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CALCULATION OF ELECTROMAGNETIC $\rho\pi$ FORMFACTOR FROM QCD SUM RULES

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A b s t r a c t

Electromagnetic $\rho\pi$ formfactor at intermediate momentum transfer, $0.7\ GeV^2 \leq Q^2 \leq 3\ GeV^2$, is calculated using QCD sum rules for the vertex function of two vector and one axial vector currents. In this region the results obtained are consistent within 25% accuracy with the vector meson dominance model predictions and can be regarded as its theoretical justification.

The QCD sum rules method suggested in the pioneering paper by Shifman, Vainshtein and Zakharov[1] proved to be extremely effective for model independant calculations of many low energy hadron physics parameters. In this method nonperturbative effects associated with large distances are accounted for phenomenologically through power corrections arising in the operator product expansion which are proportional to vacuum expectation values of various operators composed of quark and gluon fields. An essential feature of this approach is the use of the Borel transformation which allows one to suppress the higher resonance and continuum contribution. Application of this method to polarization operators of various currents resulted in determination of masses and couplings of low lying mesonic[1,2] and baryonic[3] states. In the recent papers[4,5] QCD sum rules for the vertex function of two axial vector and one vector currents were considered and pion electromagnetic formfactor was obtained in the region of intermediate momentum transfer. In doing so, the double dispersion relation for the vertex function was used and the Borel transformation in two variables was performed *). In ref.[7] the same vertex function was used to determine ρ and A_1 electromagnetic formfactors and $\rho \to 2\pi$ decay width. The vertex function of two vector and one axial vector currents was considered in ref.[8] in an analogous way and the coupling constant of $\omega \to \rho\pi$ transition was calculated. This made it

*) QCD sum rules based on a double dispersion relation but without the Borel transformation were originally employed by A.Yu.Khodjamirian[6] in the calculation of charmonium radiative transitions.

possible to determine the $\omega \to 3\pi$ decay width in a good agreement with experiment, and using the vector meson dominance (VMD) model, the $\omega \to \pi\gamma$; $\rho^0 \to \pi^0\gamma$ and $\pi^0 \to 2\gamma$ decay widths as well. In the present paper we use the same vertex function to compute the $\rho\pi\gamma$ formfactor.

Let us consider the following amplitude

$$A_{\mu\nu\lambda}(p,p',q) = -\int d^4x\, d^4y\, e^{i(p'y-px)} \langle 0| T\{ j_\mu^{(0)}(0) j_\nu^{(1)}(y)\, j_\lambda^5(x)\}|0\rangle \quad (1)$$

with $q = p - p'$ and negative p^2, p'^2, $q^2 \sim -1$ GeV^2 . Here $j_\mu^{(0)} = \frac{1}{6}(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)$ and $j_\nu^{(1)} = \frac{1}{2}(\bar{u}\gamma_\nu u - \bar{d}\gamma_\nu d)$ are isoscalar and isovector currents and $j_\lambda^5 = \frac{1}{2}(\bar{u}\gamma_\lambda\gamma_5 u - \bar{d}\gamma_\lambda\gamma_5 d)$ is isovector axial vector current. For the momenta in question the characteristic values of α_s are small, $\alpha_s \lesssim 0.3$. Therefore we consider the amplitude $A_{\mu\nu\lambda}$ in the lowest order in α_s in which it is given by the quark loop diagram of Fig.1 and by power correction diagrams of Figs.2 and 3.

The $\rho\pi\gamma$ formfactor is determined by the structure transversal in p' and q,

$$A_{\mu\nu\lambda}^{(\rho\pi\gamma)} = f(p^2, p'^2, Q^2)\, p_\lambda\, \epsilon_{\mu\nu\alpha\beta}\, p_\alpha p'_\beta , \quad (2)$$

where $Q^2 = -q^2 > 0$. The quark loop contribution to this amplitude can be calculated through the double dispersion relation in p^2 and p'^2,

$$f^{(0)}(p^2, p'^2, Q^2) = -\frac{1}{4\pi^2}\int_0^\infty ds \int_0^\infty ds' \frac{\Delta(s,s',Q^2)}{(s-p^2)(s'-p'^2)} + \text{subtr. terms} \quad (3)$$

