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DETERMINATION OF BARYON
RESONANCE
AND BARYONIC MASSES
FROM QCD SUM RULES.
STRANGE BARYONS

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Abstract

The mass differences in baryonic octet $J^P=1/2^+$, decuplet $J^P=3/2^+$ and in octet $J^P=3/2^-$ are calculated basing on the QCD sum rules. The mass differences are expressed through two QCD parameters: the strange current quark mass and the value of the quark condensate $\langle 0|\bar{s}s|0\rangle$. At the properly chosen values of these parameters all of the mass differences are in a good agreement with experiment.

I. Introduction

This paper is a continuation of paper [1] (hereafter it is cited as I) which was dedicated to study QCD sum rules for non-strange baryons. We will study here mass differences in baryonic multiplets whose nonstrange terms were considered in I.

The problem of mass splitting in baryonic multiplets has been attracted attention for a long time. In the framwrok of broken SU(3) symmetry Gell-Mann and Okubo have obtained formulae which were in a good agreement with experiment. But these formulae contain phenomenological parameters to be determined from experiment for each multiplet. Therefore of interest is a study of the same problem from viewpoint of the QCD sum rules suggested by Shifman, Vainshtein and Zakharov [2] to determine the strange baryon masses via strange quark mass and QCD vacuum properties. Such calculations hav been made in [3] for nucleon octet and decuplet of Δ-isobar. It has been shown that strange baryon masses were determined not only by the strange quark mass but by the difference of the mean vacuum values $\langle 0|\bar{s}s|0\rangle - \langle 0|\bar{u}u|0\rangle \neq 0$ too. The consideration made in [3], though it exhibited a principal possibility to calculate mass differences in baryonic multiplets basing on the QCD sum rules, was not still quite confident and its results were not sufficiently accurate since the calculations did not take into account the continuum contribution. This shortcoming has been corrected in the given paper. In addition, higher power corrections and perturbation theory corrections in the leading logarithmic approximation are taken into account. All of this made it possible to essentially improve the calculation accuracy and to get relieable results for the baryon mass splittings. The calculation of the baryon mass splitting is of interest also from another point of view: at the experimentally known mass differences it allows one to determine two parameters entering the theory, i.e. the strange quark mass and the difference of the mean vacuum values $\langle 0|\bar{s}s|0\rangle - \langle 0|\bar{u}u|0\rangle$ and to verify the selfconsistency of the whole approach.

2. Nucleon Octet. $J^P = 1/2^+$

In this Section, the method of the calculation of mass differences in the given multiplet will be described by an example of a nucleon octet. To calculate the nucleon mass one can use the following baryonic current:

$$\eta = \varepsilon^{abc}\left[(u^a C d^b)u^c - (u^a C\gamma_5 d^b)\gamma_5 u^c\right] \tag{1}$$

which differs from the current $\eta_{1\mu}$ in I by a factor γ_μ. Here a, b, c are colour indeces of quarks, ε^{abc} is antisymmetric tensor, μ - is the Lorenz index, $C=-C^T$ is the charge conjugation matrix, u and d are operators of u and d quarks.

To determine Σ and Ξ masses one needs currents with their quantum numbers. These currents can be easily obtained applying SU(3) transformation to (1)

$$\begin{aligned}\eta^{\Sigma^+} &= \varepsilon^{abc}\left[(u^a C s^b)u^c - (u^a C\gamma_5 s^b)\gamma_5 u^c\right]\\ \eta^{\Xi^0} &= \varepsilon^{abc}\left[(s^a C u^b)s^c - (s^a C\gamma_5 u^b)\gamma_5 s^c\right]\end{aligned} \tag{2}$$

where s is the strange quark operator.

Just as in [1,3], the sum rules are obtained by considering the polarization operator $\Pi(q)$ of quark currents (1) or (2), by representing the structure functions of polarization operator through dispersion relations in q^2 and by applying to such dispersion relations the Borel transformations. Referring the reader for the details of the method to our preceding paper, we dwell here on new moments appearing in account of the strange quark mass and of nonzero difference of the mean vacuum values $\langle 0|\bar{s}s|0\rangle - \langle 0|\bar{q}q|0\rangle$, q=u,d. Just as in I, u and d quarks will be assumed to be massless and, due to isotopic invariance, it is put $\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle$. In the calculations we account for the same power corrections as in I, i.e. for the same set of Feynman diagrams.

Let us introduce parameters characterizing the value of the SU(3) symmetry violation in the mean vacuum values

$$f = \frac{\langle 0|\bar{s}s|0\rangle}{\langle 0|\bar{u}u|0\rangle} - 1 ; \quad f_g = \frac{\langle 0|\bar{s}\sigma_{\mu\nu}\lambda^n G^n_{\mu\nu} s|0\rangle}{\langle 0|\bar{u}\sigma_{\mu\nu}\lambda^n G^n_{\mu\nu} u|0\rangle} - 1 \tag{3}$$

The polarization operator and, consequently, the baryon masses will be calculated in the linear approximation in m_s, f, and f_g. At phenomenological approach this corresponds to the symmetry violation due to the $(\bar{3},3)$ component of octet in the Hamiltonian $H=H_0 + H^3_3$. The hypothesis that baryon mass splittings in multiplets are described by the term H^3_3 in the linear approximation is the basis for the Gell-Mann-Okubo mass formulae. The success of these formulae justifies, thereby, our assumption.

In the linear approximation in m_s the strange quark propagator in x-representation equals to

$$-iS_F(x) = \frac{i\hat{x}}{2\pi^2 x^4} - \frac{m_s}{4\pi^2 x^2} \tag{4}$$

and proportional to m_s terms in the vacuum averages of the quark field product are

$$\langle 0|:S_\alpha^a(x),\ \bar{S}_\beta^b(0):|0\rangle = i\frac{m_s}{48}\,\delta^{ab}\hat{x}_{\alpha\beta}\Big[\langle 0|\bar{S}S|0\rangle + \frac{g x^2}{3\cdot 2^3}\langle 0|\bar{S}\sigma_{\mu\nu}\frac{\lambda^n}{2}G^n_{\mu\nu}S|0\rangle + \ldots\Big] + \tag{5}$$

$$+\ O(m_s^2)\ +\ m_s\text{-independent terms.}$$

In (5) only terms proportional to m_s are written explicitly, the m_s-dependent terms are given by formula (7.1).

