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# OPTIMAL INTERFACE BETWEEN PRINCIPAL DETERRENT SYSTEMS AND MATERIAL ACCOUNTING

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ABSTRACT

The purpose of this study is to find an optimal blend between three safeguards systems for special nuclear material (SNM), the material accounting system and the physical security and material control systems. The latter two henceforth will be denoted as principal deterrent systems. The optimization methodology employed is a two-stage decision algorithm, first an explicit maximization of expected diverter benefits and subsequently a minimization of expected defender costs for changes in material accounting procedures and incremental improvements in the principal deterrent systems. The probability of diverter success function dependent upon the principal deterrents and material accounting system variables is developed. Within the range of certainty of the model, existing material accounting, material control and physical security practices are justified.

INTRODUCTION

Large-scale analytical studies of the material accounting system for special nuclear materials (SNM) have been performed since 1976. The purpose of these studies has been (a) to directly examine the implications of deliberate diversion on nuclear material accounting, (b) consider the applicability of inventory difference data to statements about unauthorized diversion of SNM, and (c) to provide tools for assessing licensee material accounting safeguards performance standards.

These studies have ranged from advances in statistical testing techniques to simulation techniques to dynamic programming to game theory, and recently, a two-stage decision algorithm. This study employs an expanded version of the two-stage decision algorithm,[1] modified to directly incorporate interaction between three safeguards systems, the material accounting system and the physical security and material control systems. The latter two are henceforth denoted as principal deterrent systems. Unlike previous material accounting studies, this model includes an explicit analysis of the relative benefits of changes in material accounting procedures versus changes in incremental principal deterrence.

The expanded two-stage decision algorithm maintains the adversarial relationship between the defender (society, the corporate owner of the facility, and the U.S. government) and the diverter which is advantageous in modeling safeguards problems. The defender desires to minimize total societal safeguards cost, whereas the diverter is concerned with maximizing his expected benefit, two antagonistic goals. The defender cost and diverter benefit functions are explicitly modeled, both being functions of the amount diverted, the alarm threshold, and the principal deterrent systems. Application of the expanded two-stage algorithm consists of choosing the amount diverted which maximizes expected diverter benefits for every combination of safeguards expenditures on principal deterrents and material accounting. The safeguards mix adopted by the defender is assumed to be known by the diverter. Knowing these maximal diversion amounts for each possible defender safeguards strategy, the defender then chooses the safeguards strategy which minimizes his cost function.

This study, like previous material accounting studies, is concerned with decision criteria to be used in taking action in response to the recorded inventory difference, which is the difference between book inventory and physical inventory performed at the end of an accounting period. If there were no measurement errors, process errors, or human errors, the inventory difference would be zero unless diversion had occurred. Due to various random errors inherent in the system, the inventory difference is not, in general, zero and diversions can conceivably be masked by the inherent errors. Thus, the decision making problem is, given an inventory difference reading, what if any action should be taken to verify theft and attempt to recover material that may have been diverted.

The clean-out inventory and search and recovery costs are modeled in this study as a function of the alarm threshold selected. The present practice in the licensed domestic nuclear industry, regarding the material accounting system, follows regulations of the Nuclear Regulatory Commission that establish inventory periods and limits on measurement accuracy. The Nuclear Regulatory Commission has also established guidelines and operating suggestions for appropriate action limits. The operational procedure at present is to establish fixed alarm thresholds and to take action when the inventory difference exceeds these thresholds. When the inventory difference exceeds 4σ, 4.0 times the standard deviation of random errors for the facility, the facility is shut down, a clean-out inventory is conducted, and an investigation of the cause of the inventory difference is initiated.

The principal deterrent systems are modeled using a proxy variable. Additional expenditures on the principal deterrent systems result in a reduction of the maximum amount which can be diverted. An effect proportional to the quantity diverted can be incorporated into the model.

## THE MODEL

Safeguards analysis can be modeled after the adversarial nature of game theory, where both adversaries have well-defined moves. In this expanded two-stage decision algorithm , the defender sets the safeguards mix which is assumed to be under the control of the defender and known to the diverter. The move of the diverter is to divert a quantity x, between zero and the minimum total quantity of SNM that could pose a severe threat to society, K. K is assumed to be 5.0 kg. in this study. The "move" of nature is a value ε for the random value of the inventory difference due to all other causes than a potential diversion. The move of the defender is to invest in additional physical security and material control defined by the proxy variable X. The defender also presets the value of the alarm threshold a. The values of the variables a and X are assumed to be known by the diverter, and both assume values between zero and 5 kg.

