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THE FERMION BOUNDARY CONDITION AND THE θ-ANGLE IN QED_2

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ABSTRACT

The order parameter of the Schwinger model is calculated in the Euclidean functional integral approach. It is shown that the symmetry breaking angle θ is intimately connected to the boundary condition imposed on the fermions. The transition to the Euclidean description involves both imaginary time and imaginary θ.

АННОТАЦИЯ

Рассчитан параметр порядка в модели Швингера методом функционального интегрирования. Показано что угол нарушения симметрии θ тесно связан с граничным условием налагаемым на фермионы. Переход к евклидовому описанию требует как мнимого времени так и мнимого угла θ.

KIVONAT

A Schwinger modell rendparamétereit számitjuk euklidészi funkcionál-integrál segitségével. Megmutatjuk, hogy a szimmetriasértésnek megfelelő θ szög a fermion-terekre kirótt határfeltételekkel áll szoros kapcsolatban. Az euklidészi formalizmusra való áttérés szükségessé teszi mind az időkoordináta, mind a θ paraméter imagináriussá való elfolytatását.

Schwinger model [1] proved itself to be an important tool in illustrating field theoretic ideas such as mass generation, spontaneous symmetry breaking, etc. The model permits exact operator solutions in different gauges [2], [11]-[14] as well as an exact treatment in terms of functional integrals [3]-[10]. Though this letter method is in many respects simpler and more instructive than the rather involved operator solutions, apparently no completely satisfactory account of the *phase* of the order parameter $\langle\bar{\Psi}\Psi\rangle$ has been given so far within the functional integral approach. It is the purpose of the present note to fill this gap.

Operator solutions [10], [13], [14] give for the order parameters of QED_2 the following simple expressions

$$\langle\bar{\Psi}_L\Psi_L\rangle = \frac{m'}{2\pi}e^{i\theta} \quad , \qquad \langle\bar{\Psi}_R\Psi_R\rangle = \frac{m'}{2\pi}e^{-i\theta} \quad , \tag{1}$$

where $m' = \frac{m}{2}e^{\gamma}$, γ is Euler's constant and m is equal to the mass of the "physical" particle described by the model. At the first glance, the reproduction of (1) in the framework of functional integrals must be provided by the textbook formula

$$\langle\bar{\Psi}_M(0)\Psi_M(0)\rangle = \frac{1}{Z}\int[\mathcal{D}A_\mu]e^{-S[A]-\Gamma^{(1)}[A]}\,iG_{MM}(0,0,A) \quad , \tag{2}$$

which in the present case contains Gaussian integrals only.

In (2) $S[A]$ is the Euclidean action of the free photon field,

$$\Gamma^{(1)}[A] = \frac{e^2}{2\pi}\int d^2x A^{\mu}(g_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box})A^{\nu} \tag{3}$$

is equal to the negative logarithm of the determinant of the covariant Dirac operator $\not{D}$ [3], [8] and

$$G(1,2) = e^{-e\gamma_5\phi(1)}G_0(1,2)e^{-e\gamma_5\phi(2)} \tag{4}$$

is the inverse of $\not{D}$. In the last formula

$$G_o(x) = \frac{1}{2\pi r^2}\gamma_\mu x^\mu \tag{5}$$

is the free fermion propagator and the function ϕ parametrizes the vector potential (see Appendix).

Now, using (4), (5) in (2), we obtain $\langle\bar{\Psi}_M\Psi_M\rangle = 0$ which is an obviously erroneous result in the light of the operator solution (1). As shown in [3], [4], the correct magnitude of $\langle\bar{\Psi}_M\Psi_M\rangle$ can be obtained, if we start with the calculation of the expectation value

$$\langle\bar{\Psi}_L(x)\Psi_L(x)\bar{\Psi}_R(y)\Psi_R(y)\rangle = \frac{1}{Z}\int[\mathcal{D}A_\mu]e^{-S[A]-\Gamma^{(1)}[A]}G_{LR}(x,y,A)G_{RL}(y,x,A) \quad ,$$

which for $(x-y) \to \infty$ tends to $(\frac{m'}{2\pi})^2$. From this result, referring to cluster decomposition and the Minkowskian relation $\langle\bar{\Psi}_L\Psi_L\rangle = \langle\bar{\Psi}_R\Psi_R\rangle^*$, we can infer the correct magnitude of the order parameters:

$$|\langle\bar{\Psi}_L(x)\Psi_L(x)\rangle| = |\langle\bar{\Psi}_R(y)\Psi_R(y)\rangle| = \frac{m'}{2\pi} \neq 0 \quad .$$

The method does not permit us to draw any definite conclusion, regarding the phase θ. In particular, it remains obscure whether θ is some fixed number or an arbitrary parameter.