Imaginary part of the polarization operator expressed via physical states is approximated by the sum of the lowest state contributions (in this case N, Λ, Σ or Ξ) and of continuum, i.e.

$$Im\,\Pi(q) = \pi\beta_R^2(\hat{q} - m_R)\,\delta(q^2 - m_R^2) +$$

$$+\ \pi\,\theta(q^2 - W_1^2)\,\hat{q}\,Im\,\Pi_1(q^2) + \pi\,\theta(q^2 - W_2^2)\,Im\,\Pi_2(q^2) \tag{6}$$

where β_R is the baryon residue into the currents (1) and (2)

$$\langle 0|\,\eta\,|R,\, J^P = 1/2^+\rangle = \beta_R\,\gamma_5\,v(q) \tag{7}$$

W_1 and W_2 are continuum thresholds for the structures with odd and even number of γ-matrices ($\hat{q}$ and I), $Im\,\Pi_1(q^2)$ and $Im\,\Pi_2(q^2)$ are imaginary parts of polarization operators at these structures.

The sum rules for nucleon considered in I can be written as

$$\hat{q}:\quad I_1(M, W_1) = \beta_N^2\, e^{-m_N^2/M^2}$$

$$1:\quad I_2(M, W_2) = \beta_N^2\, m_N\, e^{-m_N^2/M^2} \tag{8}$$

where $\tilde{\beta}_N = (2\pi)^2 \beta_N$, m_N is the nucleon mass and continuum contribution is transferred to the l.h.s. From the left of each of the sum rule the structure is marked to which it corresponds. In case of hyperons ($Y = \Lambda, \Sigma, \Xi$) the account of corrections breaking SU(3) symmetry, in the linear in m_s, f and f_g approximation leads to replacing the sum rules (8) by

$$\hat{q}: \; I_1(M, W_1) - \delta_{1Y}(M, m_s, f, f_g, \delta W_{1Y}) = \tilde{\beta}_Y^2 e^{-m_Y^2/M^2}$$
$$1: \; I_2(M, W_2) + \delta_{2Y}(M, m_s, f, f_g, \delta W_{2Y}) = \beta_Y^2 m_Y e^{-m_Y^2/M^2} \quad (9)$$

where I_1 and I_2 are the same functions as in (8), β_Y - is the hyperon residue into the current η^Y, δ_1 and δ_2 contain terms proportional to m_s, f, f_g and corrections δW_Y related to continuum variation: $W_{1Y} = W_1 + \delta W_{1Y}$, $W_{2Y} = W_2 + \delta W_{2Y}$ (in the linear approximation in δW_Y). The fact that continuum thresholds are different for different terms of SU(3) multiplets is physically evident and it is absolutely necessary to take this effect into account. Thereby, the calculation is supplemented by additional parameters which must be determined from the same sum rules. Naturally, this leads to a deteriaration of accuracy. As will be seen, the theoretically obtained δW_Y are "reasonable" in comparison with the experimental data. Aiming to decrease the number of parameters we will put in the following $\delta W_{1Y} = \delta W_{2Y} = \delta W_Y$.

Using (8) and (9) one can readily obtain the following equalities valid in the linear in m_s, f, f_g, δW_Y and $\delta\tilde{\beta}_Y^2 = \tilde{\beta}_Y^2 - \tilde{\beta}_N^2$ approximation

$$m_Y - m_N = \frac{\delta_{2Y}(M) + \delta_{1Y}(M)\, m_N}{\tilde{\beta}_N^2\, e^{-m_N^2/M^2}} \tag{10}$$

It must be emphasized that expression (1C) is fulfilled only in the region in M where the sum rules (8) and (9) are fulfilled. From I this region Ω' is known:*) $\Omega' = (M: 0.9\,GeV \le M \le 1.2\,GeV)$. The mass, nucleon residue and continuum thresholds are also kown from I.

The further actions are sketched as follows. The calculation of corrections to polarization operator in QCD yields $\delta_{1Y}(M)$ and $\delta_{2Y}(M)$ as linear functions from m_s, f, f_g and δW_Y. The substitution of $\delta_{1Y}(M)$ and $\delta_{2Y}(M)$ into (10) leads to analogous expressions for $m_Y - m_N$

$$m_Y - m_N = a_Y(M)\, m_s + b_Y(M) f + c_Y(M) f_g + d_Y(M)\, \delta W_Y = \\ = K_{Y-N}(M) \tag{11}$$

with the known coefficients $a_Y(M)$, $b_Y(M)$, $c_Y(M)$, $d_Y(M)$. The l.h.s of (11) is independent of the Borel parameter M, the r.h.s is, generally speaking, dependent. Therefore, unknown parameters δW_Y is reasonable to determine by the requirement of minimum M dependence of the r.h.s of (11) in the region $\Omega\,(M_1 \le M \le M_2)$. We can formally treat δW_Y as linear functions of m_s, f, f_g. Then, to provide the weak dependence of $K_{Y-N}(M)$ within $M_1 \le M \le M_2$ it is enough to require

$$K_{Y-N}(M_1) = K_{Y-N}(M_2) \tag{12}$$

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*) In contrast to , (defined in the preceding paper I) is the region in M where the sum rules for neutrino baryons are fulfilled. It can be easily determined from the graphs of ref. I.

and to determine δW_Y from this condition. Substituting δW_Y into (11), we get relations expressing $m_Y - m_N$ via three QCD parameters : m_s, f and f_g. The problem thereby reduces to finding the values of m_s, f and f_g which give the best fit for the whole set of experimental data on mass splittings in the nucleon octet $J^P=1/2^+$ ($m_\Sigma - m_N$, $m_\Xi - m_N$), decuplet $J^P=3/2^+$ ($m_{\Sigma^*} - m_\Delta$) and octet $J^P=3/2^-$ ($m_{\Sigma^{**}} - m_{N^*}$, $m_{\Xi^{**}} - m_{N^*}$). As will be seen from the answers, the results turn out to be very wekkly dependent of f_g , so that practically all of the mass splittings are determined by two quantities - m_s and f .

