ITEP - 98

INSTITUTE OF THEORETICAL
AND EXPERIMENTAL PHYSICS

V.L.Eletsky, B.L.Ioffe, Ya.I.Kogan

THE $g_{\omega\rho\pi}$ COUPLING CONSTANT FROM QCD SUM RULES

MOSCOW 1982

УДК 530.I45:538.3

Abstract

QCD sum rules for the vertex function of two vector and one axial vector currents are used to calculate the $g_{\omega\rho\pi}$ coupling constant. The obtained value, $g_{\omega\rho\pi} \approx 17\ \mathrm{GeV}^{-1}$ is in a good agreement with experimental data.

Nowadays it has been established that the QCD sum rules method originally suggested in the pioneering paper by Shifman, Vainshtein and Zakharov[1] were very effective for calculation of various quantitites in low energy hadron physics. Application of this method to polarization operators resulted in determination of masses and couplings of low lying mesonic[1,2] and baryonic[3] states. In recent papers[4,5] the sum rules based on a double dispersion relation for the vertex function of two axial vector and one vector currents were used to obtain the pion formfactor in the region of intermediate momentum transfer. An essential feature of this approach is the use of the Borel transformation originally proposed in ref.[1] and generalized to the case of two variables in ref.[4]. This transformation allows one to suppress the contributions of higher resonances and continuum [1]. Here we report on the application of QCD sum rules to the calculation of $\omega \to \rho\pi$ transition coupling constant, $g_{\omega\rho\pi}$.

Let us consider the vertex function of two vector and one axial vector currents

$$A_{\mu\nu\lambda}(p,p',q) = -\int d^4x\, d^4y\, e^{i(p'y-px)} \langle 0 | T\{ j_\mu^{(\omega)}(x)\, j_\nu^{(\rho)}(y)\, j_\lambda^{5}(0)\} | 0\rangle \quad (1)$$

for negative p^2, p'^2, $q^2 \sim -1\ GeV^2$. Here $j_\mu^{(\omega)}$, $j_\nu^{(\rho)}$ are isoscalar and isovector vector currents: $j_\mu^{(\omega)} = \frac{1}{6}(\bar u \gamma_\mu u + \bar d \gamma_\mu d)$, $j_\nu^{(\rho)} = \frac{1}{2}(\bar u \gamma_\nu u - \bar d \gamma_\nu d)$ and $j_\lambda^5 = \frac{1}{2}(\bar u \gamma_\lambda \gamma_5 u - \bar d \gamma_\lambda \gamma_5 d)$ is isovector axial current. For the momenta in question the

---

1) The double dispersion relation for a vertex function without the Borel transformation was used in ref.[6] for calculation of radiative transitions in quarkonium.

characteristic values of $\alpha_s \lesssim 0.3$. In the lowest order in α_s the amplitude $A_{\mu\nu\lambda}$ is given by the diagram of Fig.1 and by the power corrections which arise in the operator product expansion and are proportional to the vacuum averages of various operators composed of quark and gluon fields[1].

In general, the amplitude $A_{\mu\nu\lambda}$ is the sum of eight structures

$$A_{\mu\nu\lambda} = f_1 p_\mu T_{\nu\lambda} + f_2 p_\nu T_{\lambda\mu} + f_3 p_\lambda T_{\mu\nu} + f_4 S_{\mu\nu\lambda} + \\ + f_1' p_\mu' T_{\nu\lambda} + f_2' p_\nu' T_{\lambda\mu} + f_3' p_\lambda' T_{\mu\nu} + f_4' S_{\mu\nu\lambda}' \tag{2}$$

where $T_{\mu\nu} = \mathcal{E}_{\mu\nu\alpha\beta} p_\alpha p_\beta'$, $S_{\mu\nu\lambda} = \mathcal{E}_{\mu\nu\lambda\alpha} p_\alpha$ and $S_{\mu\nu\lambda}' = \mathcal{E}_{\mu\nu\lambda\alpha} p_\alpha'$. Since the amplitude $A_{\mu\nu\lambda}$ is symmetric under the interchange $\mu \leftrightarrow \nu$ and $p \leftrightarrow -p'$ the structure functions f_i and f_i' satisfy the following relations

