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ATOMIC ENERGY OF CANADA LIMITED

L'ÉNERGIE ATOMIQUE DU CANADA LIMITÉE

MULTIVARIABLE CONTROL IN NUCLEAR POWER STATIONS: ORDER REDUCTION

Contrôle multivariable des centrales nucléaires Réduction de son caractère très évolué

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Chalk River, Ontario

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Résumé

Les modèles linéaires, à valeurs multiples, des systèmes employés dans les centrales nucléaires ont tendance à être très évolués, ce qui donne lieu à des difficultés dans la conception des dispositifs de contrôle faisant appel aux méthodes à valeurs multiples. Cependant, ces dispositifs n'ont accès qu'aux entrées et aux sorties de la centrale et les variables internes à réponse rapide peuvent être masquées par des variables plus lentes mais dominantes. La réduction du caractère très évolué des modèles s'applique à l'analyse modale pour éliminer les modes rapides tout en conservant le caractère de la réponse des entrées et des sorties. Deux autres techniques présentées ont fait l'objet de démonstrations dans le modèle d'un générateur de vapeur nucléaire. La technique préférée est mise en oeuvre dans le programme MVPACK destiné à la conception aidée par ordinateur.

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ABSTRACT

Linear, state-space models of power plant systems tend to be of high order, leading to difficulties in the design of control systems by state-space methods. However, the control system has access only to the plant inputs and outputs, and the fast-responding internal variables may be masked by slower, dominant variables. Model order reduction applies modal analysis to eliminate the fast modes while retaining the character of the input-output response. Two alternative techniques are presented and demonstrated on a model of a nuclear steam generator. The preferred method is implemented in MVPACK, the computer-aided design package.

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NOMENCLATURE

In general, capital letters represent matrices, while lower-case letters are used for vectors and scalars. No additional marking is used to identify vectors or matrices. Such distinctions will be clear from the context.

Symbol	Description	Defining Equation
A	Plant dynamics matrix	(1)
A^*	Reduced model dynamics matrix	(22),(33)
$A_{11},A_{12},A_{21},A_{22}$	Partitions of A	(6)
B	Plant input matrix	(1)
B^*	Reduced model input matrix	(23),(34)
C	Plant output matrix	(2)
C^*	Reduced model output matrix	(29),(36)
C_1,C_2	Partitions of C	(27)
D	Direct input-output coupling matrix	(2)
D^*	Reduced system direct input-output coupling matrix	(30),(37)
I	Identity matrix of adequate dimension	
P	Permutation matrix	(3)
r	Order of the reduced model	

NOMENCLATURE (cont'd)

Symbol	Description	Defining Equation
t	Time	
u	Input vector	(1)
U	Dual eigenvector matrix	(11)
U_1^T, U_2^T	Partitions of U^T	(12)
V	Eigenvector matrix	(5)
$V_{11}, V_{12}, V_{21}, V_{22}$	Partitions of V	(6)
x	State vector	(1)
x_1, x_2	Partitions of x	(8),(9)
x_i	$i = 1$ to 15, state variables of steam generator model	Appendix A
y	Output vector	(2)

Greek Letters

Symbol	Description	Defining Equation
Λ	Eigenvalue matrix	(5)
Λ_1	Upper left rxr block of Λ	(6)
Λ_2	Lower right (n-r)x(n-r) block of Λ	(6)

NOMENCLATURE (cont'd)

		Defining Equation
ξ	Modal state vector	(7)
ξ_1, ξ_2	Partitions of ξ	(8),(9)

1. INTRODUCTION

Multivariable methods can provide elegant solutions to the control problems of complex systems such as nuclear power plants. These methods, based on well-established mathematical theory, take into account the various phenomena and interactions in the plant and produce improved control strategies compared with those given by conventional, empirical, single-variable methods. To demonstrate the feasibility of multivariable control in nuclear power plants, MVPACK [1], a computer-aided design system for multivariable controllers, was developed in the Dynamic Analysis Laboratory at the Chalk River Nuclear Laboratories.

In the development of the MVPACK optimal control module, it was found that an order-reduction module would be useful to generate low-order plant models suitable for controller designs. For most control problems, the plant model available is too complex to meet the needs of the control designer. Order reduction can be used to extract, from a high-order linear model, a simplified plant model to meet the design requirements for

- order,
- accuracy,
- variables involved, and
- measurability of the state variables.