The corrections to polarization operator proportional to m_s are calculated by substituting into fig.1 graphs the propagators and vacuum averages according to eqs.(4),(5). The terms proportional to f and f_g can be easily taken into account if one notes that in the terms with even numbers of γ-matrices in the polarization operator the vacuum averages must be substituted by $\langle 0|\bar{s}s|0\rangle$ in case of Σ and by $\langle 0|\bar{u}u|0\rangle$ in case of Ξ , while in the terms with odd number of γ-matrices - by $\langle 0|\bar{u}u|0\rangle^2$ in case of Σ and by $\langle 0|\bar{s}s|0\rangle^2$ in case of Ξ .

The result of the calculation is

$$
\begin{aligned}
\Pi^a &= \frac{m_s}{(2\pi)^4} q^4 \ln{-q^2/\Lambda^2} \times \frac{1}{8} \; (\times 0) \\
\Pi^b &= -\frac{m_s}{(2\pi)^2} \hat{q} \langle 0|\bar{u}u|0\rangle \ln{-q^2/\Lambda^2} \times \frac{1}{4} \; (\times 0) \\
\Pi^c &= \frac{m_s}{q^2} \langle 0|\bar{u}u|0\rangle^2 \times \frac{1}{3} \; \left(\times \frac{2}{3}\right) \\
\Pi^d &= -\frac{m_s}{q^2} \hat{q} m_0^2 \langle 0|\bar{u}u|0\rangle \times \frac{1}{24} \; \left(\times \frac{1}{12}\right) \\
\Pi^e &= -f \frac{\langle 0|\bar{u}u|0\rangle}{(2\pi)^2} q^2 \ln{-q^2/\Lambda^2} \times \frac{1}{4} \; (\times 0) \\
\Pi^f &= -f \hat{q} \frac{\langle 0|\bar{u}u|0\rangle^2}{q^2} \times 0 \; \left(\times \frac{1}{3}\right) \\
\Pi^g &= 0
\end{aligned}
\qquad (13)
$$

where letters by polarization operators show which diagrams were used at their calculation; the factor without brackets corresponds to the Σ_n case, in the brackets to Ξ ; $m_0^2\langle 0|\bar{u}u|0\rangle \equiv g_s\langle 0|\bar{u}G_{\mu\nu}^n\frac{\lambda^n}{2}\sigma_{\mu\nu}u|0\rangle$. Corrections due to $\delta_{1\Sigma}(M)$ and $\delta_{2\Sigma}(M)$ are

$$\delta_{1\Sigma}(M) = m_s\left(\frac{1}{4}aM^2L^{4/9}E_2(W_1,M) + \frac{m_0^2 a}{24}\right) - \delta W_\Sigma\frac{L^{4/9}}{8}W_1 e^{-W_1^2/M^2}\left(W_1^4 + \frac{b}{2}\right)$$

$$\delta_{2\Sigma}(M) = m_s\left(\frac{M^6}{4}E_6(W_2,M) + \frac{a^2}{3}L^{8/9}\right) + f\frac{a}{4}M^4L^{8/9}E_4(W_2,M) + \delta W_\Sigma\frac{1}{2}aW_2^2L^{8/9}e^{-W_2^2/M^2} \tag{14}$$

where $L = \ln(M/\Lambda)/\ln(\mu/\Lambda)$; Λ is the strong interaction constant entering the definition of the QCD effective charge $\alpha_s(q^2) = \frac{4\pi}{9\ln(-q^2/\Lambda^2)}$ (in case of three flavours); μ is the normalization point of the operator expansion

$$E_2(W,M) = 1 - \exp\left(-\frac{W^2}{M^2}\right); \quad E_4(W,M) = 1 - \left(1 + \frac{W^2}{M^2}\right)\exp\left(-\frac{W^2}{M^2}\right)$$

$$E_6(W,M) = 1 - \left(1 + \frac{W^2}{M^2} + \frac{W^4}{2M^4}\right)\exp\left(-\frac{W^2}{M^2}\right)$$

$$a = -(2\pi)^2\langle 0|\bar{u}u|0\rangle = 0.546\ \mathrm{GeV}^3$$

$$b = (2\pi)^2\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle = 0.5\ \mathrm{GeV}^4; \quad m_0^2 = 0.8\ \mathrm{GeV}^2 \tag{15}$$

$$\Lambda = 150\ \mathrm{MeV}; \quad \mu = 0.5\ \mathrm{GeV}$$

Formulae (14) are written in the leading logarithmic approximation accounting for anomalous dimensions of the currents (1),(2) ($d = -4/9$), for the operators $\bar{\psi}\psi$ ($d = -4/9$), $\frac{\alpha_s}{\pi}G_{\mu\nu}^n G_{\mu\nu}^n$ ($d=0$), and for the quark mass m_s ($d=4/9$).

Substituting (14) into (10) and using the obtained in I values of the nucleon mass $m_N = 1$ GeV, of its residues to the quark current (1) $\tilde{\beta}_N^2 = 0.43$ GeV^6 and of the continuum thresholds $W_1 = W_2 = 1.5$ GeV, we come to the following numerical expressions for the quantity $K_Y(M)$ defined in (11) in case of Σ -hyperon

$$K_\Sigma(M=0.9) = 2.82 m_s + 0.78 f + 0.06\,\delta W_\Sigma$$
$$K_\Sigma(M=1.2) = 2.48 m_s + 0.99 f + 0.23\,\delta W_\Sigma \qquad (15a)$$

All of the values in (15) are expressed in GeV's. Excluding δW_Σ from (15) according to (12), we get the following expression for the mass difference of Σ-hyperon and nucleon:

$$m_\Sigma - m_N = 2.94 m_s + 0.71 f \quad (GeV) \qquad (16)$$

Proceeding in the same manner, in case of Ξ -hyperon we have

$$\delta_{1\Xi}(M) = m_s \frac{m_0^2 a}{12} - f\frac{a^2}{3} L^{4/9} - \frac{1}{8}\,\delta W_\Xi\, W_1 L^{4/9} e^{-W_1^2/M^2}\left(W_1^4 + \frac{b}{2}\right)$$
$$\delta_{2\Xi}(M) = m_s \frac{L^{8/9} a^2}{2} + \frac{1}{2}\,\delta W_\Xi\, a W_2^3 L^{8/9} e^{-W_2^2/M^2} \qquad (17)$$

From (10),(17) there follow numerical equalities

$$K_\Xi(M=0.9) = 1.38 m_s - 1.34 f + 0.06\,\delta W_\Xi$$
$$K_\Xi(M=1.2) = 1.29 m_s - 0.94 f + 0.23\,\delta W_\Xi \qquad (18)$$

Excluding δW_Ξ from (18) we arrive at the formula

$$m_\Xi - m_N = 2.22 m_s - 1.48 f \qquad (19)$$

It is seen from comparison of (16) and (19) that for Ξ would be heavier than Σ, it is necessary to have $f<0$ and $|f| \sim m_s$ (in GeV's). This conclusion will be confirmed by consideration of other sum rules.