The functions representing the monetary considerations of the two adversaries are now described. Total defender cost consists of material accounting cost, the cost of unretrieved SNM, and principal deterrent cost, while total diverter benefit is the societal value of SNM successfully appropriated. Complete derivation of material accounting cost, the societal cost of SNM, and diverter benefit is reported in reference 1. The functions in reference 1 were simplified by allowing only non-negative values of the alarm threshold. The cost of incremental principal deterrent systems is represented by C(X) and is added to the defender function developed in reference 1. Successful defeat of the principal deterrent systems is assumed for all $x < X$. C(X) is defined in such fashion that no diversion occurs for $x \geq X$. The defender and diverter monetary functions are the expected value for the random value of the causes of a non-zero inventory difference other than diversion. No account is yet taken for the likelihood of success of the diverter. The diverter and defender monetary functions are presented as follows.

$$M_1(x,a,X) = c_3 \cdot x \cdot \int_{-\infty}^{a-x} \rho(\varepsilon)\,d\varepsilon + c_3 \cdot \begin{cases} 0 & a > x \geq 0 \\ \int_0^{a-x} \varepsilon\rho(\varepsilon)\,d\varepsilon & 0 \leq a \leq x \end{cases} \qquad x < X$$

$$= 0 \qquad x \geq X$$

$$M_2(x,a,X) = c_1 \cdot \int_{a-x}^{\infty} \rho(\varepsilon)\,d\varepsilon + c_2 \cdot x \cdot \int_{a-x}^{\infty} \rho(\varepsilon)\,d\varepsilon) + c_2 \cdot \int_{a-x}^{\infty} \varepsilon\rho(\varepsilon)\,d\varepsilon + M_1 + C(X), \qquad x < X$$

$$= C(X) \qquad x \geq X$$

Where:

- $M_1(x,a,X)$ = the benefit to the diverter of SNM successfully diverted by the diverter;
- $M_2(x,a,X)$ = total safeguards cost to the defender, including societal considerations, of SNM successfully diverted;
- $x$ = quantity of SNM diverted by the diverter; $0 \leq x \leq K$, where K is the quantity of SNM that could pose a severe threat to society; K assumed to be 5.0 kg.;
- $a$ = alarm threshold preset by the defender and known to the diverter, $0 \leq a \leq K$;
- $c_1$ = fixed shutdown inventory cost;
- $c_2$ = unit search and recovery cost;
- $c_3$ = unit cost to the defender and society of unretrieved SNM;
- $\varepsilon$ = value of the inventory difference due to all other causes than a potential diversion;
- $\rho(\varepsilon)$ = associated normal distribution of ε, centered around zero with a standard deviation of the inventory difference σ;

X = the upper bound on the amount successfully divertable, is a proxy variable representing the incremental principal deterrent systems, preset by the defender and known to the diverter, $0 \leq x < X \leq K$;

C(X) = incremental cost to society of initiating principal deterrent systems, where the acquisition cost has been allocated using straight-line depreciation over an estimated useful life of 30 years, assuming no salvage value.

The probability of diverter likelihood of success function P(a,X,X-x) is specified by four factors:

$P_1$ (X) = probability of diverter presence related to the principal deterrent systems;

$P_2$(a) = probability of diverter presence related to material accounting;

$P_3$(X-x,X) = probability of successfully defeating the principal deterrent systems;

$P_4$ = probability of diverter being present at all, assumed a constant and conservatively set equal to one in this study.

Since these events are independent, the final probability of successful diversion is the product of the four functions. A fifth probability, that of successfully defeating the material accounting system, is already implicit in the monetary cost functions.