This is a reasonable alternative to building a completely new model because one takes advantage of the experience gained in the development of the larger model.

This report presents two order-reduction techniques. Each is a different development of the same basic idea.

These methods were compared on a 15th-order steam generator model. Only one of the two proved to be acceptable, and the corresponding module, MVREDN, has been implemented in MVPACK.

2. ORDER-REDUCTION TECHNIQUES

The order-reduction techniques presented here are based on the modal properties of the system. Both result in reduced models that retain the dominant modal characteristics of the original model. The first technique was introduced by Skira and DeHoff [2], while the second one expands on a strategy introduced by Davison [3]. The notation used follows that adopted earlier [4] for the development of MVPACK.

2.1 The Order-Reduction Problem

A prerequisite for controller design is a mathematical model of the plant, suitable for the application of established control theories. This model is usually in the form of differential equations describing the fundamental physical phenomena of the plant. Once tuned to adequately describe the behavior of the actual plant, such a model offers valuable insight into process dynamics. The state-space representation of all or part of a nuclear power station is usually very complex with a large number of parameters and state variables. Also, the model is often non-linear, so that linearization about a nominal operating point is needed to yield the linear model required for design.

In theory, multivariable techniques can handle high-order linear systems with complex internal interactions. However, two limitations appear in actual application. First, the designer must adjust a large number of parameters and may have difficulty interpreting the various effects obtained. Second, most multivariable techniques lead to a state-feedback controller. An observer or Kalman filter is then required to estimate the complete state vector, with a corresponding loss of simplicity.

A way to circumvent these limitations is to build a low-order model of the plant in which most state variables are those that are amenable to measurement. The controller design method then leads to a low-order controller. For the favorable cases in which the reduced state vector corresponds to plant measurements, it results in an output-feedback controller directly applicable to the plant.

Order reduction is an analytical procedure that extracts the matrices of the desired low-order model from the given high-order model. The philosophy of the two techniques presented relies on the same basic idea as the simplification of high-order, single-variable systems. It drops the fast, stable modes, while retaining all the unstable modes and the slow modes that lie in the bandwidth of interest for the controller. Care is required in the multivariable case because of the coupling between the state variables. Order reduction requires the definition of a reduced state vector. As seen below in Sections 2.5 and 2.6, the two techniques presented differ at this point. However, they both derive the new state vector from a reduced set of the actual plant state variables. This is the crucial point at which the designer must make a decision. Finally, the designer should simulate the reduced model to verify that it is suitable for design.

2.2 Description of the Complete Model

The system under study is described by a linear model about the operating point chosen by the designer, with the state-space formulation

$$\frac{d}{dt} x = Ax + Bu \qquad (1)$$

$$y = Cx + Du \qquad (2)$$

where

x is the state vector, of dimension n

u is the input vector, of dimension p

y is the output vector, of dimension m

A,B,C and D are constant matrices of appropriate dimensions

This formulation leads to the block diagram shown in Figure 1.

As mentioned previously, the reduced state vector is derived from a selection of the actual state variables. Thus, the designer must first determine the state variables on which the control strategy is to be based. They are preferably measurable variables that dominate the dynamics of the system. Once this selection has been made, the chosen state variables can be placed at the beginning of the state