The Gell-Mann-Okubo formula for the baryon octet enables one to obtain from (16) and (19) the mass difference of Λ-hyperon and nucleon

$$m_\Lambda - m_N = \frac{1}{3}\left[2(m_\Xi - m_N) - (m_\Sigma - m_N)\right] =$$
$$= 0.5\, m_s - 1.22 f \quad (GeV) \qquad (20)$$

Comparing (16),(19),(20) with the experimental values of the mass splittings in the nucleon octet $(m_\Lambda - m_N)_{exp.} = 0.165$ GeV, $(m_\Sigma - m_N)_{exp.} = 0.250$ GeV, $(m_\Xi - m_N) = 0.380$ GeV, one can determine the strange quark masses (at the normalization point $\mu = 0.5$ GeV) and of the parameter f (3)

$$m_s = 108 \text{ MeV}, \qquad f = -0.096 \qquad (21)$$

and the variations of the continuum thresholds

$$\delta W_\Lambda = 230 \text{ MeV}, \qquad \delta W_\Sigma = 340 \text{ MeV}, \qquad \delta W_\Xi = 520 \text{ MeV} \qquad (22)$$

The values δW_Y (22) seem to be reasonable in the sense they coincide up to 100-150 MeV with the mass differences $(m_{Y^*} - m_Y) - (m_{N^*} - m_N)$ where Y^* and N^* are the lowest resonance states with the same quantum numbers as Y and nucleon N (except for parity, see [5]). Even if δW_Y (22) contain a noticeable error, this error weakly affects m_s and f, since δW_Y enter (16) and (18) with small coefficients. The value of the strange quark mass appeared to be somewhat smaller than usually adopted m_s=150 MeV. Therefore, it is reasonable to understand if it is possible to get this standard value of m_s at the expence of uncertainties in δW_Y , and of rejecting accurate fulfillment of eq.(12). The analysis shows that using these two circumstances only, one can draw m_s up to 130 MeV (then $f = -0.13$) but it is hardly possible to get it larger.

3. Decuplet $J^P = 3/2^+$

In this Section, following the method described in Sec.2, we consider the mass splittings in the decuplet $J^P=3/2^+$. As there is the relation

$$m_{\Sigma^*} - m_{\Delta} = m_{\Xi^*} - m_{\Sigma^*} = m_{\Omega} - m_{\Xi^*} \tag{23}$$

then to describe the masses in the baryon decuplet it is enough to find only the difference $m_{\Sigma^*} - m_{\Delta}$.

Papers [1,3] used the current

$$\eta_\mu^{\Delta} = \varepsilon^{abc} (u^a C \gamma_\mu u^b) u^c \tag{24}$$

to determine the Δ-isobar mass. Applying to (24) the SU(3) transformation, it is easy to find the current with the Σ^* quantum numbers

$$\eta_\mu^{\Sigma^*} = \frac{1}{\sqrt{3}} \varepsilon^{abc} \left[2 (u^a C \gamma_\mu s^b) u^c + (u^a C \gamma_\mu u^b) s^c \right] \tag{25}$$

With the help of the current (25) we calculate $m_{\Sigma^*} - m_{\Delta}$ by the above described method. The corrections due to the polarization operator in our approximation are

$$\Pi_{\mu\nu}^{a} = \frac{m_s}{4(2\pi)^4} q^4 \ln\frac{-q^2}{\Lambda^2} \left\{ g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3}\frac{q_\nu\gamma_\mu - q_\mu\gamma_\nu}{q^2}\hat{q} - \frac{2}{3}\frac{q_\mu q_\nu}{q^2} \right\}$$

$$\Pi_{\mu\nu}^{b} = -\frac{m_s}{(2\pi)^2} \langle 0|\bar{u}u|0\rangle \left\{ \ln\frac{-q^2}{\Lambda^2} \left[g_{\mu\nu}\hat{q} - \frac{3}{8}\gamma_\mu\gamma_\nu\hat{q} + \frac{3}{8}(q_\nu\gamma_\mu - q_\mu\gamma_\nu) - \frac{1}{8}(q_\nu\gamma_\mu + q_\mu\gamma_\nu\right] - \frac{q_\mu q_\nu}{q^2} \right\}$$

$$\Pi_{\mu\nu}^{c} = \frac{2}{3} m_s \frac{\langle 0|\bar{u}u|0\rangle^2}{q^2} \left\{ g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3}\frac{q_\nu\gamma_\mu - q_\mu\gamma_\nu}{q^2}\hat{q} - \frac{2}{3}\frac{q_\mu q_\nu}{q^2} \right\} \tag{26}$$

$$\Pi_{\mu\nu}^{d} = \frac{1}{2} m_s \frac{m_0^2 \langle 0|\bar{u}u|0\rangle}{(2\pi)^2 q^2} \left\{ g_{\mu\nu}\hat{q} - \dots \right\}$$

$$\Pi^{e}_{\mu\nu} = -\frac{4}{9} f \frac{\langle 0|\bar{u}u|0\rangle}{(2\pi)^2} q^2 \ln{-q^2/\Lambda^2} \left\{ g_{\mu\nu} - \frac{5}{16}\gamma_\mu\gamma_\nu + \frac{1}{4}\frac{q_\nu\gamma_\mu - q_\mu\gamma_\nu}{q^2}\hat{q} - \frac{1}{2}\frac{q_\mu q_\nu}{q^2} \right\}$$

$$\Pi^{f}_{\mu\nu} = \frac{8}{9} f \frac{\langle 0|\bar{u}u|0\rangle^2}{q^2} \left\{ g_{\mu\nu}\hat{q} - \frac{3}{8}\gamma_\mu\gamma_\nu\hat{q} + \frac{3}{8}(q_\nu\gamma_\mu - q_\mu\gamma_\nu) - \frac{1}{8}(q_\nu\gamma_\mu + q_\mu\gamma_\nu) \right\}$$

$$\Pi^{g}_{\mu\nu} = \frac{2}{9} f_g \frac{m_0^2\langle 0|\bar{u}u|0\rangle}{(2\pi)^2} \left\{ \ln{-q^2/\Lambda^2} \left[g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu \right] + \frac{1}{2}\frac{q_\nu\gamma_\mu - q_\mu\gamma_\nu}{q^2}\hat{q} - \frac{q_\mu q_\nu}{q^2} \right\}$$