Taking into account the likelihood of success function, the expected benefit to the diverter is the diverter monetary function ($M_1$) decreased by the probability factor:

$$B_1(x,a,X-x) = M_1(x,a,X)\ P(a,X,X-x)$$

In the first stage of the two-stage algorithm, the diverter selects a strategy $x_{max}$ (a, X) which maximizes expected diverter benefits $B_1(x,a,X,X-x)$ for all combinations of defender strategies (a,X) which are fixed and known to the diverter. In the second stage, knowing $x_{max}(a,X)$, the defender chooses the strategy which minimizes his expected total costs, i.e., he chooses the ordered pair (a,X) which minimizes the expected defender cost:

$$B_2(x,a,X,X-x) = M_2(x_{max}(a,X),a,X) \cdot P(X,a,X-x) + M_2(0,a,X) \cdot [1 - P(X,a,X-x)]$$

A small facility was modeled, for which 10σ = 5kg. The parameters $c_1$, $c_2$, and $c_3$ are provided in terms of ratios to $c_1$ = \$5,000 for a small facility. The ratio chosen was $c_1:c_2:c_3$ = .01:.01:1, with the ratios $c_1:c_2:c_3$ = .01:.01.1 and .01:.01:5 also considered.

The functional form considered most representative of the cost of incremental principal deterrence is exponential, because the exponential function has the desirable property that an approximately infinite marginal cost must be accrued to prevent all diversions. Therefore $C(X) = C(0)e^{-X}$ was used with C(0) = .1, .2, and .3. Assuming that X = 0 would require 10% of a facility cost of \$1,000,000,000 and using straight line depreciation, 30 years useful life, and no salvage value, C(0) is approximately 0.3 when normalized to $c_1$.

Only integer values of X are considered in view of computer constraints, physical safeguards systems considerations, and the dubious value of additional information obtained utilizing a smaller increment size. X is a single variable representation of numerous safeguards mixes which can provide identical bounds on theft, and C(X) will not distinguish between these various mixes. X = 0 is considered unachievable.

$P_1(X)$ exists because, in general, any hardware and personnel system protecting an asset provides a deterrent effect to potential purloiners. $P_2(a)$ exists due to the deterrence function and assurance function of the material accounting system. The functions used are $P_1(X)$ = M (0.25 + 0.75X), M < 1, and $P_2(a)$ = 0.7 + 0.03a. One bias built into these functions is a greater deterrent effect attributed to the principal deterrent systems than the material accounting system. Reasons supporting this include immediacy of diverter detection by the principal deterrent systems and lack of concrete evidence against a diverter if diversion is suspected by material accounting and not the principal deterrent systems.

The function $P_3$(X-x) has been introduced to represent the authors' belief that the chance of diversion is proportional to the quantity diverted, for x up to the maximum which can be diverted. The hyperbolic tangent was chosen to represent this function because it allows a slow decline in the ease of diversion with amounts slightly about zero and a slow increase in the ease of diversion for amounts slightly less than the maximum allowed. The exact form of the hyperbolic tangent is given by:

$$P_3(X-x) = \begin{cases} \dfrac{\mathrm{Tanh}\left(\dfrac{X-x}{1.5}\right) - 3 + 1}{2}, & x \leq X \\ 0, & x > X \end{cases}$$

The functions C(X), $P_1(x)$, $P_2(a)$ and $P_3$(X-x) have also been represented by straight lines

$C(X) = .1226 - .01226X$; $P_1(X) = M$, $P_2(a) = 1.$, and $P_3(X-x) = 0.1(X-x)$ for $x \leq X$ and $P_3(X-x) = 0$ for $x > X$. These straight line representations of the functions are believed to represent the boundary value of the family of curves which might represent the true situation.

RESULTS

The resultant optimal alarm threshold and incremental increase of principal deterrent systems were determined for a range of parameters and for the functions and the bounding described in the model described above. Specifically, the magnitude of the cost of SNM to society ($c_3$) and the cost of incremental principal deterrent ($C(X)$) and the likelihood of diverter success function due to material control and physical security ($M$) were varied over a likely range. An investigation of all the variations performed indicates that: 1) the optimal alarm threshold is inversely related to the cost of the incremental principal deterrent systems; 2) in cases where the expected cost of the incremental principal deterrent systems is relatively large compared to other costs, then it is cost effective to depend upon material accounting, and not increase principal deterrence; this effect is increased at lower values of the likelihood of diverter success function; 3) the optimal alarm level is directly related to the ratio of fixed shutdown inventory cost ($c_1$) to the value to society of unretrieved SNM in cases where the principal deterrence remains constant; 4) increasing the value to society of unretrieved SNM either requires greater principal deterrence or leaves it unchanged, but if the principal deterrent remains unchanged, the alarm threshold is lowered; 5) in cases where additions to the principal deterrent systems are too costly to undertake, the optimal alarm threshold is inversely proportional to the likelihood of diverter success function.

