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Information on pion-nucleus optical potentials from elastic scattering

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Abstract

Data on the elastic scattering of pions by nuclei between 20 and 230 MeV is analyzed in an almost model-independent fashion. The real part of the potential, which is described by a bias-free Fourier-Bessel series, is found to have the typical Kisslinger or Laplacian-like shape between 30 and 80 MeV.

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With the availability of accurate and extensive experimental results on the elastic scattering of low to medium energy pions by nuclei[1-9] it has become feasible to analyze such data with pion-nucleus optical potentials.[10-14] The pion-nucleon interaction in this energy region is dominated by the (3,3) resonance and, therefore, analyses have been performed using the Kisslinger potential[15] or some variants of it, with generally reasonable success. This potential is usually written in terms of the densities of neutrons and protons in the target nucleus (and their derivatives), and it contains as many as 13 parameters which cannot all be determined from the scattering and reaction data. Therefore, several assumptions are often made and χ^2 fits to scattering data as well as to pionic atom data[14,16-18] provide the values of the remaining parameters of the potential. Obviously, making a fit to the data with the help of a model does not necessarily reveal the true information content of the data; some information could be masked out in such a procedure or alternatively one may be led to conclusions which result from the model itself rather than from the data. This problem is particularly interesting in the case of the pion-nucleus interaction because of the peculiar properties of the Kisslinger potential which could have pathological behavior under quite realistic conditions.[19]

In this letter I report on a "model-independent" analysis of elastic scattering of pions by nuclei between 20 and 230 MeV. With very little *a priori* assumptions on the shapes of the potentials, complex optical potentials together with their uncertainties are constructed from fits to the data over the interesting region from the lowest energies feasible to just beyond the (3,3) resonance.

So-called "model-independent" methods have been most successful in analyses of the elastic scattering of medium energy protons and alpha particles by nuclei,[20-23] where one determines either strengths and shapes of optical potentials or shapes of nuclear densities by performing χ^2 fits to elastic scattering data. In the latter case a specific interaction model must be introduced in order to relate the optical potentials to nuclear densities. These analyses of hadron scattering follow similar studies of electron scattering[24,25] where the interaction is known and only the shape of the charge distribution is obtained from fits to the data. In the present work one looks for an optical potential U which is inserted into the Klein-Gordon equation,

$$\hbar^2c^2(\nabla^2+k^2)\psi = \left[2E(U+V_c) - V_c^2\right]\psi ,$$

where V_c is the Coulomb potential due to the finite charge distribution of the nucleus, E and $\hbar k$ are the total energy and momentum of the pion, respectively. Quadratic terms involving the potential U are usually[10-14] excluded. The real part of the optical potential is parametrized in terms of a Fourier-Bessel (FB) series [20,21] as follows:

$$\mathrm{Re}U(r) = \sum_{n=1}^{N} a_n j_o(n\pi r/R_c) ,$$

where R_c is a cut-off radius beyond which ReU is set to zero. The imaginary part of the potential is parametrized as a conventional Woods-Saxon potential

$$\mathrm{Im}U(r) = -W/\left[1 + \exp\left\{(r-R_I)/a_I\right\}\right]$$

so as to avoid the undesirable occurrence of a flux-creating potential over some regions of the nucleus, as occasionally happens with the Kisslinger potential. The free parameters of the potential (W, R_I, a_I

and the coefficients a_n) are determined by a χ^2 fit procedure. An attractive feature of the FB method[20,21] is the possibility to calculate, from the covariance matrix, the uncertainties of any quantity which is obtained from the coefficients a_n and in particular the uncertainty of ReU(r) as a function of position.

In the present work fits were started with no nuclear potential at all and R_I was kept close to the radius of the nuclear charge distribution at the initial stages of the parameter searches. It is emphasized that as the real potential was zero at the beginning of the fit procedure it could converge to whatever shape was required in order to fit the data. Calculations were made for several values of the cut-off radius R_c and several number of terms N, to ensure convergence to a well-defined potential. Typical values of R_c were between 4.5 and 10 fm (depending on the target nucleus) and the number of terms was usually 3 or 4. Some fits were also made with different choices of initial values to ascertain that the final potentials did not depend on the procedure used.