In [3] it was shown that the sum rules for the structures $g_{\mu\nu}\hat{q}$ and $g_{\mu\nu}$ are contributed only by the resonances with spin 3/2. Applying to these structures the above developed methods, for $\delta_{1\Sigma^*}(M)$ and $\delta_{2\Sigma^*}(M)$ determined by the formulae analogous to (9) we have

$$\delta_{1\Sigma^*}(M) = m_s \left[\frac{1}{2} a m_0^2 L^{-16/27} - a M^2 L^{-4/27} E_2(W_1, M) \right] -$$
$$- \frac{8}{9} f a^2 L^{20/27} - \frac{\delta W_{\Sigma^*}}{10} W_1 L^{-4/27} e^{-W_1^2/M^2} \left[W_1^4 - \frac{25}{18} b \right]$$
$$\delta_{2\Sigma^*}(M) = m_s \left[\frac{1}{2} M^6 E_6(W_1, M) L^{-16/27} + \frac{2}{3} a^2 L^{8/27} \right] +$$
$$+ \frac{4}{9} f a M^4 L^{8/9} E_4(W_2, M) + \delta W_{\Sigma^*} a W_2 e^{-W_2^2/M^2} \left[\frac{8}{3} W_2^2 L^{8/27} - \right. \tag{27}$$
$$\left. - \frac{4}{3} m_0^2 L^{-4/27} \right] - \frac{2}{9} f_g m_0^2 a M^2 L^{-4/27} E_2(W_2, M)$$

Expressions (27) take into account the anomalous dimension of the currents (22),(25) which is +2/27. They should be substituted into equality analogous to (10)

$$m_{\Sigma^*} - m_\Delta = \frac{\delta_{2\Sigma^*}(M) + \delta_{1\Sigma^*}(M)\, m_\Delta}{\tilde{\lambda}_\Delta^2\, e^{-m_\Delta^2/M^2}} \tag{28}$$

where m_Δ and $\tilde{\lambda}_\Delta$ are the mass and residues of the isobar Δ.

According to I: $\Omega' = (M: 1.1\ \mathrm{GeV} \leq M \leq 1.4\ \mathrm{GeV})$, $m_\Delta = 1.35$ GeV, $\tilde{\lambda}_\Delta^2 = 2.5\ \mathrm{GeV}^6$, $W_1 = 2.1$ GeV, $W_2 = 2.2$ GeV. As a result we come to the following numerical expressions

$$K_{\Sigma^*}(1.1) = 0.25\, m_s - 0.24 f - 0.15 f_g + 0.78\, \delta W_{\Sigma^*}$$

$$K_{\Sigma^*}(1.4) = 0.42\, m_s + 0.23 f - 0.12 f_g + 1.07\, \delta W_{\Sigma^*} \qquad (29)$$

Excluding from (28) δW_{Σ^*}, we have

$$m_{\Sigma^*} - m_\Delta = -0.21\, m_s - 1.5 f - 0.23 f_g \qquad (30)$$

At previously determined values of m_s and f (21) the mass splitting in the decuplet is

$$m_{\Sigma^*} - m_\Delta = (0.124 - 0.23 f_g)\ \mathrm{GeV} \qquad (31)$$

in comparison with the experimental value $m_{\Sigma^*} - m_\Delta = 0.150$ GeV. It is natural to think that f_g is of the same order as f, i.e. of the scale of a usual SU(3) violation, $|f_g| \sim 0.1$. Then the second term in (31) is inessential and (31) is in a good agreement with experiment.

In the mass splitting in decuplet the continuum contribution is much more than in the mass splitting in the nucleon octet (cf. (15),(16) with (29)), therefore the resulting uncertainty is larger as well. The value of the continuum threshold change obtained from (29) is, however, quite a reasonable

$$\delta W_{\Sigma^*} = -0.59\, m_s - 1.62 f - 0.1 f_g = (0.090 - 0.1 f_g)\ \mathrm{GeV} \qquad (32)$$

at the same values of m_s and f. (Experimentally [5], the first excite d states with the quantum numbers Σ^* are only slightly higher than those with quantum numbers Δ).

4. Baryonic Octet $J^P = 3/2^-$

In I, making use of the current

$$\eta_{2\mu} = \varepsilon^{abc}\left[(u^a C \sigma_{\rho\lambda} d^b)\sigma_{\rho\lambda}\gamma_\mu u^c - (u^a C \sigma_{\rho\lambda} u^b)\sigma_{\rho\lambda}\gamma_\mu d^c\right] \qquad (33)$$

the values of the mass and residue of resonance $N^*(J^P=3/2^-, J=1/2)$ and of the continuum thresholds

$$M_{N^*}=1.6 \text{ GeV}, \qquad \tilde{\lambda}^2_{N^*}= 60 \text{ GeV}^6, \; W_1=2.6 \text{ GeV}, \; W_2=2.2 \text{ GeV} \qquad (34)$$

have been found. The baryonic currents belonging to the same octet as (33) with the quantum numbers Σ^{**} and Ξ^{**} are

$$\eta^{\Sigma^{**}}_{2\mu} = \varepsilon^{abc}\left[(u^a C \sigma_{\rho\lambda} s^b)\sigma_{\rho\lambda}\gamma_\mu u^c - (u^a C \sigma_{\rho\lambda} u^b)\sigma_{\rho\lambda}\gamma_\mu s^c\right]$$

$$\eta^{\Xi^{**}}_{2\mu} = \varepsilon^{abc}\left[(s^a C \sigma_{\rho\lambda} u^b)\sigma_{\rho\lambda}\gamma_\mu s^c - (s^a C \sigma_{\rho\lambda} s^b)\sigma_{\rho\lambda}\gamma_\mu u^c\right] \qquad (35)$$