Table 1 presents the values of the variables a and X found by the two stage algorithm for three magnitudes of the exponential form of the cost of deterrence function $C(X)$. The value of M is varied from .01 to .0001. The value of the variable a is between $3\sigma$ and $4\sigma$ but varies in a complex fashion dependent upon the selection of X.

Table 2 provides the variation of X for fixed values of the variable a. Table 3 provides the variation of the variable a for fixed value of X.

Table 4 provides the change in solution when $P_3(a)$ is varied to the limiting case of no dependence upon alarm threshold. Table 5 shows the limiting case of the hyperbolic tangent replaced by a straight line. Table 6 shows the limiting case of $P_1(X)$ being a straight line. Finally, Table 7 shows the effect of replacing the $C(X)$ exponential function by a straight line.

CONCLUSION

This study's purpose was to provide a generic model to better comprehend the relationships that exist among the various safeguards systems and to determine the optimal mix between these safeguards systems. Most of the relationships among the functions and parameters were expected. Justification of the existing material accounting practice of setting the alarm threshold to $4\sigma$ can be accomplished in either of two ways. If additions to existing principal deterrent systems are relatively inexpensive, the defender will increase the level of principal deterrence, lowering the probability of diverter success enough to allow the alarm level to be set between $3\sigma$ and $4\sigma$. Assuming additions to principal deterrent systems are too costly to undertake, and assuming existing principal deterrence is effective enough to lower the probability of diverter success to between .001 and .0001, then any alarm threshold between $3\sigma$ and $4\sigma$ is justified, even assuming very high values for the unit cost of unretrieved SNM to society. Finally, if existing principal deterrent systems are effective enough to lower the probability of diverter success to between .001 and .0001, then almost any additions to principal deterrent systems will not be cost-effective.

REFERENCES

1. J.H. Opelka, W.B. Sutton, "Two-Stage Decision Approach to Material Accounting," Journal of the Institute of Nuclear Materials Management, Proceedings Issue, Volume XI (1982).

TABLES FOLLOW

Table 1. A Comparison of the Defender Minimum Solutions (a, X) and Current Practice (4,10) for Three Values of the Exponential Representation of C(X) and Values of M = .01, .001, .0001

| M | a | X | $x_{max}$ | Defender Minimum | Defender (4,10) | % Change |
|---|---|---|---|---|---|---|
| | | | $C(X) = .3e^{-X}$ | | | |
| .01 | 3.07 | 5 | 1.05 | .00371940 | .02006883 | 81.4 |
| .001 | 3.02 | 7 | 1.76 | .00103395 | .00202063 | 48.8 |
| .0001 | 3.35 | 10 | 2.55 | .00019085 | .00021581 | 11.5 |
| | | | $C(X) = .2e^{-X}$ | | | |
| .01 | 3.07 | 5 | 1.05 | .00304500 | .02006429 | 84.8 |
| .001 | 3.55 | 6 | 1.45 | .00092060 | .00201609 | 54.3 |
| .0001 | 3.42 | 9 | 2.49 | .00018244 | .00021127 | 13.6 |
| | | | $C(X) = .1e^{-X}$ | | | |
| .01 | 3.07 | 5 | 1.05 | .00237180 | .02005975 | 88.1 |
| .001 | 3.55 | 6 | 1.45 | .00067275 | .00201155 | 66.5 |
| .0001 | 3.61 | 8 | 2.33 | .00016028 | .00020673 | 22.4 |

Table 2. Defender Minimal Solutions X for fixed a = 1, 4, 10, for Values of M = .01, .001, .0001

| | a = 1 | | a = 4 | | a = 10 | |
|---|---|---|---|---|---|---|
| M | X | $x_{max}$ | X | $x_{max}$ | X | $x_{max}$ |
| .01 | 5.8 | .9 | 5.1 | 1.1 | 5.0 | 1.1 |
| .001 | 9.1 | 2.3 | 6.6 | 1.8 | 6.4 | 1.7 |
| .0001 | 10 | 1.3 | 9.1 | 2.8 | 8.2 | 2.9 |