The reason for the failure of (2) to reproduce $\langle\bar{\Psi}_M\Psi_M\rangle$ becomes clear if, by means of compactification of the Euclidean plane [7], [8], one makes the Dirac operator $\not{D}$ to possess a discrete set of eigenfunctions. Actually, fermion functional integrals have no meaning unless in a discrete basis identified with the eigenfunctions of $\not{D}$ [7], [17]. Now, for the validity of (2), $\not{D}$ must not have eigenfunctions with vanishing eigenvalues. On the other hand, in a certain domain of the external field $\not{D}$ does possess zero modes so in the context of compactification or in any equivalent approach the application of (2) must be abandoned. In [7], [8] a beautiful though tedious alternative route still within compactification has been elaborated which reproduces the absolute value of $\langle\bar{\Psi}_M\Psi_M\rangle$. The correct θ dependence can also be obtained provided the additional term

$$\delta S_\theta = i\theta\frac{e}{4\pi}\int d^2x\epsilon^{\mu\nu}F_{\mu\nu}(x)$$

is included into the action.

However, beside compactification the Euclidean plane the Dirac operator $\not{D}$ can be made to have a discrete spectrum also by means of a suitable boundary condition on a circle $\mathcal{C}_R$ whose radius R will be sent to infinity at the final stage of the calculations. The most general local Lorentz-invariant self-adjoint boundary condition is given by the equation[1]

[1] The condition (6) expresses the absence of current across the boundary. In the special case of $\theta = 0$ it has been extensively used in the bag model calculations, see [18], [19]. The symmetry of (6) is easily established. We conjecture that it is also self-adjoint.

$$\Psi = \pm B\gamma_r \Psi \Big|_{r = R} \quad , \tag{6}$$

where

$$B = \begin{pmatrix} -e^{\theta} & 0 \\ 0 & +e^{-\theta} \end{pmatrix}$$

and the Dirac matrix γ_r is defined in the Appendix. The parameter θ is an arbitrary real constant. In what follows we confine ourselves to the upper sign in (6) since boundary conditions of either sign lead to identical final conclusions.

When subjected to the condition (6) the Dirac operator will have only nonzero eigenvalues. In order to show this, we note that the solutions of the equation $\not{D}\Psi = 0$ which are finite within $\mathcal{C}_R$ are of the form

$$H_{Lj} = \frac{c_{Lj}}{\sqrt{r}}\left(\frac{r}{R}\right)^{|j|} e^{e\phi} \chi_{Lj} \qquad (j \le -\tfrac{1}{2}) \quad ,$$

$$H_{Rj} = \frac{c_{Rj}}{\sqrt{r}}\left(\frac{r}{R}\right)^{j} e^{-e\phi} \chi_{Rj} \qquad (j \ge \tfrac{1}{2}) \quad . \tag{7}$$

Here is c_{Mj}-s are arbitrary constants and

$$\chi_{Lj} = \frac{1}{\sqrt{2\pi}} e^{i(j+1/2)\varphi}\binom{1}{0} \quad , \qquad \chi_{Rj} = \frac{1}{\sqrt{2\pi}} e^{i(j-1/2)\varphi}\binom{0}{1} \quad .$$

On the boundary of $\mathcal{C}_R$ an arbitrary zero mode H of $\not{D}$ takes the form

$$H(R,\varphi) = \sum_{j\le -1/2} a_j e^{e\phi(R,\varphi)} \chi_{Lj} + \sum_{j\ge 1/2} b_j e^{-e\phi(R,\varphi)} \chi_{Rj} \quad . \tag{8}$$

We will show that $a_j = b_j = H = 0$.

The equation (8) can be simplified in a gauge in which the function ϕ defined in the Appendix only up to an arbitrary solution of the equation $\Box u = 0$ is fixed by the condition $\phi(R,\varphi) = 0$. In this gauge which will be employed throughout the exponentials drop out of (8). Substituting (8) into (6) and using the relations

$$B\gamma_r \chi_{Lj} = e^{-\theta}\chi_{Rj} \quad , \qquad B\gamma_r \chi_{Rj} = e^{\theta}\chi_{Lj} \quad ,$$

we obtain

$$e^{-\theta}\sum_{j\le -1/2} a_j \chi_{Rj} + e^{\theta}\sum_{j\ge 1/2} b_j \chi_{Lj} = \sum_{j\le -1/2} a_j \chi_{Lj} + \sum_{j\ge 1/2} b_j \chi_{Rj} \quad .$$

The functions χ_{Mj} constitute a complete orthonormal set on the boundary, therefore, all the coefficients a_j, b_j must indeed vanish. Since the eigenvalues of $\not{D}$ are gauge invariant the absence of zero modes is a gauge independent property which permits us to employ the original textbook formula (2) in a calculation of the vacuum expectation value of $\bar{\Psi}\Psi$.