Here $\Delta(s,s',Q^2)$ is the double discontinuity of $f^{(0)}(p^2,p'^2,Q^2)$ on the cuts $0<s<\infty$ and $0<s'<\infty$. The discontinuity $\Delta(s,s',Q^2)$ is calculated in the standard way substituting the denominators of quark propagators by δ-functions, $k^{-2}\to -2\pi i\,\delta(k^2)$. It should be emphasized that in general the amplitude (1) contains six linearly independent structures and the discontinuity $\Delta(s,s',Q^2)$ as well as the function $f(p^2,p'^2,Q^2)$ is a certain combination of the corresponding structure functions[8]. Explicitly,

$$\Delta(s,s',Q^2)=\frac{1}{4}\,\varkappa^{-5/2}\Big[ss'(s-s')^2+Q^2(-s^3-4s'^3+6s^2s'-3ss'^2)+$$

$$+\;Q^4(-2s^2-8s'^2+3ss')+Q^6(-s-4s')\Big], \qquad (4)$$

$$\varkappa=(s-s')^2+2Q^2(s+s')+Q^4$$

Aiming to saturate the sum rules by ρ and π mesons and, following refs.[4,5,7,8], let us perform the Borel transformation in two variables p^2 and p'^2,

$$f_B(M^2,M'^2,Q^2)=\lim_{\substack{n\to\infty,\,-p^2\to\infty\\ n'\to\infty,\,-p'^2\to\infty\\ -p^2/n\to M^2\\ -p'^2/n'\to M'^2}}\frac{1}{(n-1)!(n'-1)!}(-p^2)^n\Big(\frac{d}{dp^2}\Big)^n(-p'^2)^{n'}\Big(\frac{d}{dp'^2}\Big)^{n'}\times f(p^2,p'^2,Q^2) \qquad (5)$$

This transformation is also necessary to nullify the unknown subtraction terms which persist in (3) and are polynomials in one of the two variables p^2 or p'^2 and arbitrary functions of two other, p'^2 and Q^2, or p^2 and Q^2. In general, the Borel parameters M^2 and M'^2 are independent and should be chosen in such a way as to warrant stability of the ultimate result against their variation. In the calculations of the pion formfactor[4] and $g_{\omega\rho\pi}$ coupling constant[8] the Borel transformation was also performed in two variables. In both cases the two variables corresponded to two similar channels, namely, axial vector channels in ref.[4] and vector channels in ref.[8]. Thus, in both cases the problem was symmetric and the two Borel parameters were naturally put to be equal from the very beginning. The following permissible intervals were obtained: 0.8 GeV$^2 \leq M^2 \leq$ 1.5 GeV^2 for the axial vector channel[4] and 0.6 GeV$^2 \leq M'^2 \leq$ 1.1 GeV^2 for the vector channel[8]. The essential overlapping of these intervals indicates that it is not altogether wrong to put $M^2 = M'^2$ in the case of $\rho\pi\gamma$ formfactor and thus deal with one parameter only. As is seen from what follows this step is justified by stability of the results against the variation of the Borel parameter M^2. Besides, the use of symmetric Borel transform substantially simplifies the calculation of gluon condensate power corrections.

Now, approximating the continuum contribution by the quark loop diagram and transferring it to the other side of the sum rule we obtain after the symmetric Borel transformation,

$$f_B^{(0)}(Q^2, M^2) = -\frac{1}{4\pi^2 M^4} \int_0^{s_0} ds \int_0^{s_0'} ds' \exp\left(-\frac{s+s'}{M^2}\right) \Delta(s, s', Q^2) \qquad (6)$$

where s_0 = 0.8 GeV^2 and s_0' = 1.5 GeV^2 are duality intervals in the axial vector and vector channels, respectively[1].