$$f_1'(p^2, p'^2, Q^2) = -f_2(p'^2, p^2, Q^2), \; f_2'(p^2, p'^2, Q^2) = -f_1(p'^2, p^2, Q^2), \\ f_3'(p^2, p'^2, Q^2) = -f_3(p'^2, p^2, Q^2), \; f_4'(p^2, p'^2, Q^2) = f_4(p'^2, p^2, Q^2) \tag{3}$$

where $Q^2 = -q^2 > 0$. In fact, of eight structures $\sim f_i$, f_i' only six are linearly independent since there are two kinematical relations

$$p_\mu T_{\nu\lambda} + p_\nu T_{\lambda\mu} + p_\lambda T_{\mu\nu} = -(pp') S_{\mu\nu\lambda} + p^2 S_{\mu\nu\lambda}', \\ p_\mu' T_{\nu\lambda} + p_\nu' T_{\lambda\mu} + p_\lambda' T_{\mu\nu} = -p'^2 S_{\mu\nu\lambda} + (pp') S_{\mu\nu\lambda}' \tag{4}$$

Choosing, for instance, $S_{\mu\nu\lambda}$, $S'_{\mu\nu\lambda}$, $p_\mu T_{\nu\lambda} - p_\nu T_{\lambda\mu}$, $p'_\mu T_{\nu\lambda} - p'_\nu T_{\lambda\mu}$, $p_\lambda T_{\mu\nu}$ and $p'_\lambda T_{\mu\nu}$ as independent structures[2)] and using eq.(3) we can write the amplitude corresponding to $\omega \to \rho\pi$ transition in the form

$$
\begin{aligned}
A^{(\pi)}_{\mu\nu\lambda} &= f(p^2, p'^2, Q^2)\, q_\lambda\, \varepsilon_{\mu\nu\alpha\beta}\, p_\alpha p'_\beta , \\
f &= \frac{1}{2}\left(f_3 + \tilde{f}_3\right) - \frac{1}{4}\left(f_1 + \tilde{f}_1 + f_2 + \tilde{f}_2\right) ,
\end{aligned}
\tag{5}
$$

where $\tilde{f}_i(p^2, p'^2, Q^2) = f_i(p'^2, p^2, Q^2)$ and $q = p - p'$. Note that the structure functions f_4 and f'_4 contribute only to $\omega \to \rho A_1$ transition and thus are absent in the amplitude (5).

The contribution to this amplitude from the ground term - the quark loop diagram of Fig.1 can be obtained using the double dispersion relation in p^2 and p'^2,

$$
f^{(0)}(p^2, p'^2, Q^2) = -\frac{1}{4\pi^2}\int_0^\infty ds \int_0^\infty ds' \frac{\Delta(s, s', Q^2)}{(s-p^2)(s'-p'^2)} + \text{subtr. terms}, \tag{6}
$$

where $\Delta(s, s', Q^2)$ is the double discontinuity of $f^{(0)}(p^2, p'^2, Q^2)$ on the cuts $0 < s < \infty$ and $0 < s' < \infty$. The discontinuity $\Delta(s, s', Q^2)$ is calculated in the standard way substituting denominators of quark propagators by δ -functions, $k^{-2} \to -2\pi i\, \delta(k^2)$. The subtraction terms in eq.(6) are polynomials in one of the two variables p^2 or p'^2 , and arbitrary functions of two other, p'^2 and Q^2, or p^2 and Q^2 .

In order to enhance the contributions of ω and ρ mesons in the vector channels we perform the Borel transformation in

---

2.) We emphasize that structures $S_{\mu\nu\lambda}$ and $S'_{\mu\nu\lambda}$ must be included into linear independent set of structures. Otherwise, there arise some troublesome kinematic singularities.

two variables p^2 and p'^2

$$f_B(M^2, M'^2, Q^2) = \lim_{\substack{n\to\infty, -p^2\to\infty \\ n'\to\infty, -p'^2\to\infty \\ -p^2/n \to M^2 \\ -p'^2/n' \to M'^2}} \frac{1}{(n-1)!(n'-1)!} (-p^2)^n \left(\frac{d}{dp^2}\right)^n (-p'^2)^{n'} \left(\frac{d}{dp'^2}\right)^{n'} \times f(p^2, p'^2, Q^2) \tag{7}$$