The boundary condition (6) is Euclidean Lorentz-invariant since $[J, B\gamma_r] = 0$, where $J = -i\frac{\partial}{\partial\varphi} + \frac{1}{2}\gamma_5$ is the boost gererator. It is, however, not invariant under separate γ_5 transformations. Therefore, it plays the same role as a γ_5-breaking driving term of the form $m\bar{\Psi}\Psi$. However, from the technical point of view, there are important difference between the two cases. First, when $\not{D}$ is made self-adjoint by means of restricting it to a finite part of the Euclidean plane then as we have just seen the corresponding boundary condition *automatically* breaks γ_5 symmetry. Second, the boundary condition (6) permits an exact treatment while the Schwinger model with a mass-term like driving term is not soluble by any known method.

In order to apply (2), the Green's function $G(1,2)$, obeying (6), has to be found. Obviously, $G(1,2)$ must be of the form

$$G(1,2) = G(1,2) + \sum_{M,j} \varepsilon_{Mj} H_{Mj}(1) H^{+}_{Mj}(2)$$

with $\varepsilon_{Mj} = \pm 1$ and H_{Mj} taken from (7). The boundary condition $G = G\gamma_r G|_R$ fixes uniquely ε_{Mj} and the coefficients c_{Mj} in (7) and we obtain the following result:

$$\begin{aligned} G(1,2) = &- \frac{1}{2\pi r_{12}^2} e^{-e\gamma_5\phi(1)} \gamma_\mu (x_1^\mu - x_2^\mu) e^{-e\gamma_5\phi(2)} - \\ &- \frac{e^{\Theta}}{2\pi R} e^{e[\phi(1)+\phi(2)]} \, \frac{1}{2}(1-\gamma_5) \frac{R^2}{R^2 - r_1 r_2 e^{i(\varphi_1-\varphi_2)}} + \\ &+ \frac{e^{-\Theta}}{2\pi R} e^{-e[\phi(1)+\phi(2)]} \, \frac{1}{2}(1+\gamma_5) \frac{R^2}{R^2 - r_1 r_2 e^{-i(\varphi_1-\varphi_2)}} \quad . \end{aligned} \tag{9}$$

Substituting this expression into (2) we arrive at the formulae:

$$\begin{aligned} \langle \bar{\Psi}_L \Psi_L \rangle &= - \frac{ie^{\Theta}}{2\pi R} \frac{1}{Z} \int [\mathcal{D} A_\mu] e^{-S[A] - \Gamma^{(1)}[A] + 2e\phi(o)} \quad , \\ \langle \bar{\Psi}_R \Psi_R \rangle &= + \frac{ie^{-\Theta}}{2\pi R} \frac{1}{Z} \int [\mathcal{D} A_\mu] e^{-S[A] - \Gamma^{(1)}[A] - 2e\phi(o)} \quad . \end{aligned} \tag{10}$$

Here the effective action $\Gamma^{(1)}[A]$ is the same as before since under (6) its derivation as done e.g. in [7], [8] goes through without change in the gauge $\phi|_R = 0$. The functional integrals in (10) are of Gaussian type so they can be treated by the well known method of shifting the integration variable.

In this way we obtain

$$\langle\bar{\Psi}_L\Psi_L\rangle = -\frac{ie^{\Theta}}{2\pi R}e^{e\bar{\phi}(o)} \quad ; \quad \langle\bar{\Psi}_R\Psi_R\rangle = \frac{ie^{-\Theta}}{2\pi R}e^{-e\bar{\phi}(o)} \quad ,$$

where $\bar{\phi}$ is the shift in the variable ϕ which is connected to the shift in E through the equation $\bar{E} = -\Box\bar{\phi}$. The function $\bar{E}$ which makes the linear term in the exponent of (10) to vanish is found to be

$$\bar{E} = \frac{e}{\pi}K_o(mr) \quad ; \quad m^2 = \frac{e^2}{\pi} \quad .$$

Hence,

$$e\bar{\phi}(o) = \frac{e}{2\pi}\int d^2x \ln(\frac{R}{r})\bar{E}(x) = \ln\frac{mR}{2} + \gamma + \mathcal{O}(e^{-mR}) \quad .$$

