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DOES THE SU(5) MONOPOLE CATALYZE PROTON DECAY? *

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ABSTRACT

The role of Higgs induced mass for the fermions in the SU(5) monopole catalysis of the baryon decay problem is investigated. We find that the inclusion of such a mass does not rule out the Rubakov effect but it does suppress the catalysis cross-section.

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In a recent paper[1], it was claimed that the monopole catalysis of proton decay[2] is absent for SU(5) monopoles [3], because of the non-existence of the zero-energy fermion-monopole bound state in the theory for (Higgs induced) massive fermions. Due to the far-reaching implications of this claim we reanalyze the problem for the SU(5) fundamental monopole by paying due attention to the geometrical structure of the monopole. Our results disagree with that of reference 1.

It was shown by Jackiw and Rebbi [4] that if the fermion mass is induced by the Higgs field that created the SU(2) monopole, there exist normalizable zero energy s-wave bound states. Then one may wonder why it is necessary to reconsider the problem for the SU(5) monopole again, given that their gauge field strucures are the same. However, this is a well-posed problem; because although the gauge field structures are the same, the Higgs field structure of the SU(5) is a lot richer. For instance, it is the <u>24</u>-Higgs which generates the monopole, whereas the fermion mass generation is induced by the <u>5</u>-Higgs. Therefore, there is no a priori reason why the problem of zero-energy bound states should be the same for the SU(5) monopole. However, we shall prove that the zero-energy bound states still exist for the SU(5) monopole.

We start by briefly reviewing the SU(5) monopoles. In the standard scenario, the SU(5) symmetry is broken down to $SU(3)_c \times SU(2)_L \times U(1)_Y$, at an energy scale of 10^{15} Gev, by a superheavy Higgs multiplet Φ in the adjoint representation <u>24</u>. At the Salam-Weinberg stage the symmetry is further broken down to $SU(3)_c \times U(1)_{e.m.}$ by another Higgs, H, in the fundamental representation <u>5</u>.

A monopole is a configuration carrying non-trivial topology in a U(1) subgroup generated by the Abelian diagonal generators. The simplest realisation [3] can be obtained by a transposition of the 't Hooft - Polyakov monopole[5] to an appropriate SU(2) subgroup of SU(5). The stable (least massive) corresponds to the 1+1+2+1 imbedding of a SU(2) into SU(5) given by

$$\vec{T} = \frac{1}{2}\,\mathrm{diag}\,(0,0,\vec{\tau},0) \tag{1}$$

where $\vec{\tau}$ are the 2 x 2 Pauli matrices. The general form of this monopole is

$$W_i = (\vec{T}\times\hat{r})_i \, \frac{K(r)-1}{g\,r} \tag{2a}$$

$$W_0 = 0 \tag{2b}$$

$$\Phi = \frac{1}{g}\left(\begin{array}{cc|c|c} \phi_1(r) & & & \\ & \phi_1(r) & & \\ \hline & & \phi_2(r)+\phi(r)\,\hat{r}\cdot\vec{\tau} & \\ \hline & & & -2(\phi_1+\phi_2) \end{array}\right) \tag{2c}$$

$$H = \frac{1}{g}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ h(r) \end{pmatrix}. \tag{2d}$$

The analog of the Julia – Zee dyon is obtained as well by the same method :

$$W_0 = \frac{1}{g}\left(\begin{array}{cc|c|c} J_1(r) & & & \\ & J_1(r) & & \\ \hline & & J_2(r)+\frac{1}{2}\,J(r)\,\hat{r}\cdot\vec{\tau} & \\ \hline & & & -2(J_1+J_2) \end{array}\right). \tag{3}$$

This configuration will be useful later. The functions $K(r)$, $J_i(r)$ and are real as required by the Hermiticity of W_μ and Φ . We can also take $h(r)$ to be real as this minimizes the energy.

The boundary conditions at infinity are defined by requiring the finiteness of the energy, which in turn requires the fields to approach the Higgs vacuums away from the origin. In the phenomenologically interesting two-stage breaking case, the Φ-Higgs vacuum is attained at $O(10^{15})$ GeV, whereas the H-Higgs vacuum is attained at $O(10^2)$ GeV. Therefore, geometrically, the fundamental monopole of SU(5) has the following nested structure : There is

a core of radius m_X^{-1} in which all the SU(5) degrees of freedom are excited. The Salam-Weinberg phase is located in the shell $m_X^{-1} < r < m_W^{-1}$. In this region, Φ has attained its v.e.v. but H is still zero. It attains its v.e.v. at about $m_W^{-1} \sim O(10^2)$ GeV. Outside m_W^{-1} radius, only colour and ordinary magnetic fields survive. These long range fields are superpositions of two ordinary single Dirac-unit monopoles.