The data analyzed consists mostly of published results[1-5,7,8] but it includes also some unpublished data,[6,9] particularly for π^-. The nuclei studied are ^{12}C, ^{16}O, ^{40}Ca, Fe, ^{90}Zr and ^{208}Pb at energies between 20 and 230 MeV. The quality of the fits was always very good, with values of χ^2 per degree of freedom in the range of 0.5–2. Figure 1 shows an example for the dependence of the resulting optical potential on the bombarding energy for π^+ elastically scattered from ^{12}C. The error bars for the real potential are obtained from the covariance matrix of the least-squares fit procedure.[20,21] The imaginary potential is shown by the continuous curves and is plotted with its sign reversed, for clarity of display. The local spread in its values is typically within 10%. It is seen that at 30 MeV the real part of the potential is poorly determined, but it becomes better determined as the energy increases. The shape of the potential is completely different from the shapes of optical potentials found for nucleons and light ions over the same energy range, and it has the characteristic shape of the Kisslinger or Laplacian potentials, resulting from the p-wave term in the pion-nucleon interaction. The absorptive part of the potential is seen to increase with energy in this energy range, as expected. Figure 2 shows the potentials for π^+ and π^- scattered from ^{40}Ca. At 65 MeV the real potentials have the characteristic shape mentioned above, with evidence for an energy dependence observed via the Coulomb effect. Similar energy dependence is observed also in other cases. At the two higher energies for which results are shown, the absorption becomes very strong and the real potential, as extracted from the scattering data, loses its unique shape. It appears that the scattering below 80 MeV is dominated by the real potential whereas at higher energies the imaginary potential is dominant.

The last point may be made clearer by considering a mean free path derived from the optical potential, $\lambda = -\hbar^2c^2k/2E\,\mathrm{Im}U$, with $\hbar k$ the local momentum and disregarding for this argument corrections due to effective mass and nonlocality. It is seen from Figs. 1,2 that below 80 MeV the penetration into the nucleus is indeed determined by the real potential. In fact, at the lowest energies it is limited by repulsion and not by absorption. The situation is reversed at the higher energies. The above picture is reflected also in the values of the reaction cross section σ_R (the total reactive content) calculated from the best fit potentials. An unusually large sensitivity of σ_R to the details of the real potential is observed at the lower energies. For the very few cases where experimental results for σ_R are available,[26] using these in the χ^2 fits greatly improves the accuracy of the deduced real potential. Extending measurements of σ_R to low energies could improve our knowledge of the real part of the pion-nucleus optical potential.

The question of how much residual dependence on the procedure is present in these potentials is a difficult one. Because of the finite number of terms in the FB series there obviously is some coupling between the values of ReU at different radii (as can also be verified with the help of the covariance matrix). The two figures shown are typical of the results obtained in all the other cases and this applies to the detailed shapes as well as to the uncertainties. Where in the nuclear interior the accuracy is lost is not precisely determined.

Another important question is the relationship between the potentials obtained in the present work and those obtained when fits to the data are made using potentials which explicitly depend on some theoretical considerations. The most widely used is the Kisslinger potential[15]

$$U_k(r) = \left(\hbar^2c^2/2E\right)\left[q(r) + \underset{\sim}{\nabla}\cdot\alpha(r)\underset{\sim}{\nabla}\right] ,$$

where $q(r)$ and $\alpha(r)$ are the components resulting from the s-wave and p-wave terms in the pion-nucleon interaction. This potential can be transformed[27] into an equivalent local but energy-dependent potential

$$U_{eff} = (\hbar^2c^2/2E)\left[q - k^2\alpha - \frac{1}{2}\,\nabla^2\alpha - \frac{(\underset{\sim}{\nabla}\alpha)^2}{4(1-\alpha)}\right]/(1-\alpha)$$

and U_{eff} could be identified with U, at least for the purpose of calculating elastic scattering. (U_{eff} is obtained for the transformed[27] wave function $\tilde{\psi} = (1-\alpha)^{1/2}\psi$.) However, the imaginary part of U_{eff} does not have the smooth shape assumed for ImU and it often changes sign thus creating flux locally. U cannot be used directly in distorted waves calculations because it provides $\tilde{\psi}$ and not ψ. This last difficulty does not arise in the Laplacian interpretation of U because then $\tilde{\psi} = \psi$ (and also the $1-\alpha$ denominator as well as the terms with $(\nabla\alpha)^2$ do not appear in U_{eff}).

In conclusion, "model-independent" analyses of elastic scattering of pions by nuclei show, for the first time, that the typical shape of the pion-nucleus optical potential expected from the p-wave term of the pion-nucleon interaction is indeed obtained from the data without any prior assumptions regarding its existence. This is so for energies below ≈80 MeV where the penetration of pions into the nucleus is determined by the real potential. Further studies of these potentials, which are rather well determined experimentally near the nuclear surface, could provide new information on the pion-nucleus interaction.

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<u>Figure Captions</u>

1. The real part (dots with error bars) and the negative of the imaginary part (smooth curves) for the π^{+}-^{12}C optical potential. The data used are from Refs. 4,5,9.