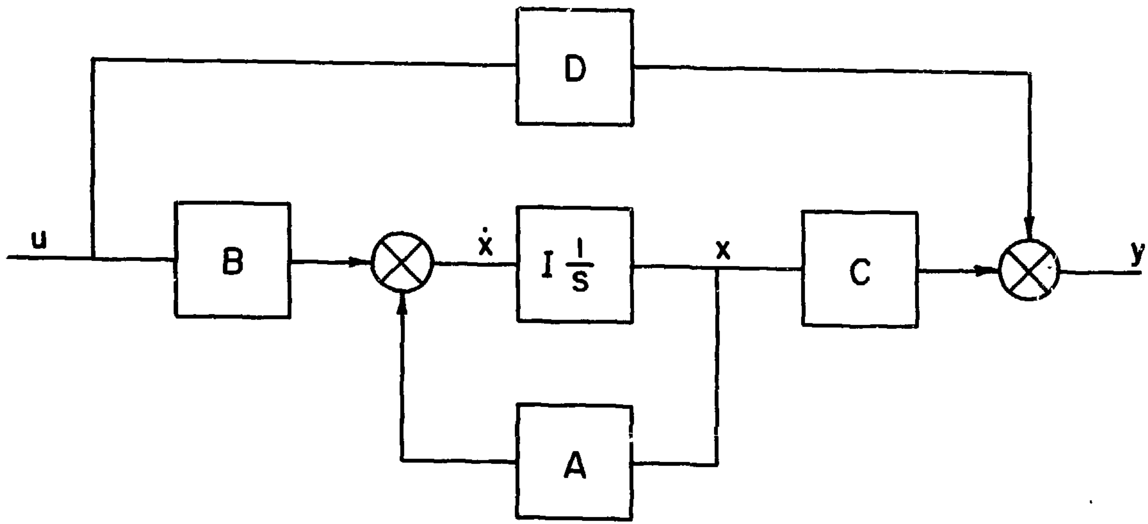

FIGURE 1 BLOCK DIAGRAM OF A LINEAR MULTIVARIABLE SYSTEM

vector, using an appropriate permutation, to yield the revised model equations.

$$\frac{d}{dt}(Px) = (PAP^{-1})Px + (PB)u \qquad (3)$$

$$y = (CP^{-1})Px + Du \qquad (4)$$

where P is an nxn permutation matrix, and

Px is the new state vector.

Note that for a permutation matrix $P^{-1} = P^T$.

This permutation simplifies the formulation of reduction techniques using matrix algebra. For ease in the sequel, it is assumed that the model given by equations (1) and (2) has already been suitably permuted. Thus, if the order of the desired reduced model is r, the first r state variables are those selected by the designer.

The modal decomposition of the plant dynamics matrix A is defined by

$$A \cdot V = V \cdot \Lambda \qquad (5)$$

where

Λ is the diagonal matrix of the eigenvalues of A

V is the matrix whose columns are the eigenvectors of A.

It is assumed that the eigenvalues of A are distinct. The case of repeated eigenvalues without independent eigenvectors occurs infrequently and will not be considered further; consequently, V is assumed nonsingular. Practically, this calculation is performed with real matrices, replacing complex pairs by their real and imaginary parts [4]. In this case, Λ becomes a block-diagonal matrix.

For the theoretical development, it is convenient to partition matrices and vectors such that the subscript 1 corresponds to the r first elements. The modal decomposition of A then becomes

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \begin{pmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{pmatrix} \quad (6)$$

In the rest of the report, the index 1 is associated with the r first elements of the vector, row or column.

To satisfy further requirements in the reduction analysis, eigenvalues and eigenvectors are ordered in such a way that Λ_1 corresponds to the r slowest modes, and the following assumptions are made

- Λ_2 and V_{11} are nonsingular,

- a complex pair must not be split in the partitioning, so the choice of r is not completely free.

In practice these conditions are easily satisfied when the designer makes an appropriate and reasonable choice of the state variables.

2.3 Mode Simplification

A modal state vector ξ can be defined by

$$V\xi = x \tag{7}$$

and partitioned as

$$V_{11}\xi_1 + V_{12}\xi_2 = x_1 \tag{8}$$

$$V_{21}\xi_1 + V_{22}\xi_2 = x_2 \tag{9}$$

In the modal state space, the behavior of the system is now described by the modal state equation

$$\frac{d}{dt}\xi = \Lambda\xi + U^T Bu \tag{10}$$

where U is the dual eigenvector matrix [4] whose columns are the eigenvectors of the transpose of A, normalized such that

$$U^T V = I \tag{11}$$

i.e.

$$V^{-1} = U^T = \begin{pmatrix} U_1^T \\ \\ U_2^T \end{pmatrix} \tag{12}$$

Equation (10) is partitioned to separate slow and fast modes, yielding the differential equations

$$\frac{d}{dt}\,\xi_1 = \Lambda_1 \xi_1 + U_1^T Bu \qquad (13)$$

$$\frac{d}{dt}\,\xi_2 = \Lambda_2 \xi_2 + U_2^T Bu \qquad (14)$$

The choice in Λ_1 of the r slowest modes justifies the key assumption on which order reduction is based: the vector ξ_2 involving the fastest modes is assumed to be always in steady state relative to the vector ξ_1 involving the slowest modes. So, ξ_2 is supposed to have little effect on system dynamics and

$$\frac{d}{dt}\,\xi_2 = 0 \qquad (15)$$

is assumed.