The calculation of the power corrections was carried out analogously to refs.[4,7,8,10] in the fixed point gauge[9], $x_\mu A_\mu(x) = 0$ which is very convenient when dealing with gluon condensate corrections and quark condensate corrections given by diagrams with soft gluon exchange. In the chiral limit the dominant contribution comes from the terms proportional to $\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle_0$ and $\langle \bar\psi \Gamma \psi \bar\psi \Gamma \psi \rangle$, the latter under factorization hypothesis being reduced to $\langle \bar\psi \psi \rangle_0^2$. In the fixed point gauge these corrections are given by the diagrams of the types depicted in Figs.2 and 3, the coordinate origin being chosen according to eq.(1), in the isoscalar vector vertex corresponding to photon. After the symmetric Borel transformation the power corrections take the form

$$f_B^{(q)} = \frac{4\pi\alpha_s \langle \bar\psi\psi \rangle_0^2}{486\, M^8} \left(45 \frac{M^4}{Q^4} + 27 \frac{M^2}{Q^2} + 5 + 5 \frac{Q^2}{M^2} \right), \qquad (7)$$

$$f_B^{(g)} = \frac{\alpha_s}{\pi} \frac{\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle_0}{576\, M^4} \left(-12 \frac{M^2}{Q^2} + 5 \right) \qquad (8)$$

Saturating the physical part of the sum rules by π and ρ mesons and parametrizing the matrix element of electromagnetic current,

$$\langle \pi | j_\mu^{em} | \rho \rangle = e\, F(Q^2)\, \varepsilon_{\mu\alpha\beta\gamma}\, P_\alpha P'_\beta\, \epsilon_\gamma , \qquad (9)$$

where $e^2 = 4\pi\alpha = 4\pi/137$ and ϵ_γ is the ρ meson polarization vector, we get after the Borel transformation,

$$f_B^{(phys.)} = \frac{\sqrt{4\pi\alpha}\, f_\pi\, m_\rho^2}{g_\rho} M^{-4} e^{-m_\rho^2/M^4} F(Q^2) \qquad (10)$$

Here g_ρ and f_π are defined in the standard way,

$\langle 0 | j_\nu^{em} | \rho \rangle = m_\rho^2 g_\rho^{-1} \epsilon_\nu$ and $\langle 0 | j_\lambda^5 | \pi^0(p) \rangle = f_\pi p_\lambda$,
f_π = 93 MeV. Finally, putting $f_B^{(phys.)} = f_B^{(0)} + f_B^{(q)} + f_B^{(g)}$
we obtain the formfactor,

$$F(Q^2) = -M^4 \frac{e^{m_\rho^2/M^2}}{m_\rho^2 f_\pi \sqrt{4\pi\alpha}} \left(f_B^{(0)} + f_B^{(q)} + f_B^{(g)} \right) \qquad (11)$$

with $f_B^{(0)}$, $f_B^{(q)}$ and $f_B^{(g)}$ determined by eqs.(6), (7) and (8), respectively. The formfactor $F(Q^2)$ is calculated using the following values of parameters[1,4]: $4\pi^2/g_\rho^2$ = 0.41, m_ρ^2 = 0.6 GeV^2, $\alpha_s/\pi \langle G_{\mu\nu}^a G_{\mu\nu}^a \rangle_0$ = 0.012 GeV^4, $\alpha_s \langle \bar\psi\psi \rangle_0^2$ = 8·10^{-5} GeV^6 . The integral in eq.(6) was done numerically. The whole approach is consistent only in the region of intermadiate Q^2 since at $Q^2 \to 0$ and $Q^2 \to \infty$ the ratio of the power corrections (7) and (8) to the main term (6) increases infinitely. The dependence of $F(Q^2)$ on the Borel parameter M^2 for different values of Q^2 is shown in Fig.4. As is seen from eqs.(6),(7) and (8), the contributions of the power corrections and continuum rise with decrease and increase of M^2, respectively. Therefore, only the values of M^2 at which each of these contributions is less than 50% of the total results were considered, i.e. 0.6 GeV$^2 \leq M^2 \leq$ 0.8 GeV^2 . As is seen from Fig.4, for M^2 in this interval (and) 0.7 GeV$^2 \leq Q^2 \leq$ 3 GeV^2 the M^2 dependence of $F(Q^2)$ is rather weak. The Q^2 dependence of the formfactor for the central value $M^2 \approx$ 0.7 GeV^2 is presented in Fig.5 by the solid curve. The accuracy of the result is about 20% and is mainly governed by the model used to account for the continuum contribution.