We find, therefore, the following final result:

$$\langle\bar{\Psi}_L\Psi_L\rangle = -ie^{\Theta}\cdot\frac{m'}{2\pi} \quad ; \quad \langle\bar{\Psi}_R\Psi_R\rangle = +ie^{-\Theta}\cdot\frac{m'}{2\pi} \tag{11}$$

Except for Θ which in the Minkowski space solution [11], [13], [14] is a pure imaginary quantity rather than a real one (11) gives the correct value of the order parameter $\langle\bar{\Psi}_M\Psi_M\rangle$. Note that the drop out of R of the final result is the consequence of the relation

$$\frac{e}{2\pi}\int d^2x\bar{E}(x) = 1 \quad .$$

It is straightforward to extend the calculation of $\langle\bar{\Psi}_M\Psi_M\rangle$ to the case when a background charge density is also present. Assume that a pair of charges $\pm q$ is sitting at the points $x^1 = \pm a$ $(am >> 1)$. In Euclidean formulation this situation may be represented by the current density $j_\mu = \epsilon_{\mu\rho}\partial^\rho j$ with $j = q\Theta(a-r)$. If we include the additional term $A_\mu j^\mu = \epsilon_{\mu\nu}\partial^\nu\phi\cdot j^\mu$ into the exponents of (10) and repeat the calculation of $\langle\bar{\Psi}_M\Psi_M\rangle$ we obtain the previous result except for Θ which is replaced by $\Theta - 2\pi\frac{q}{e}$. Exactly this rule was obtained earlier in the Minkowski-space operator solution [15], [13]. This coincidence suggests that the parameter Θ in the boundary condition (6) can in fact be identified with the Θ parameter of the operator solution.

If Ψ obeys the condition (6) then the chiral transformed $\Psi' = e^{\alpha\gamma_5}\Psi$ with real α also fulfills the boundary condition with the modified parameter $\Theta' = \Theta + 2\alpha$. In the Euclidean functional integral formulation the Lagrangian is invariant under this chiral rotation since Ψ and $\bar{\Psi}$ are handled as independent variables. However, the eigenvalues of $\not{D}$ depend on Θ so the effective action $\Gamma^{(1)} = -\ln\det\not{D}$ changes under chiral rotations. As it is easy to see, the change of the eigenvalue λ_ℓ under an infinitesimal chiral transformation is equal to

$$\delta\lambda_\ell = 2\lambda_\ell(\varphi_\ell,\gamma_5\varphi_\ell)\delta\alpha \quad ,$$

where φ_ℓ is the corresponding eigenfunction. Hence,

$$\frac{\delta\Gamma^{(1)}}{\delta\alpha} = -\sum_\ell \frac{1}{\lambda_\ell}\frac{\delta\lambda_\ell}{\delta\alpha} = -2\sum_\ell(\varphi_\ell,\gamma_5\varphi_\ell) \quad .$$

Using suitable regularization, this sum has been calculated by Fujikawa [17] in four dimensions. In our case his method gives the following result:

$$\sum_\ell(\varphi_\ell,\gamma_5\varphi_\ell) = \frac{e}{2\pi}\int d^2x E(x) \tag{12}$$

and we arrive at the well known relation

$$\frac{\delta\Gamma^{(1)}}{\delta\theta} = \frac{1}{2}\frac{\delta\Gamma^{(1)}}{\delta\alpha} = -\frac{e}{2\pi}\int d^2x E(x) \quad .$$

This result supports the interpretation of θ and α as the Euclidean counterparts of the Minkowskian θ-parameter and chiral rotation angle respectively in spite of the fact that they differ from the customary usage in an imaginary unit.

The formula (12) might lead to apparent contradictions if no proper account is made of the domain of $\not{D}$. To give an example, we notice that since the eigenvalues λ_ℓ are all different from zero and $\not{D}$ anticommutes with γ_5 φ_ℓ and $\gamma_5\varphi_\ell$ belong to different eigenvalues of $\not{D}$. The equation (12), therefore, seems to express the vanishing of the integral of an arbitrary function $E(x)$. The resolution of this contradiction consists in the observation that one of φ_ℓ and $\gamma_5\varphi_\ell$ obeys (6) with the upper the other with the lower sign. Therefore, they are in fact eigenfunctions of different Hermitean operators and need not be orthogonal to each other.[2]

The formal rule for the continuation of a fermion Green's function from the Euclidean space into the Minkowskian one consists in the replacement of the coordinate x^2 by ix^0 (see Appendix). On the basis of this rule the Minkowskian value of $\langle\bar{\Psi}_M\Psi_M\rangle$ must simply coincide with the Euclidean one. However, our result (11) does not fit completely into this picture since the Minkowskian relation $\langle\bar{\Psi}_L\Psi_L\rangle = \langle\bar{\Psi}_R\Psi_R\rangle^*$ requires purely imaginary θ while in (6) and hence in (11) too θ is a real parameter. On the other hand, we have made plausible that the parameter θ in the boundary condition plays indeed the role of the θ-parameter of the Minkowski space operator solution. Therefore, in the particular case considered in this note the transformation from the Euclidean space to the Minkowskian one requires the continuation of both x^2 and θ to imaginary values. This result casts some doubt on the naive hope that a simple universal connection exists between the two descriptions.[3]

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[2] The analogous problem in a different context has been considered by Kiskis [20].