The interpretation of the approximation (15) requires care; indeed, it can be objected that although the fastest modes are the first to reach steady state, they are also those that can have the largest derivatives, in apparent contradiction of equation (15). Use of equation (15) transforms equation (14) to

$$\Lambda_2 \xi_2 + U_2^T Bu = 0 \qquad (16)$$

so ξ_2 follows the input directly, and in a transient generated by a rapidly varying input, $d\xi_2/dt$ can be very large.

Finally, the reduced modal equations of the plant are

$$\frac{d}{dt}\xi_1 = \Lambda_1\xi_1 + U_1^T Bu \qquad (17)$$

$$\xi_2 = -\Lambda_2^{-1} U_2^T Bu \qquad (18)$$

2.4 First Reduction Technique

The first technique, introduced by Skira and DeHoff [2], defines a reduced state vector x_1 containing the first r elements of the complete state vector x. Partitioning equation (1) gives

$$\frac{d}{dt}x_1 = A_{11}x_1 + A_{12}x_2 + Bu \qquad (19)$$

Elimination of ξ_1 between (8) and (9) and of ξ_2 with (18) yields

$$x_2 = V_{21}V_{11}^{-1}x_1 - (V_{22}-V_{21}V_{11}^{-1}V_{12})\Lambda_2^{-1}U_2^T Bu \qquad (20)$$

so that the neglected states x_2 are in equilibrium with x_1 and u.

Then, substitution of equation (20) into (19) forms the reduced model state equation

$$\frac{d}{dt}x_1 = A^* x_1 + B^* u \qquad (21)$$

where

$$A^* = A_{11} + A_{12}V_{21}V_{11}^{-1} \qquad (22)$$

$$B^* = B_1 - A_{12}(V_{22}-V_{21}V_{11}^{-1}V_{12})\Lambda_2^{-1}U^TB \qquad (23)$$

The modal decomposition of A, equation (6), yields alternative expressions for A^* and B^* as

$$A^* = V_{11}\Lambda_1V_{11}^{-1} \qquad (24)$$

$$B^* = (V_{11}\Lambda_1V_{11}^{-1}V_{12}\Lambda_2^{-1}U_2^T + V_{11}U_1^T)B \qquad (25)$$

These forms require only the eigenvectors and eigenvalues of A, so they are more suitable for computation.

With the same partitioning of the state vector x, the complete model output equation becomes

$$y = C_1x_1 + C_2x_2 + Du \qquad (26)$$

where

$$C = (C_1,C_2) \qquad (27)$$

Then, replacing x_2 by expression (20), the output equation of the reduced model is

$$y = C^*x_1 + D^*u \qquad (28)$$

where

$$C^* = C_1 + C_2 V_{21} V_{11}^{-1} \tag{29}$$

$$D^* = D + C_2 (V_{21} V_{11}^{-1} V_{12} - V_{22}) \Lambda_2^{-1} U_2^T B \tag{30}$$

In this method, the steady state of the neglected modes ξ_2 appears in the first term of B^*, equation (25). The retained states normally include all the outputs, so C_2 is 0 and D^* contains only D. This ensures that the resulting transfer-function matrix has acceptable high-frequency roll-off, but the nature of B^* completely alters the low-frequency response.

2.5 Second Reduction Technique

This technique is based on a method developed by Davison [3]. The reduced state vector is defined as

$$x_r = V_{11} \xi_1 \tag{31}$$

Then, using equation (17), the reduced state-space model is

$$\frac{d}{dt} x_r = A^* x_r + B^* u \tag{32}$$

where

$$A^* = V_{11} \Lambda_1 V_{11}^{-1} \tag{33}$$

$$B^* = V_{11} U_1^T B \tag{34}$$

In this case the reduced model output equation is

$$y = C^{*}x_{r} + D^{*}u \tag{35}$$

where

$$C^{*} = C_{1} + C_{2}V_{21}V_{11}^{-1} \tag{36}$$

$$D^{*} = D - (C_{1}V_{12} + C_{2}V_{22})\Lambda_{2}^{-1}U_{2}^{T}B \tag{37}$$

This order-reduction technique usually introduces an extra direct input-output coupling. Davison [3] chose to ignore this term, which is a direct consequence of the assumption that the fast modes follow the inputs. As a result, he obtained incorrect steady-state gains.