The double Borel transformation is also absolutely necessary to nullify unknown subtraction terms which persist in dispersion relation (6). Since $m_\omega^2 \approx m_\rho^2$, it is natural to use a symmetric Borel transformation, i.e. take $M^2 = M'^2$. As usual, in the QCD sum rule approach the sum over the physical states will be approximated by the contribution of the lowest resonances (ω and ρ) plus continuum. The calculations give for the Borel transforms of the ground term (Fig.1) in the massless quark approximation

$$f_B^{(0)}(M^2, Q^2) = -\frac{1}{4\pi^2 M^4} \iint_{0<s+s'<s_0} ds\, ds' \exp\left(-\frac{s+s'}{M^2}\right) \Delta(s, s', Q^2) =$$

$$= \frac{1}{32\pi^2 M^2} \left\{ (1+x) e^x \left[E(x) - E(x + s_0/M^2)\right] + \right. \tag{8}$$

$$\left. + \exp(-s_0/M^2)\left[1 - \ln(1 + 2s_0/Q^2)\right] - 1 \right\}$$

where $x = Q^2/2M^2$, $E(x) = \int_x^\infty t^{-1} e^{-t} dt$. Restricting the integration region within the limits $0 < s+s' < s_0$, we have taken into account the continuum contribution transferring it to the other side of the sum rule.

We wish to point out that the $1/Q^2$ term which arises[7] due to the Adler-Bell-Jackiw triangular anomaly is absent in eq.(8).

It can be shown that it is of the form of the subtraction terms in eq.(6) and thus vanishes under the Borel transformation (7).

Saturating the axial vector channel by the contributions of π and A_1 mesons we obtain for the amplitude under consideration

$$f_B^{(res.)} = f_\pi m_\omega^2 m_\rho^2 g_\omega^{-1} g_\rho^{-1} g_{\omega\rho\pi} \exp\left(-\frac{m_\omega^2+m_\rho^2}{M^2}\right)\left(\frac{1}{Q^2} - \frac{1}{Q^2+m_{A_1}^2}\right) \quad (9)$$

the constants f_π, [illegible] and g_ρ are defined in the standard way, $\langle 0|j_\mu^5|\pi\rangle$ [illegible] f_π [illegible], $\langle 0|j_\mu|V\rangle =$ [illegible] $m_V^2 g_V^{-1} e_\mu$ [illegible] where e_μ is the polarization vector [illegible]. The [illegible] $g_{\omega\rho\pi}$ is defined by the amplitude of $\omega\to\rho\pi$ transition $g_{\omega\rho\pi}\varepsilon_{\mu\nu\alpha\beta}p_\alpha p'_\beta e_\mu e'_\nu$, where p, p' and e, e' are the momenta and polarization vectors of ω and ρ mesons, respectively. In approximation of massless quarks we put $m_\pi^2 = 0$. The residues of π and A_1 poles in $f_B^{(res.)}(Q^2)$ are put to be equal in absolute values but opposite in sign for the absence of Q^{-2} terms in asymptotics. (It can be shown from the quark counting rule with account of helicity conservation asymptotically $f_B^{(res.)}(Q^2) \sim Q^{-6}$). The resulting Q^2-dependence of $f_B^{(res.)}(Q^2)$ agrees very well with that following from QCD calculation (see below eq.(11) and Fig.4).

Now we turn to calculation of power corrections. From dimensional arguments it is clear that since in the chiral limit the operators $\bar\psi\psi$ and $\bar\psi\sigma_{\mu\nu}\psi G_{\mu\nu}$ drop out, the dominant contribution comes from the terms proportional to $\langle G_{\mu\nu}^a G_{\mu\nu}^a\rangle_0$ and $\langle\bar\psi\Gamma\psi\bar\psi\Gamma\psi\rangle_0$, the latter under factorization hypothesis being reduced to $\langle\bar\psi\psi\rangle_0^2$. The calculation of these corrections was carried out along the lines of ref. [4] in the fixed point gauge, $x_\mu A_\mu(x)=0$, which is very convenient for computation of gluon corrections and quark corrections given by diagrams with soft gluon exchange [4, 8, 9]. In this gauge the gluon condensate corrections are given by diagrams depicted in Fig.2 (in accord with eq.(1) the coordinate origin is taken in the axial vector vertex). The fermion condensate corrections come from the diagrams of Fig.3. After the symmetric Borel transformation the power corrections to $f_B(M^2, Q^2)$ are