[3] The authors are indebted to V.N. Gribov for a comment on this point. On the ambiguity in the chiral transformation see also [16].

We note finally that the relation (6) with

$$\gamma_r = \frac{1}{r} x_\mu \gamma_\mu \quad ; \qquad e^{\pm\theta} \to e^{\pm\theta}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

remains a Hermitean boundary condition in four dimensions too and the transformation rule $\theta \to \theta + 2\alpha$ under global chiral rotations is also preserved. This same rule is valid for the θ-parameter of the instanton physics [21], [22] introduced via the gauge field sector rather than through a boundary condition imposed on the fermions. The idea that the two θ parameters are the same naturally suggests itself but the proof would require further study.

APPENDIX

The γ-matrices in the Minkowski space are

$$\gamma^0 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}; \quad \gamma^1 = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}; \quad \gamma_5 = \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix} \quad ; \quad \Psi = \begin{pmatrix}\Psi_L\\ \Psi_R\end{pmatrix}$$

$$g_{00} = -g_{11} = 1 \quad ; \qquad \epsilon_{01} = -\epsilon_{10} = 1 \quad .$$

In Euclidean space γ^0 is replaced by $-i\gamma^2$ while γ^1 and γ_5 remain unaltered. The Euclidean metric is negative definite: $g_{\mu\nu} = -\delta_{\mu\nu}$ and $\epsilon_{12} = -\epsilon_{21} = 1$. The Euclidean matrix γ_r is defined as

$$\gamma_r = \gamma_1 \cos\varphi + \gamma_2 \sin\varphi = \begin{pmatrix}0 & -e^{i\varphi}\\ e^{-i\varphi} & 0\end{pmatrix} \quad ,$$

where r, φ are polar coordinates in the Euclidean plane.

The vector potential in both Minkowskian and Euclidean spaces is parametrized as

$$\begin{aligned} A_\mu &= \partial_\mu \chi + \epsilon_{\mu\nu}\partial^\nu \phi \\ E &= -\Box\phi \end{aligned} \tag{A1}$$

with the transformation rules

$$A_1^E(ix^0,x^1) = A_1^M(x^0,x^1) \quad , \qquad \chi^E(ix^0,x^1) = \chi^M(x^0,x^1) \quad ,$$

$$A_2^E(ix^0,x^1) = -iA_0^M(x^0,x^1) \quad , \qquad \phi^E(ix^0,x^1) = i\phi^M(x^0,x^1) \quad .$$

We choose for simplicity $\chi = 0$ (Landau gauge). The remaining potential ϕ is defined by (A1) up to a solution of the homogeneous equation $\Box u = 0$.

The Feynman-propagator in the Minkowski space is defined as [23]

$$S(x,y) = -i\langle o|T\Psi(x)\bar{\Psi}(y)|o\rangle \qquad \text{(A2)}$$

and obeys the equation

$$\gamma^{\mu}(i\partial_{\mu} - e\epsilon_{\mu\nu}\partial^{\nu}\phi)S(x,y) = \delta(x^{o}-y^{o})\delta(x^{1}-y^{1}) \quad .$$

The transition to Euclidean metric is performed by the replacements described above together with the substitutions $x^{o} = -ix^{2}$, $\delta(x^{o}) = i\delta(x^{2})$. In this manner we obtain the Euclidean Green's function equation

$$\gamma^{\mu}(\partial_{\mu} - ie\epsilon_{\mu\nu}\partial^{\nu}\phi)G(x,y) = \delta(x^{1}-y^{1})\delta(x^{2}-y^{2}) \quad .$$

with $G(x^{1},x^{2};y^{1},y^{2}) = S(x^{1},-ix^{2};y^{1},-iy^{2})$. Owing to the appearance of the imaginary unit in the definition (A2), we have $\langle\bar{\Psi}\Psi\rangle \sim iG$. Notice that the equal time anticommutator of Ψ_{M} and $\bar{\Psi}_{M}$ vanishes.

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