This method ensures an acceptable low-frequency response, but the introduction of D^{*} means that the transfer-function matrix has a finite gain and zero phase shift at high frequencies. If D^{*} is small, it can be neglected, to yield a reduced transfer function that is acceptable at all frequencies.

2.6 Selection of State Variables

The designer is free to choose the state variables defining the reduced state vector. However, once the order of the reduced model has been decided, the usefulness of the model depends strongly on the definition of its state vector. No exact rule can be given about this choice, but the following guidelines are useful:

- Measurable state variables of the complete model should be selected. The ideal solution is a reduced state vector that is an approximation of the complete system output factor. In this case, output feedback and state feedback are equivalent and the controller is simplified.

- A property of the reduced model is that the modal decomposition of its dynamics matrix is

$$A^{*}V_{11} = V_{11}\Lambda_{1} \qquad (38)$$

  Thus, the dynamics of the reduced model involve only the r slowest modes of the complete model, and their distribution can be deduced easily from the modal distribution of the complete model.

- Generally, despite the complex internal interactions encountered in a high-order system, the dominant modes of a system can be related to a small set of state variables describing the major physical phenomena involved. These variables are the important ones in control design problems. So, knowledge of the plant physics is a key factor in order reduction.

3. IMPLEMENTATION IN MVPACK

The calculations involved in multivariable methods require an interactive computer-aided design system. MVPACK [1] is an interactive system of computer programs developed in the Dynamic Analysis Laboratory at CRNL, for the purpose of evaluating multivariable control design methods.

MVREDN is the order-reduction design module of MVPACK. This program accepts a linear state-space model, presented in standard form, and produces a reduced-order model using the second method as presented in Section 2.5. For reasons discussed below, the first method was rejected after testing.

MVREDN performs an eigenanalysis of the complete model by calling the copy of EISPACK [7] included in MVPACK, and displays the modes of the system at the terminal. Then, the user is asked for the state variables to keep in the reduced model; any attempt to split a complex pair or other inconsistency is rejected immediately. The result generated by MVREDN is presented in a new data file, as a model in standard form. An additional vector is created to identify the retained state variables.

4. EXAMPLE: REDUCTION OF A 15th-ORDER STEAM GENERATOR MODEL

4.1 Model Analysis

The example of a complex control problem presented here is related to a nuclear steam generator [5]; the reduction techniques are applied to the 15th-order model previously studied to demonstrate the modal control design program [6] in MVPACK. Since it is not the aim of this report to design a specific controller for this given system, the accuracy of the reduced models is evaluated only in open loop.

The state variables, inputs and outputs of the model are identified in Appendix A. The state-space matrices A, B and C were given in reference [6] and they will not be repeated here. When the steam generator is isolated from a reactor, the primary inlet temperature T_{pi} is uncontrollable and the primary outlet temperature T_{po} is unobservable, so these states have no significance for control. However, these variables are associated with a repeated eigenvalue, making analysis difficult. To eliminate this problem, element (1,1) of matrix A was changed from 1.43 to 10. Because the state involved is uncontrollable, this change does not affect the control problem.

Reduced models of 6th and 4th orders were derived using both techniques presented. So, 6 state variables were chosen to define the new state vector. Of the 15 variables involved in the complete model, 3 are potentially meaningful for control. They are:

x_{11}, L_d	Downcomer level
x_{13}, P_s	Steam pressure
x_{15}, T_d	Downcomer temperature

Actual measurements, x_{11} and x_{13}, are always selected as noted in Section 2.6. The existence of a complex pair of eigenvalues as modes 5 and 6 means that the reduced model cannot be fifth order, so we consider models of orders 4 and 6. The three other variables selected are

x_{14}, x_e	Boiling section exit quality
x_{12}, L_{s1}	Subcooled region length
x_{10}, T_{m4}	Fourth tube metal lump temperature

Table 1 gives the eigenvalues of the 15th-order model, and partial mode shapes of the six slowest modes are shown in Figure 2.