The corrections to the polarization operators for these currents calculated according to diagrams of fig.1, are

$$\Pi^a_{\mu\nu} = \frac{m_s}{(2\pi)^4} q^4 \ln\frac{-q^2}{\Lambda^2}\left\{g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3}\frac{q_\nu\gamma_\mu - q_\mu\gamma_\nu}{q^2}\hat{q} - \frac{2}{3}\frac{q_\mu q_\nu}{q^2}\right\} \times 3 \; (\times 12)$$

$$\Pi^b_{\mu\nu} = m_s\frac{\langle 0|\bar{u}u|0\rangle}{(2\pi)^2}\Big\{\ln\frac{-q^2}{\Lambda^2}\Big[g_{\mu\nu}\hat{q} - \frac{3}{8}\gamma_\mu\gamma_\nu\hat{q} + \frac{3}{8}(q_\nu\gamma_\mu - q_\mu\gamma_\nu) - $$
$$- \frac{1}{8}(q_\nu\gamma_\mu + q_\mu\gamma_\nu)\Big] - \frac{1}{5}\left(\frac{1}{2}\right)\frac{q_\mu q_\nu}{q^2}\hat{q}\Big\} \times 40 \; (\times 32)$$

$$\Pi^c_{\mu\nu} = \frac{m_s}{q^2}\langle 0|\bar{u}u|0\rangle^2\left\{g_{\mu\nu} - \ldots\right\} \times 16 \; (\times 32) \qquad (36)$$

$$\Pi^d_{\mu\nu} = -\frac{m_s}{(2\pi)^2}\frac{m_0^2\langle 0|\bar{u}u|0\rangle}{q^2}\left\{g_{\mu\nu}\hat{q} - \ldots\right\} \times \frac{73}{3}\left(\times\frac{-9}{3}\right)$$

$$\Pi^e_{\mu\nu} = -f\frac{\langle 0|\bar{u}u|0\rangle}{(2\pi)^2} q^2 \ln\frac{-q^2}{\Lambda^2}\left\{g_{\mu\nu} - \ldots\right\} \times \frac{16}{3}\left(\times\left(-\frac{64}{3}\right)\right)$$

$$\Pi^{f}_{\mu\nu} = -f\,\frac{\langle 0|\bar{u}u|0\rangle^2}{q^2}\left\{ g_{\mu\nu}\hat{q} - \ldots \right\} \times \frac{64}{3}\left(\times\left(-\frac{32}{3}\right)\right)$$

$$\Pi^{g}_{\mu\nu} = f_g\,\frac{m_0^2\langle 0|\bar{u}u|0\rangle}{(2\pi)^2}\left\{ \ln -q^2/\mu^2 \left[g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu \right] + \ldots \right\} \times \frac{8}{3}\left(\times\left(-\frac{32}{3}\right)\right)$$

where factors without brackets correspond to the case of Σ^{**}, in the brackets to Ξ^{**}. For the sake of simplicity, in some of expressions (36) we leave only the terms proportional to $g_{\mu\nu}\hat{q}$, $g_{\mu\nu}$.

Just as in the decuplet case, in this Section we consider the sum rules for the structures $g_{\mu\nu}$ and $g_{\mu\nu}\hat{q}$.

From (36) there follow the expressions for $\delta_1(M)$ and $\delta_2(M)$ in Σ^{**} and Ξ^{**} case

$$\delta_{1\Sigma^{**}}(M) = m_s\left[40\,a M^2 L^{-4/27} E_2(W_1,M) - \frac{73}{3} a\, m_0^2 L^{-16/27}\right] +$$
$$+ f\,\frac{64}{3} a^2 L^{20/27} - \delta W_{\Sigma^{**}} W_1 L^{-4/27} e^{-W_1^2/M^2}\left[\frac{24}{5} W_1^4 - \frac{4}{3} b\right]$$
$$\delta_{2\Sigma^{**}}(M) = m_s\left[-6M^6 L^{-16/27} E_6(W_2,M) - 16\, a^2 L^{8/27}\right] -$$
$$- f\,\frac{16}{3} a M^4 L^{8/27} E_4(W_2,M) + f_g\,\frac{8}{3} a\, m_0^2 M^2 L^{-4/27} E_2(W_2,M) +$$
$$+ \delta W_{\Sigma^{**}} W_2\, e^{-W_2^2/M^2}\left[32\, a W_2^2 L^{8/27} - 16\, a\, m_0^2 L^{-4/27}\right]$$

(37)

$$\delta_{1\Xi^{**}}(M) = m_s\left[32\, a M^2 L^{-4/27} E_2(W_1,M) - \frac{59}{3} a\, m_0^2 L^{-16/27}\right] -$$
$$- f\,\frac{32}{3} a^2 L^{20/27} - \delta W_{\Xi^{**}} W_1\, e^{-W_1^2/M^2} L^{-4/27}\left[\frac{24}{5} W_1^4 - \frac{4}{3} b\right]$$
$$\delta_{2\Xi^{**}}(M) = m_s\left[24 M^6 L^{-16/27} E_6(W_2,M) - 32\, a^2 L^{8/27}\right] +$$
$$+ f\,\frac{64}{3} a M^4 L^{8/27} E_4(W_2,M) - f_g\,\frac{32}{3} a\, m_0^2 M^2 L^{-4/27} E_2(W_2,M) +$$
$$+ \delta W_{\Xi^{**}} W_2\, e^{-W_2^2/M^2}\left[32\, a W_2^2 L^{8/27} - 16\, a\, m_0^2 L^{-4/27}\right]$$

In (37), the anomalous dimension of the current $\eta_{2\mu}$ (+2/27) was taken into account.