The formfactor $F(Q^2)$ can be also obtained using the VMD model assuming the electromagnetic current to be coupled to

ω meson which turns into photon,

$$F(Q^2) = \frac{g_{\omega\rho\pi}}{g_\omega} \frac{m_\omega^2}{m_\omega^2 + Q^2} \qquad (12)$$

Using here the coupling $g_{\omega\rho\pi} \simeq 17\ \mathrm{GeV}^{-1}$ obtained earlier in ref.[8] from QCD sum rules and the value[1] $4\pi/g_\omega^2 = 0.046$ we get the dashed curve in Fig.5. It is seen that the VMD model agrees with the QCD sum rules calculation within 25% accuracy in the whole interval, $0.7\ \mathrm{GeV}^2 \leq Q^2 \leq 3\ \mathrm{GeV}^2$ in which the latter is consistent. Thus, the applicability of the VMD model is grounded by QCD.

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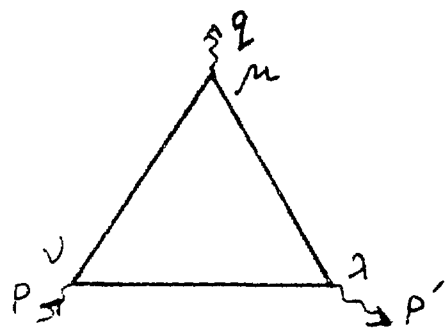

Fig.1. Main diagram for the amplitude $A_{\mu\nu\lambda}(p,p',q)$.
Wavy lines correspond to external currents.

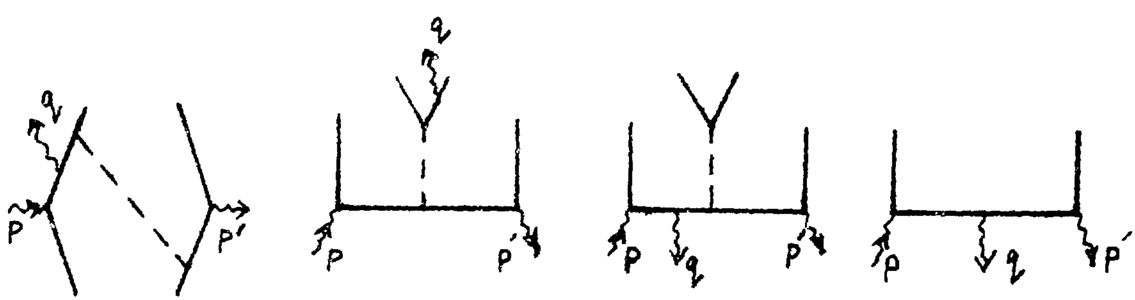

Fig.2. Quark condensate corrections.

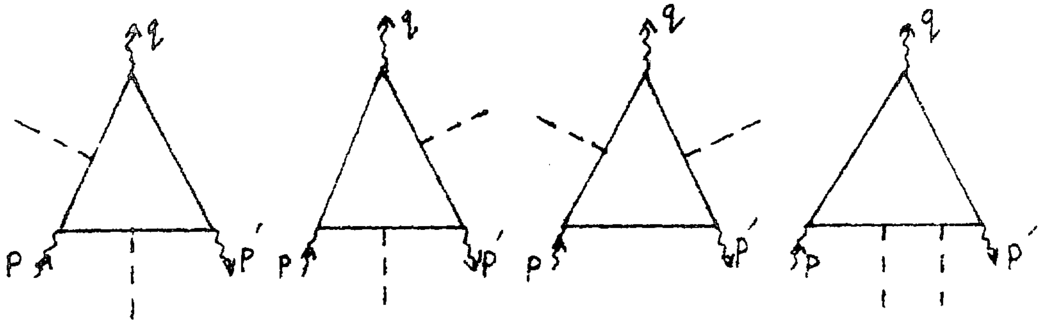

Fig.3. Gluon condensate corrections.