$$f_B^{(g)} = \frac{\alpha_s}{\pi} \frac{\langle G^a_{\mu\nu} G^a_{\mu\nu}\rangle_0}{288 M^6}\left(\frac{1}{x^2}+\frac{1}{x}-1\right),$$

$$f_B^{(q)} = \frac{4\pi\alpha_s \langle\bar\psi\psi\rangle_0^2}{27 M^8}\left(-\frac{1}{4x^2}+\frac{2}{x}-\frac{13}{18}+\frac{2}{9}x\right) \qquad (10)$$

Finally, putting $f_B^{(res.)} = f_B^{(0)} + f_B^{(g)} + f_B^{(q)}$ we obtain from eq.(9)

$$g_{\omega\rho\pi} = \frac{g_\omega g_\rho M^4}{f_\pi m_\omega^2 m_\rho^2}\exp\left(\frac{m_\omega^2+m_\rho^2}{M^2}\right)\frac{Q^2(Q^2+m_{A_1}^2)}{m_{A_1}^2}\left(f_B^{(0)}+f_B^{(g)}+f_B^{(q)}\right) \qquad (11)$$

Choosing the values [1] $4\pi^2/g_\rho^2 = 0.41$, $4\pi/g_\omega^2 = 0.046$, taking vacuum averages [4] $\alpha_s/\pi\langle G^a_{\mu\nu} G^a_{\mu\nu}\rangle_0 = 0.012\ \mathrm{GeV}^{-4}$ and $\alpha_s\langle\bar\psi\psi\rangle_0^2 = 8\cdot 10^{-5}\ \mathrm{GeV}^{-6}$ and putting $m_\omega^2 \simeq m_\rho^2 \simeq 0.6\ \mathrm{GeV}^2$, $m_{A_1}^2 = 1.44\ \mathrm{GeV}^2$ we calculate $g_{\omega\rho\pi}$ at fixed $Q_0^2 = 2\ \mathrm{GeV}^2$ and varying M^2. The contributions of power corrections and continuum rise with decrease and increase of M^2, respectively. Therefore only those values of M^2 were considered at which each of these contributions is less than 40% of the total result, namely, $0.6\ \mathrm{GeV}^2 < M^2 < 1.1\ \mathrm{GeV}^2$. The whole approach is consistent only in the region of intermediate Q^2 since for small and large Q^2 the ratio of power corrections (10) to the main term (8) increases infinitely. Therefore, it is reasonable to determine $g_{\omega\rho\pi}$ from eq.(11) taking for Q^2 some intermediate value, say $Q^2 = 1\ \mathrm{GeV}^2$, at which both the power corrections and continuum contributions are small. Fig.4 shows the dependence of $g_{\omega\rho\pi}$ on M^2 for different Q^2. It is seen that the result weakly depends on M^2 and Q^2 in the region $0.6\ \mathrm{GeV}^2 < M^2 < 1.1\ \mathrm{GeV}^2$, $0.5\ \mathrm{GeV}^2 < Q^2 < 1.5\ \mathrm{GeV}^2$. Taking $M^2 = 0.8\ \mathrm{GeV}^2$ and $Q^2 = 1\ \mathrm{GeV}^2$ we find the best value $g_{\omega\rho\pi} = 17\ \mathrm{GeV}^{-1}$. The accuracy of our calculation is about 20% and is mainly governed by the approximations made taking into account continuum in the vector channels and the A_1 contribution in the axial vector channel and can be improved if the asymptotics of $f_B^{(res.)}(Q^2)$ will be calculated.