The quarks and leptons have both ordinary charge and colour hypercharge. Because Φ breaks $SU(2)_m$ also, $SU(2)_m \to U(1)_{\bar{Q}}$, outside the main core, quarks and leptons move as particles of total charge $\bar{Q} = Q + Y_c$ in the field of a simple Abelian Dirac monopole.

The standard choice of fermions is a right – handed $\underline{5}$, denoted by ψ and a left – handed $\underline{10}$, denoted by M. $\bar{Q} = 0$ fermions cannot interact with monopoles. The others with charge – monopole strength $\bar{Q}g = \pm\frac{1}{2}$ form the doublets

$$\begin{pmatrix} d^3 \\ e^+ \end{pmatrix}_L \quad \begin{pmatrix} e^- \\ -\bar{d}^3 \end{pmatrix}_L \quad \begin{pmatrix} \bar{u}^1 \\ u^2 \end{pmatrix}_L \quad \begin{pmatrix} -\bar{u}^2 \\ u^1 \end{pmatrix}_L \tag{4}$$

under the $SU(2)_m$.

The fermion-monopole interaction is usually treated by taking the monopole as an external field, i.e. monopole fields are assumed not to be perturbed by the fermions. We study the zero energy solutions of the Dirac equation in the monopole field as Rubakov has shown that it is these zero energy states which control the asymptotic behaviour of the Fermion Green's function. The Rubakov effect depends on the existence of these zero energy states.

We have to also stress that the anomaly plays a central role in the Rubakov effect. Thus in order to have an understanding of the zero energy bound state problem in this context, we have to consider configurations which can supply the anomaly effect as a topologically non – trivial background. Dyons are thus the right objecys. In any case, quantum monopoles are dyons.[6)] Thus, we shall carry our analysis for the dyon configuration depicted in equations (2,3). Quantum mechanically, the fermion condensate which forms around the dyon can be taken as the "zero-energy" bound state.[1)] Thus the "size" of this bound state really determines the "size" of the mono-fermion hadron. We shall see shortly that for the fundamental monopole of SU(5) the size is of the order $O(m_W^{-1})$, rather than $O(m_f^{-1})$ as is commonly believed.

Now let us consider the Dirac equation for one of the doublets in Eq.4, say (d, e^+), in the background of a SU(5) fundamental dyon and look for zero energy solutions.

$$\left(i \not{\partial} - g \not{W} - \frac{1}{\sqrt{2}} G h(r)\right) \begin{pmatrix} d_3 \\ e^+ \end{pmatrix} = 0 \tag{5}$$

where G is the standard Higgs coupling strength of the $\underline{5}$ and $\underline{10}$ fermions $(G H^\alpha \bar{\Psi}^\beta M_{\alpha\beta})$ Introducing the 2 x 2 matrices $\mathcal{M}^\pm$ (+ and - stand for the upper and lower components of the Dirac spinor),

$$\psi^\pm_{in} = \mathcal{M}^\pm_{im} \tau^2_{mn} \qquad i,m,n = 1,2. \tag{6}$$

we can reduce the Dirac equation, for E = 0, into the following form

$$\vec{\tau}.\vec{\nabla}\mathcal{M}^\pm - \frac{1}{2}W(r)(\hat{r}\times\vec{\tau})\mathcal{M}^\pm.\vec{\tau} \mp i m(r)\mathcal{M}^\pm - J_2(r)\mathcal{M}^\mp + \frac{1}{2}J(r)\mathcal{M}^\mp(\hat{r}.\vec{\tau}) = 0$$

where $m(r) = \frac{1}{\sqrt{2}} Gh(r)$ and $W(r) = \frac{1}{r}(K(r)-1)$.

Next we decompose the matrices $\mathcal{M}^\pm$ as

$$\mathcal{M}^\pm = \phi^\pm(r) + \chi^\pm_a \tau_a \qquad a = 1,2,3. \tag{8}$$

and go through the same manipulations as in reference(1) to get

$$\pm \int d^3x\, m(r)\left[|\phi^\pm|^2 + |\chi^\pm_a|^2\right] + i\varepsilon_{abc}\int d^3x\, \chi^\pm_a \partial_b \chi^{\pm *}_c + i\int d^3x\, J_2(r)\left[\chi^\pm_a \chi^{\mp *}_a - \phi^{\pm *}\phi^\mp\right] + \frac{1}{2}i\int d^3x\, J(r)\,\hat{r}_a\left[\phi^{\pm *}\chi^\mp_a - \phi^\mp \chi^\pm_a\right] = 0 .$$