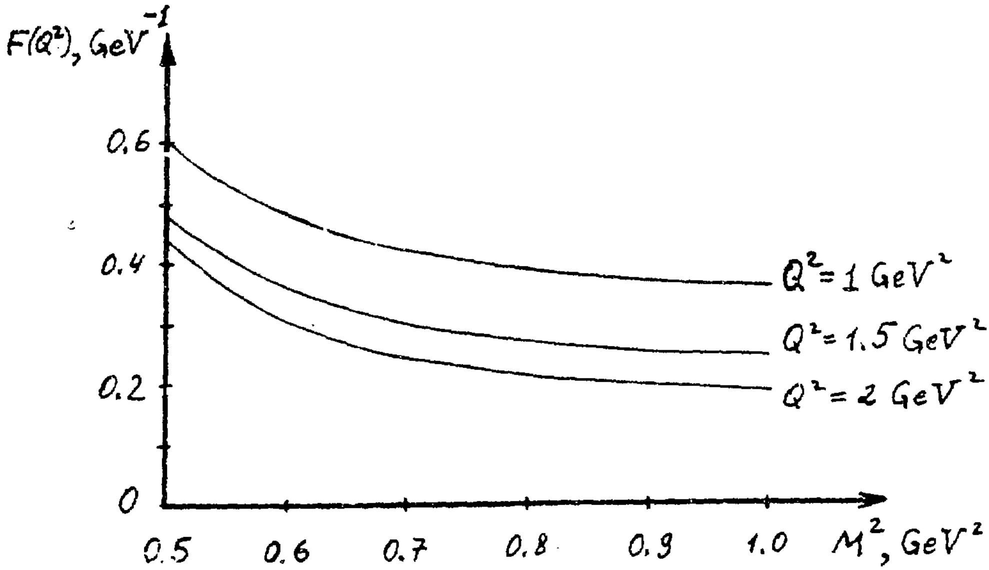

Fig.4. Formfactor $F(Q^2)$ versus the Borel parameter M^2 .

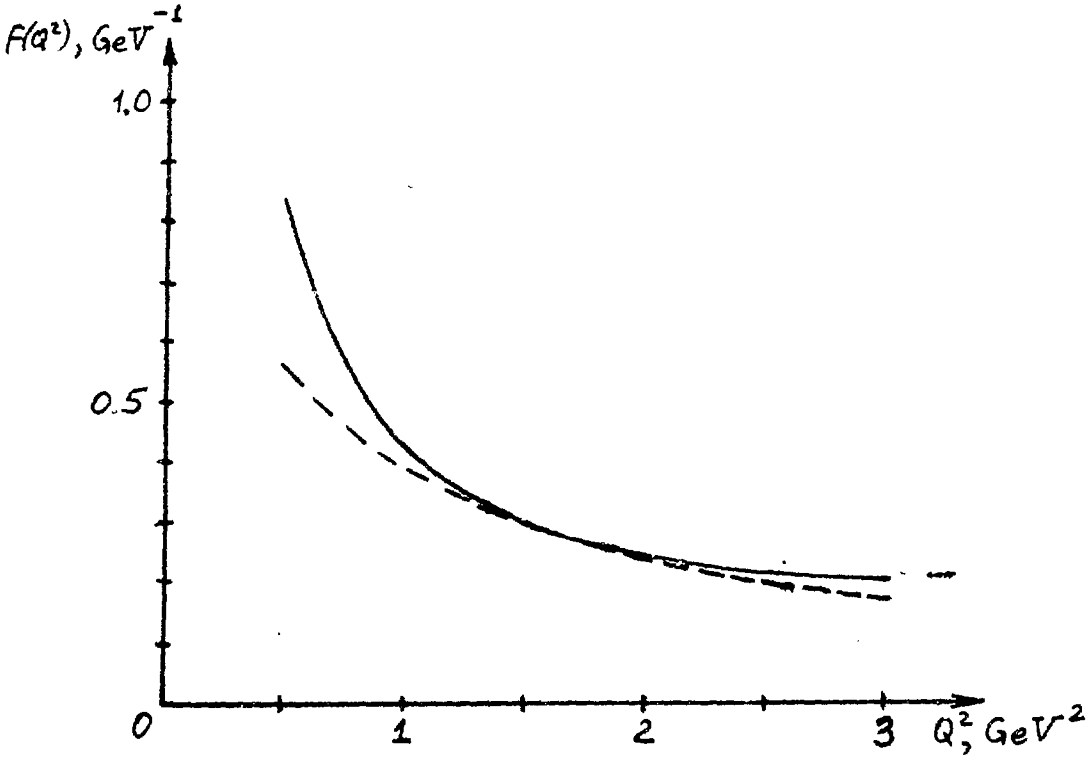

Fig.5. Q^2 dependence of the formfactor. The QCD sum rule result is presented by the solid curve. The dashed curve corresponds to the VMD model predictions.

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