Relations (11) written at the points M=1.5 and 1.8 GeV which, according to I, belong to Ω', now have the form

$$K_{\Sigma^{**}}(1.5) = 1.73 m_s + 0.28 f + 0.11 f_g - 0.85\, \delta W_{\Sigma^{**}}$$
$$K_{\Sigma^{**}}(1.8) = 1.36 m_s + 0.03 f + 0.1 f_g - 1.15\, \delta W_{\Sigma^{**}}$$
$$K_{\Xi^{**}}(1.5) = 5.23 m_s + 1.91 f - 0.44 f_g - 0.85\, \delta W_{\Xi^{**}} \qquad (38)$$
$$K_{\Xi^{**}}(1.8) = 5.56 m_s + 2.13 f - 0.39 f_g - 1.15\, \delta W_{\Xi^{**}}$$

Hence from requirement $\delta m(M) = const$ for $M \in \Omega'$ we get the mass formulae

$$m_{\Sigma^{**}} - m_{N^*} = 2.77 m_s + f + 0.1 f_g$$
$$m_{\Xi^{**}} - m_{N^*} = 4.3 m_s + 1.3 f - 0.5 f_g \qquad (39)$$
$$m_{\Lambda^{**}} - m_{N^*} = 1.94 m_s + 0.53 f - 0.37 f_g$$

The last formula in (35) is obtained from the two first using the Gell-Mann-Okubo formula for the octet.

Substituting the values of m_s and f into (21) we find

$$m_{\Sigma^{**}} - m_{N^*} = (0.2 + 0.1 f_g)\ GeV$$
$$m_{\Xi^{**}} - m_{N^*} = (0.34 - 0.5 f_g)\ GeV \qquad (40)$$
$$m_{\Lambda^{**}} - m_{N^*} = (0.16 - 0.37 f_g)\ GeV$$

Experimental identification of hyperon resonances in the octet $J^P=3/2^-$ is not quite unambiguous. There exist two resonances with $J^P=3/2^-$ and with quantum numbers of Λ-hyperon:

Λ^{**}(1520) and Λ^{**}(1670). The most suitable candidates for is sufficiently well established resonance Σ^{**}(1670), so that $m_{\Sigma^{**}} - m_{N^*}$ =150 MeV, if m_{N^*} =1520 MeV. For Ξ^{**} the experimental situation is not quite definite, most of all, $m_{\Xi^{**}}$=1820 MeV. Then $m_{\Xi^{**}} - m_{N^*}$ = 300 MeV. At such an identification of N^*, $\Sigma^{'**}$, and Ξ^{**}, the Gell-Mann-Okubo formula corresponds to the resonance Λ^{**}(1690). Then $m_{\Lambda^{**}} - m_{N^*}$=170 MeV. It is seen that the relation (40) within expected accuracy (~30%) agrees with experiment.

5. Discussion

The result of investigation of the sum rules for strange baryons is stated by the formulae for the mass differences of usual and strange baryons(16),(19) and (20) for nucleon octet; (30)-for decuplet Δ and (39) for the octet J^P=3/2$^-$. When deriving these formulae the results of I were used, in particular,for the values of the residue squared β_N^2 , λ_Δ^2 , and $\lambda_{N^*}^2$. The accuracy of the β_N^2 and λ_Δ definition was 20-30%, for $\lambda_{N^*}^2$ the accuracy is worse and is characterized by the factor $2^{\pm 1}$ in the denominator (partly compnnated by the exponent). Taking this into account and bearing in mind other sources of errors, we estimate the accuracy of our calculations of the mass splittings to be $\pm$30% (and, may be, somewhat worse in case of the octet J^P = 3/2$^-$). Requiring for the theoretical relations to agree with experiment up to 30%, one can find the region of permissible values of m_s and f . The simplest way to do it is to draw these regions on the digrams in (m_s, f) plane. In order to obviate the f_g dependence in cases of decuplet and octet J^P=3/2$^-$ let us include the results of variation into the error, assuming $|f_g|$< 0.2. Then we obtain the graph depicted in fig.2. It is seen from fig.2 that our consideration requires the following values of m_s and f :

$$m_s = 0.1 \pm 0.02 \text{ GeV}, \qquad f = -0.11 \pm 0.05 \tag{41}$$

(The value of m_s is given at the normalization point μ = 0.5 GeV).

Note that the upper limit on the strange quark mass follows from the sum rules for the octet $J^P=3/2^-$, specifically, for Σ^{**}. But the accuracy of relations for this octet is lower than for other multiplets under consideration (see the preceding Section). Therefore it is apt to give the estimate of m_s disregarding the mass formulae for Σ^{**}. In this case

$$m_s = 0.105 \pm 0.030 \text{ GeV} \tag{42}$$

The best agreement with the whole set of experimental data is at m_s =0.105 GeV, f = -0.11, f_g= 0.02, and the theoretical values of the baryon mass differences are:

$$\begin{aligned} m_\Sigma - m_N &= 0.23 \text{ GeV} \\ m_\Xi - m_N &= 0.4 \text{ GeV} \\ m_\Lambda - m_N &= 0.19 \text{ GeV} \\ m_{\Sigma^*} - m_\Delta &= 0.14 \text{ GeV} \\ m_{\Xi^{**}} - m_{N^*} &= 0.3 \text{ GeV} \\ m_{\Sigma^{**}} - m_{N^*} &= 0.18 \text{ GeV} \\ m_{\Lambda^{**}} - m_{N^*} &= 0.14 \text{ GeV} \end{aligned} \tag{43}$$

The first three values differ only slightly from those following from the data on nucleon octet (eq.(21)).

Concluding, let us repeat that the QCD sum rules for baryons give consistent and coinciding with experiment description of the mass splittings in the lowest baryonic multiplets. With such a description only two QCD parameters are used: the bare mass of strange quark and determined in (I) quantity f which characterizes the difference of vacuum condensate of strange and usual quarks. The final values of these parameters obtained in the paper are presented in (41). Note that the consideration made allowed us to determine the value of f . The current strange quark mass appeared to be somewhat smaller than the usually adopted $m_s = 150$ MeV[6] . But at the available accuracy of the mass splitting calculations it is difficult to say whether this discrepancy is real.