In contrast to ref.(1), we definitely cannot conclude from this expression that the fermion wave function vanishes identically, because of the extra terms due to the dyons. Thus the existence of the zero energy states is not ruled out by this argument. However, let us see whether we get the result of ref.(1) by restricting ourselves to a static monopole configuration. In this case, eq.(9) reduces to eq.(3.8) of ref.(1), due to the pure imaginary nature of the second term in eq.(9) :

$$\int d^3x\; h(r)\left[|\phi^\pm|^2 + |\chi^\pm_a|^2\right] = 0 \tag{10}$$

Indeed, h(r) does not change sign ; it increases exponentially from zero to its vacuum value around $r \sim m_W^{-1}$. Thus the lower limit of the integral starts around m_W^{-1}. Therefore, all that eq.(10) is telling us is that $\phi^\pm, \chi^\pm$ vanish identically for $r \gtrsim m_W^{-1}$, but can be nonzero for $r < m_W^{-1}$, a huge region surrounding the monopole core (extends over twelve orders of magnitude). Thus the equations for the zero energy s-wave are the same as those of Jackiw and Rebbi (without their Higgs term) inside the Salam-Weinberg domain,

Although the dyon equations are more complicated, the W_0 being like a second Higgs in the adjoint representation (which does not participate in the mass generation), we expect the massive fermion case for the dyon to follow the same pattern as in the monopole case.

We have just demonstrated that the Higgs induced mass for the fermions does not effect the results on the existence of zero modes which were obtained for the massless case, although Rubakov himself was worried that in the case of massive fermions higher order corrections could destroy his boundary conditions, thus invalidating his analysis[2]. Our results have a very simple explanation if we recall the meaning of spontaneously generated mass for the fermions. Mass for fermions is a well-defined concept only in the $\underline{5}$-Higgs vacuum far away from the monopole. But the monopole's neighbourhood is a region where different symmetry phases appear in a nested fashion. So a "massive" fermion approaching a monopole will experience changes in this attribute as it moves closer, by crossing different phase boundaries. As it crosses the Salam - Weinberg boundary, at m_W^{-1}, it quickly loses its mass and from then on, in a huge region, moves as a massless particle. Our analysis shows, however, that the fermion mass is not entirely without effects on physics in the neighbourhood of a monopole. Although the condensate was extending out to infinity like r^{-6} in the massless case[2], now it is confined inside $r \lesssim m_W^{-1}$. So a monofermion system is not a hadron with size m_f^{-1}, but rather the size is $\sim m_W^{-1}$. Therefore, the cross section for the fermion number violating process will be $O(m_W^{-2})$, agreeing with a previous result of Wilczek[7].

The implications of a catalysis cross section of the order of m_W^{-2} are important in the context of recent attempts to measure monopole induced proton decay [8]. If we take our arguments as correct, then the geometrical cross section for catalysed proton decay is

$$\sigma_c \simeq \frac{\pi}{m_W^2} \simeq 1.2 \times 10^{-31}\ \mathrm{cm}^2 . \tag{11}$$

Combining this with the Parker bound [9] for the monopole flux

$$F_m < 6 \times 10^{-16} \text{ cm.}^{-2} \text{ sr}^{-1} \text{ s}^{-1} \tag{12}$$

we get

$$F_m \times \sigma_c < 7.2 \times 10^{-47} \text{ sr}^{-1} \text{ s}^{-1} \tag{13}$$

which is orders of magnitude lower than the recently reported IMB result [8]

$$F_m \times \sigma_c < 6.6 \times 10^{-40} \text{ sr}^{-1} \text{ s}^{-1} \tag{14}$$

for $\sigma_c \leq 0.1$ mb. The bound eq.(13) on $F_m \times \sigma_c$ corresponds to an effective proton lifetime of

$$\tau_c > 3.5 \times 10^{37} \text{ yrs.} \tag{15}$$

Such a long lifetime would seem to be beyond the capabilities of current experiments.

Here we would like to comment on the IMB conclusion that the flux is

$$F_m < 7.2 \times 10^{-15} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}. \tag{16}$$

We feel that this upper limit on the flux cannot be taken seriously, as what is actually measured in the experiment is $F_m \times \sigma_c$, and F_m is deduced by assuming a strong interaction value for σ_c. If we use our value for σ_c, we find that the IMB experiment would yield a much less stringent limit on the flux

$$F_m < 5.4 \times 10^{-8} \text{ cm}^{-2} \text{sr}^{-1} \text{ s}^{-1}. \tag{17}$$

We see that this bound is just about Cabrerra's present limit [10] of

$$F_m < 10^{-9} \text{ cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}. \tag{18}$$

Both these limits are far above the Parker bound. However, we feel that the calculation of the flux from a measurement of $F_m \times \sigma_c$ is unreliable unless we have a better handle on σ_c. Ideally, it would be best to measure F_m from a Cabrerra type experiment and $F_m \times \sigma_c$ from monopole catalysis and the two results combined to give σ_c, which would then be a model independent number.

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