The experimental value of $g_{\omega\rho\pi}$ can be extracted from the following widths $\Gamma(\omega\to 3\pi) = 8.9$ MeV(as was originally proposed in [10] this decay proceeds through $\rho\pi$), $\Gamma(\omega\to\pi\gamma) = 0.84$ MeV, $\Gamma(\rho^0\to\pi^0\gamma) = 0.067$ MeV and $\Gamma(\pi^0\to 2\gamma) = 8$ eV. The values of $g_{\omega\rho\pi}$ obtained from these numbers using the vector meson dominance model (for the last three decays) are as follows: $g_{\omega\rho\pi}^{(\omega\to 3\pi)} = 15\ \mathrm{GeV}^{-1}$, $g_{\omega\rho\pi}^{(\omega\to\pi\gamma)} = 14\ \mathrm{GeV}^{-1}$, $g_{\omega\rho\pi}^{(\rho^0\to\pi^0\gamma)} = 12\ \mathrm{GeV}^{-1}$ and $g_{\omega\rho\pi}^{(\pi^0\to 2\gamma)} = 17\ \mathrm{GeV}^{-1}$. Note that $g_{\omega\rho\pi}^{(\omega\to\pi\gamma)}$ and $g_{\omega\rho\pi}^{(\rho^0\to\pi^0\gamma)}$ are somewhat underestimated (about 20%) due to neglect of higher resonances in VDM. (As follows from asymptotic behaviour of the corresponding formfactors, the signs of ρ'' and ρ contribution are opposite to one another). Thus, the general conclusion is that our value of $g_{\omega\rho\pi}$ agrees very well with all the data.

We are very grateful to A.V.Smilga for numerous discussions of the details of the calculations in the fixed point gauge, to M.A.Shifman for permanent interest and careful reading of the manuscript and to V.A.Khoze for the information on the experimental data.

R e f e r e n c e s

1. M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, Nucl.Phys., B147 (1979), 385, 448.

2. L.J.Reinders, S.Yazaki, H.R.Rubinstein, Nucl.Phys. B196(1982) 125.

3. B.L.Ioffe, Nucl.Phys. B188 (1981) 317; B191 (1981) 591;
V.M.Belyaev, B.L.Ioffe, Preprint ITEP-59, 1982;
Y.Chung, H.G.Dosch, M.Kremer, D.Shall, Nucl.Phys. B197 (1982) 55.

4. B.L.Ioffe, A.V.Smilga, Preprint ITEP-27, 1982.

5. V.A.Nesterenko, A.V.Radyushkin, Pisma ZhETF, 35 (1982) 395.

6. A.Yu.Khodjamirian, Phys.Lett. B90 (1980) 460.

7. A.D.Dolgov and [illegible] Zakharov, Nucl.Phys. B27 (1971) 525

8. J.Schwinger, Particles, Sources, Fields, Addison-Wesley, 1973.

9. M.S.Dubovikov and A.V.Smilga, Nucl.Phys. B[illegible] (1981) 109.
A.V.Smilga, Yad.Fiz. 35 (1982) 473.

10. M.Gell-Mann, D.Sharp and W.G.Wagner, Phys.Rev.Lett., 8 (1962) 261.

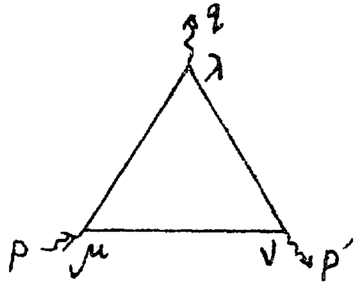

Fig.1. The main diagram for the amplitude $A_{\mu\nu\lambda}(p,p',q)$.
Wavy lines correspond to external currents.

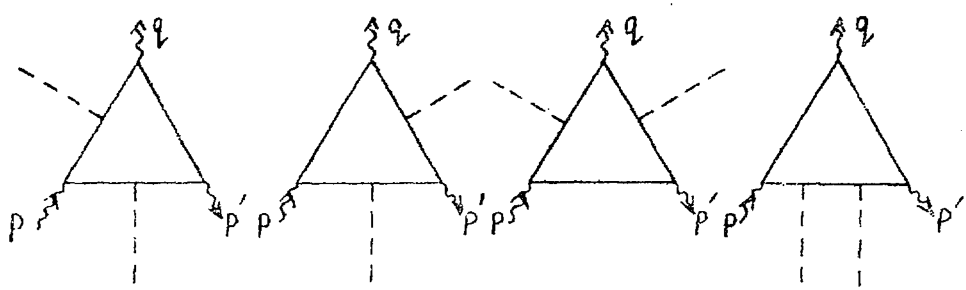

Fig.2. Gluon condensate corrections.