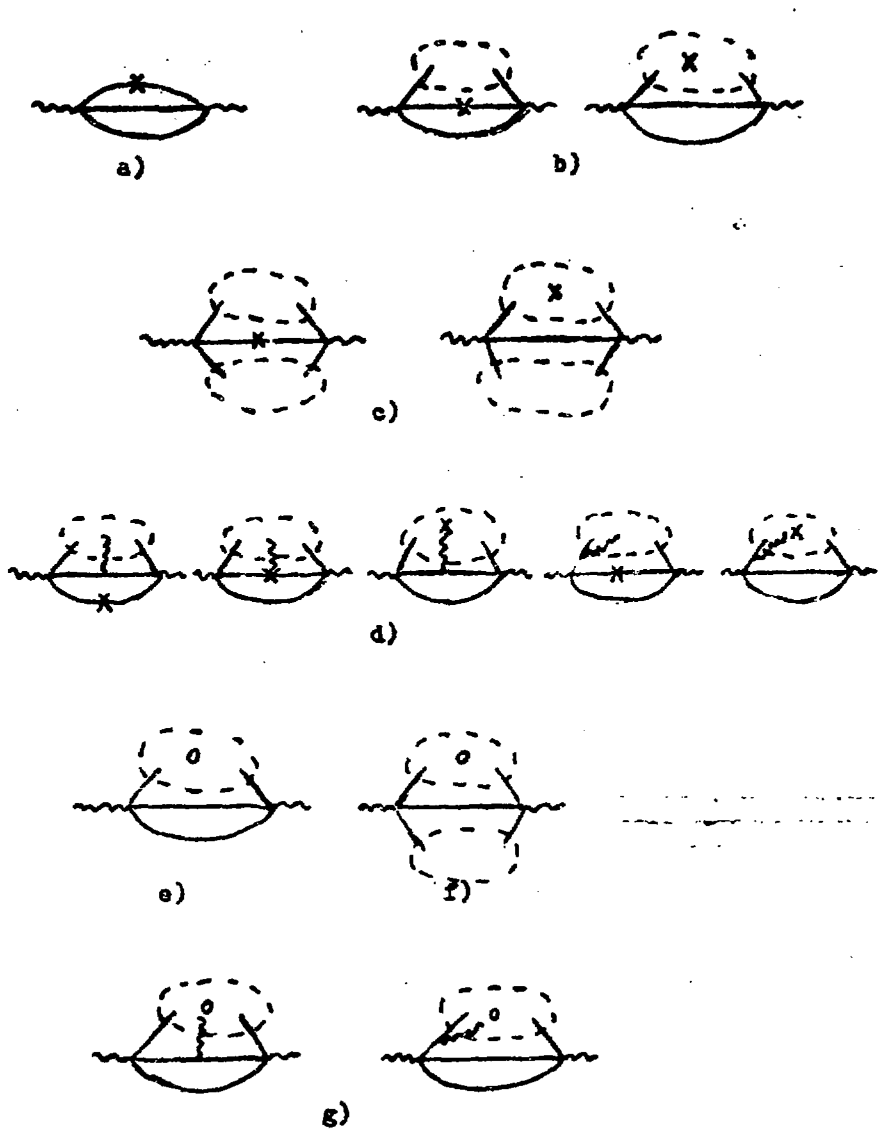

Fig.1. Feynman graphs for the calculation of polarization operator corrections. Free tails correspond to the emission of soft quark or gluon into vacuum. The account of quark mass and of factors f (or f_g) is denoted by (x) and (0), respectively.

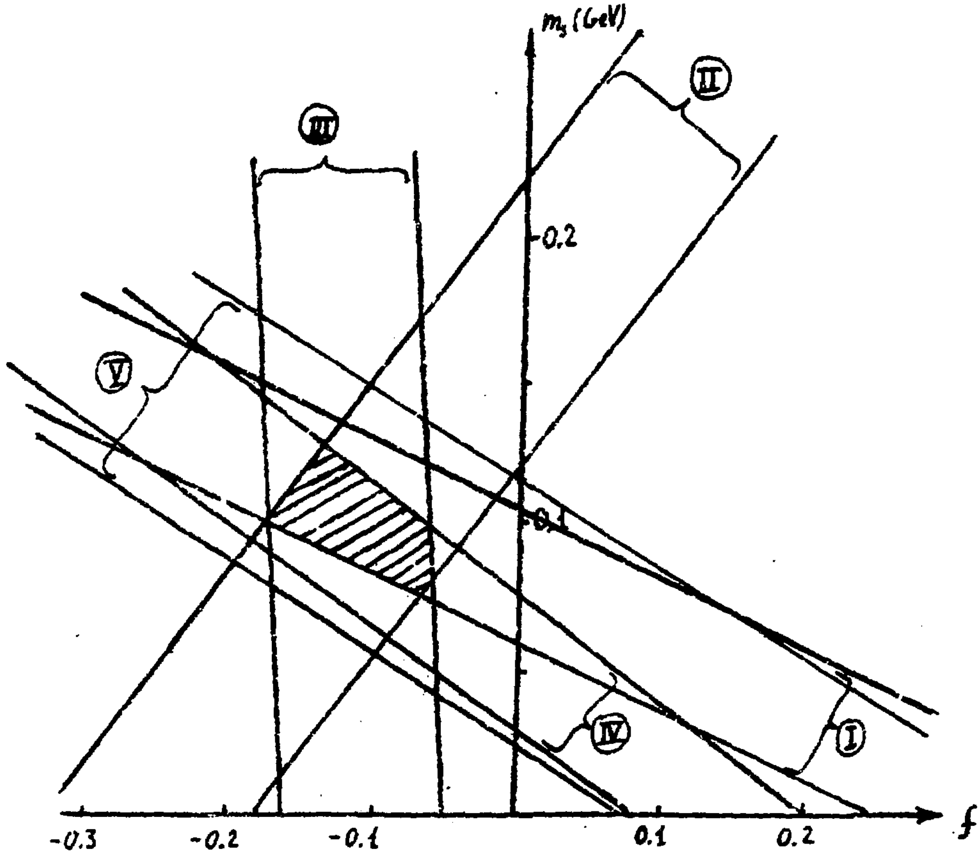

Fig. 2. Regions of permissible values of f and m_s .

I - the region determined by the sum rules for $m_\Sigma - m_N$

II - -"- $m_\Xi - m_N$

III - -"- $m_{\Sigma^*} - m_\Delta$

IV - -"- $m_{\Sigma^{**}} - m_{N^*}$

V - -"- $m_{\Xi^{**}} - m_{N^*}$

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