Faddeev-Popov ghost effects. Unfortunately due to the non exponential time dependence in the G- and D functions stochastic diagrams cannot be calculated in an easy way (as e.g. in the scalar case) so that the origin of the ghost effects is somewhat obscured [6].

The idea (suggested in [1])along which we proceed is to separate the gauge field $A_\mu = A^a_\mu T^a$ in a set of gauge independent fields $B_\mu = B^a_\mu T^a$ which are constrained by some gauge condition of our choice (as e.g. $\partial_\mu B^a_\mu = 0$) and in a gauge dependent part [8]

$$A_\mu = e^{-ig\alpha}\, B_\mu\, e^{ig\alpha} - \frac{i}{g}(\partial_\mu\, e^{-ig\alpha})\, e^{ig\alpha} \,. \tag{5.5}$$

When we are calculating gauge invariant quantities we can just use our gauge independent fields B^a_μ. We are actually "choosing" the gauge, not "fixing" it.

It is easy to see that the Langevin equation for the B-field is [7,9]

$$\frac{\partial B^a_\mu}{\partial t} = - \frac{\delta S[B]}{\delta B^a_\mu} - D^{ab}_\mu \Lambda^b + \eta^{a\prime}_\mu \tag{5.6}$$

where

$$\eta'_{\mu} = e^{-ig\alpha}\, \eta\, e^{ig\alpha} \qquad (5.7)$$

$$\Lambda = \frac{i}{g}\left(\frac{\partial}{\partial t}\, e^{ig\alpha}\right) e^{-ig\alpha} \; . \qquad (5.8)$$

In our case it follows trivially from $\partial_{\mu} B^{a}_{\mu} = 0$ that $\partial_{\mu} \dot{B}^{a}_{\mu} = 0$ which allows to calculate Λ perturbatively (for details and a more general discussion see [10]). We obtain that the longitudinal parts in the G- and D-functions are all projected away so that the time integration rules are applicable. We checked explicitly that the Faddeev-Popov ghost effects are correctly reproduced in $\langle F^{a}_{\mu\nu} F^{b}_{\rho\lambda} \rangle$ to $O(g^2)$; they result from the new type of interaction which is present now in the Langevin equation for the B-field.

In summary we believe that the diagrammatical method can successfully be applied also to gauge theories, allowing a simple evaluation of stochastic diagrams.

Referen

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[2] V.
[3] W.
[4] E.
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