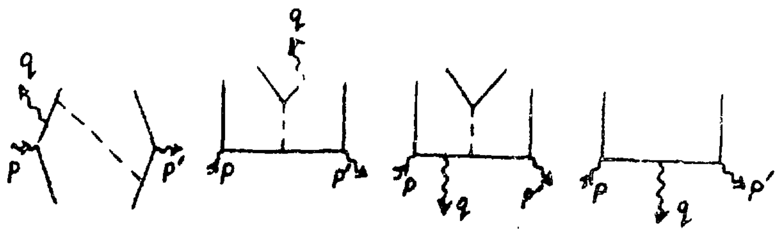

Fig.3. Quark condensate corrections.

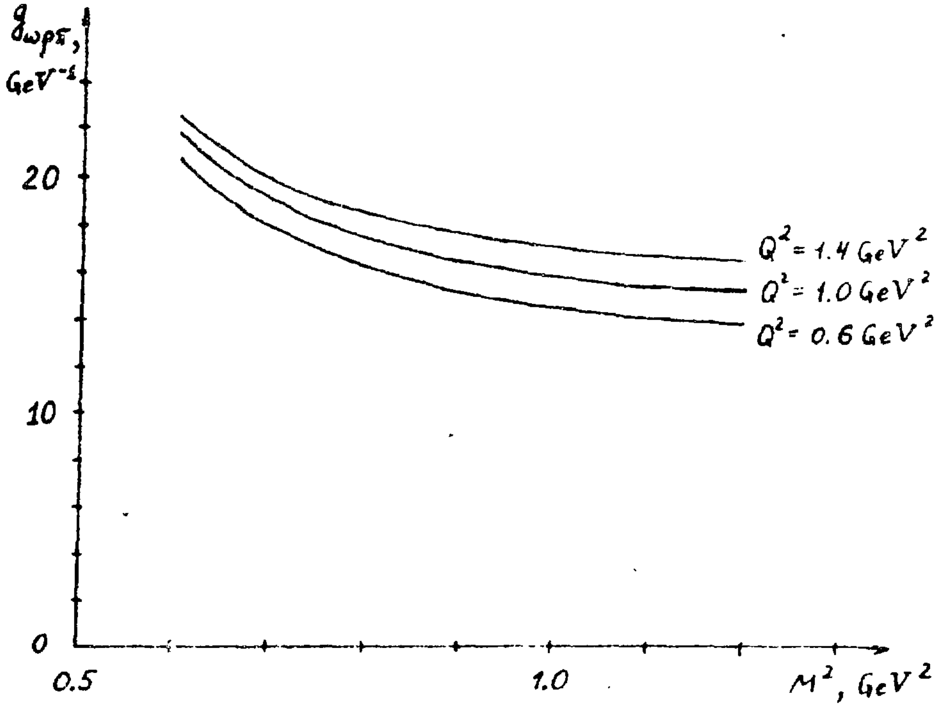

Fig.4. Coupling $g_{\omega\rho\pi}$ versus Q^2 and the Borel transformation parameter M^2 .

В.Л.Елецкий, Б.Л.Иоффе, Я.И.Коган
Вычисление константы $g_{\omega\rho\pi}$ из дисперсионных правил сумм КХД
Работа поступила в ОНТИ I.07.82

Подписано к печати 8.07.82. TI339I Формат 60 x 90 I/I6
Офсетн.печ. Усл.-печ.л.0,75 Уч.-изд.л.0,5. Тираж 290 экз.
Заказ 98. Индекс 3624 Цена 7 коп.

Отпечатано в ИТЭФ, II7259, Москва, Б.Черемушкинская, 25

ИНДЕКС 3624

М., ПРЕПРИНТ ИТЭФ, 1982, № 98, с. 1-12