Table 3. Defender Minimum Solutions for the Variable a for Fixed X = 1 through X = 10, C(X) = .2-.02 X, for M = .001

| X | C(x) | a | Defender Minimum | $x_{max}$ |
|---|---|---|---|---|
| 1 | .18 | 5.3 | .18000072 | .8 |
| 2 | .16 | 4.9 | .16000331 | .8 |
| 3 | .14 | 4.6 | .14001429 | .8 |
| 4 | .12 | 4.3 | .12005540 | .9 |
| 5 | .10 | 4.0 | .10017610 | 1.1 |
| 6 | .08 | 3.5 | .08042400 | 1.4 |
| 7 | .06 | 3.0 | .06076000 | 1.8 |
| 8 | .04 | 2.7 | .04103200 | 1.9 |
| 9 | .02 | 2.6 | .02119890 | 2.0 |
| 10 | .00 | 2.5 | .00130870 | 2.0 |

Table 4. Defender Minimum Solutions (a, X) for $P_2(a) = 0.7 + .03a$ and the Bounding Case $P_2(a) = 1.0$, for M = .01, .001, .0001

| M | a | X | $x_{max}$ |
|---|---|---|---|
| | $P(a) = .7 + .03a$ | | |
| .01 | 3.07 | 5 | 1.05 |
| .001 | 3.02 | 7 | 1.76 |
| .0001 | 3.35 | 10 | 2.55 |
| | $P(a) = 1$ | | |
| .01 | 3.40 | 5 | 1.07 |
| .001 | 3.00 | 7 | 1.76 |
| .0001 | 3.39 | 9 | 2.47 |

Table 5. A Comparison of Defender Minimum Solutions (a, X) for $P_3$ (X-x) as the Hyperbolic Tangent and $P_3$ (X-x) as a Straight Line Bounding Case

| M | a | X | $x_{max}$ | Defender Minimum |
|---|---|---|---|---|
| | $P_3(X-x) = \frac{\text{Tanh}\left(\frac{X-x}{1.5} - 3\right) + 1}{2}$ | | | |
| .01 | 3.07 | 5 | 1.05 | .0037194 |
| .001 | 3.02 | 7 | 1.76 | .00103395 |
| .0001 | 3.35 | 10 | 2.55 | .00019085 |
| | $P_3(X-x) = .1(X-x)$ | | | |
| .01 | 2.14 | 6 | 1.50 | .003999753 |
| .001 | 2.89 | 8 | 2.00 | .000837060 |
| .0001 | 3.59 | 9 | 2.48 | .000150029 |

Table 6. Defender Minimal Solutions (a, X) for $P_1(X) = M(.25 + .075\ X)$ and the Bounding Case $P_1$ (X) = M for M = .01, .001, and .0001

| M | a | X | $x_{max}$ | Defender Minimum |
|---|---|---|---|---|
| | $P_1(x) = M(.25 + .075\ X)$ | | | |
| .01 | 3.07 | 5 | 1.05 | .00371940 |
| .001 | 3.02 | 7 | 1.76 | .00103395 |
| .0001 | 3.35 | 10 | 2.55 | .00019085 |
| | $P_1(X) = M$ | | | |
| .01 | 2.78 | 5 | 1.02 | .00469507 |
| .001 | 2.89 | 7 | 1.72 | .00123592 |
| .0001 | 3.35 | 10 | 2.55 | .00019086 |

Table 7. A Comparison of Defender Minimum Solutions (a, X) for $C(X) = .3e^{-X}$ and the Bounding Case $C(X) = .1226 - .01226\ X$

| M | a | X | $x_{max}$ | Defender Minimum |
|---|---|---|---|---|
| | $C(X) = .3\ e^{-X}$ | | | |
| .01 | 3.07 | 5 | 1.05 | .0037194 |
| .001 | 3.02 | 7 | 1.76 | .00103395 |
| .0001 | 3.35 | 10 | 2.55 | .00019085 |
| | $C(X) = .1226 - .01226\ X$ | | | |
| .01 | 1.30 | 10 | 1.27 | .00756083 |
| .001 | 2.53 | 10 | 1.99 | .00130798 |
| .0001 | 3.35 | 10 | 2.55 | .000177237 |

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