2. The $\pi^{\pm}$-^{40}Ca optical potentials. See also caption to Fig. 1. The data used are from Refs. 7,8.

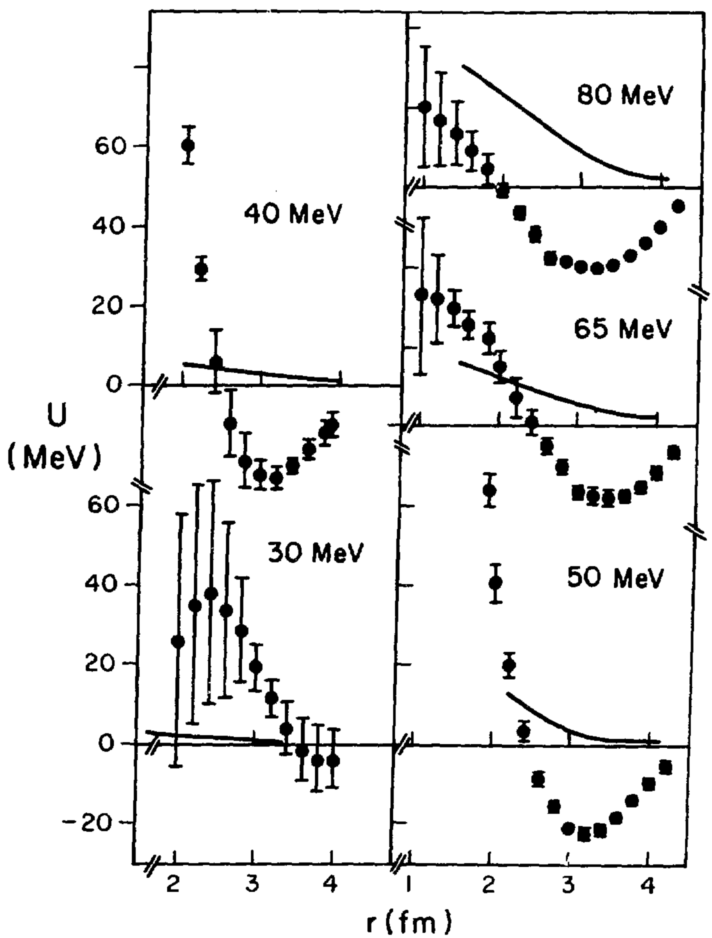
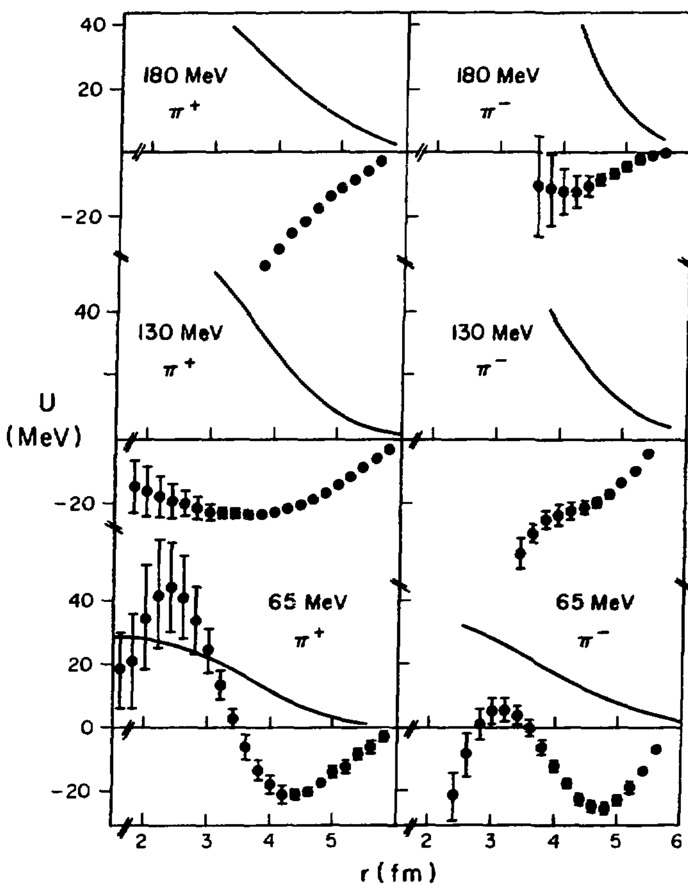


Fig. 1



180 MeV
π^+
180 MeV
π^-
130 MeV
π^+
130 MeV
π^-
65 MeV
π^+
65 MeV
π^-
U
(MeV)
40
20
-20
40
-20
40
20
0
-20
2
3
4
5
2
3
4
5
6
r (fm)

Fig. 2