TABLE 1

COMPLETE MODES OF A 15th-ORDER MODEL OF A STEAM GENERATOR

1	4.52×10^{-5}
2	-0.0518
3	-0.239
4	-0.331
5	-0.886 + j 0.255
6	-0.886 - j 0.255
7	-1.30
8	-1.43
9	-3.20
10	-3.28
11	-5.84
12	-5.98 + j 0.0671
13	-5.98 - j 0.0671
14	-6.30
15	-10.0

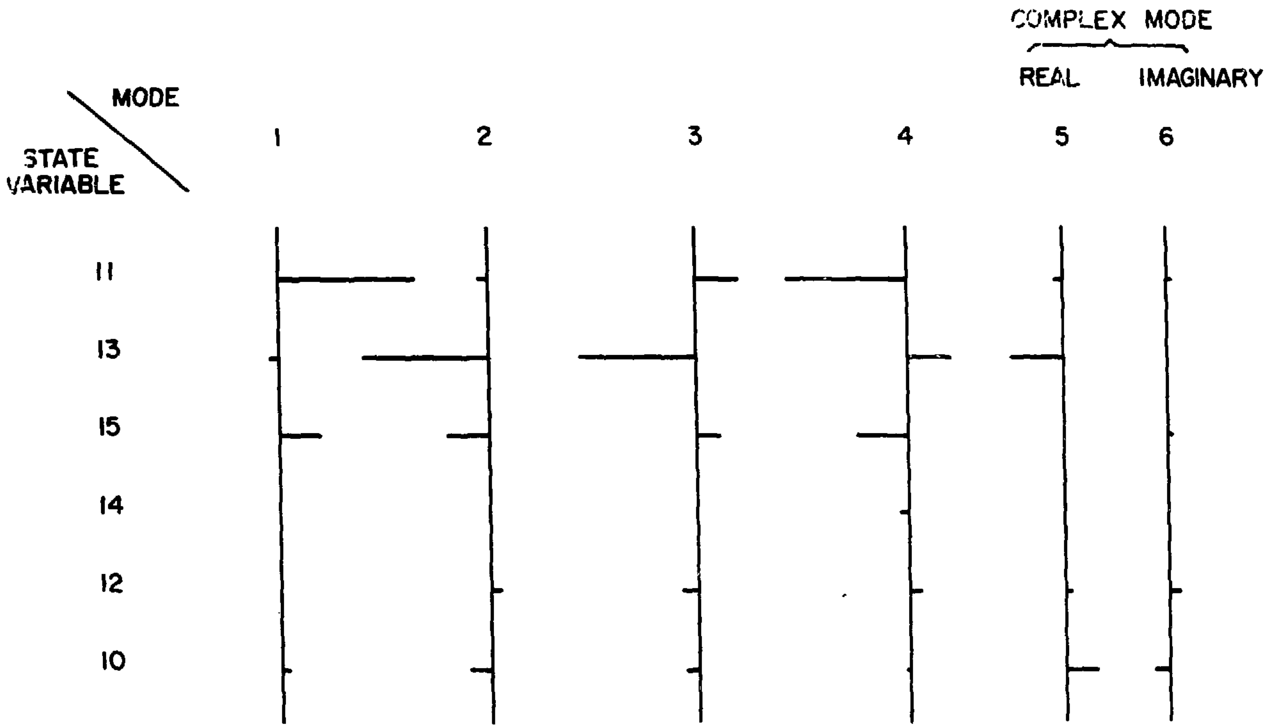

FIGURE 2 STEAM GENERATOR PARTIAL MODE SHAPES

4.2 First Reduction Technique

The first technique was used to produce several reduced models, and it was found to be inconsistent. Indeed, for a given order of reduction, the reduced model is extremely dependent on the selection of state variables. Figure 3 shows the results of simulating two 4th-order reduced models. Model A was obtained by retaining x_{11}, x_{13}, x_{15} and x_{14} from the complete model, while in model B x_{14} was replaced by x_{12}.

The system matrices for both models are given in Appendix B. The forcing function is a trapezoidal pulse on the steam valve lift. In both cases the output variables are steam pressure and downcomer level. It can be seen that the downcomer level trajectory is strongly dependent on the choice of the 4th variable retained.

4.3 Second Reduction Technique

Simulation and theoretical verification shows that the second technique does not induce the problem seen with the first technique. The following Figures 4 and 5 show results from simulations of:

- the complete 15th-order model,
- the 6th-order reduced model obtained by retaining x_{11}, x_{13}, x_{15}, x_{14}, x_{12} and x_{10},
- the 4th-order reduced model obtained by selecting x_{11}, x_{13}, x_{15} and x_{14}.

Reduced system matrices are given in Appendix C. The forcing function is a trapezoidal pulse on either the steam valve lift or the feedwater flow.

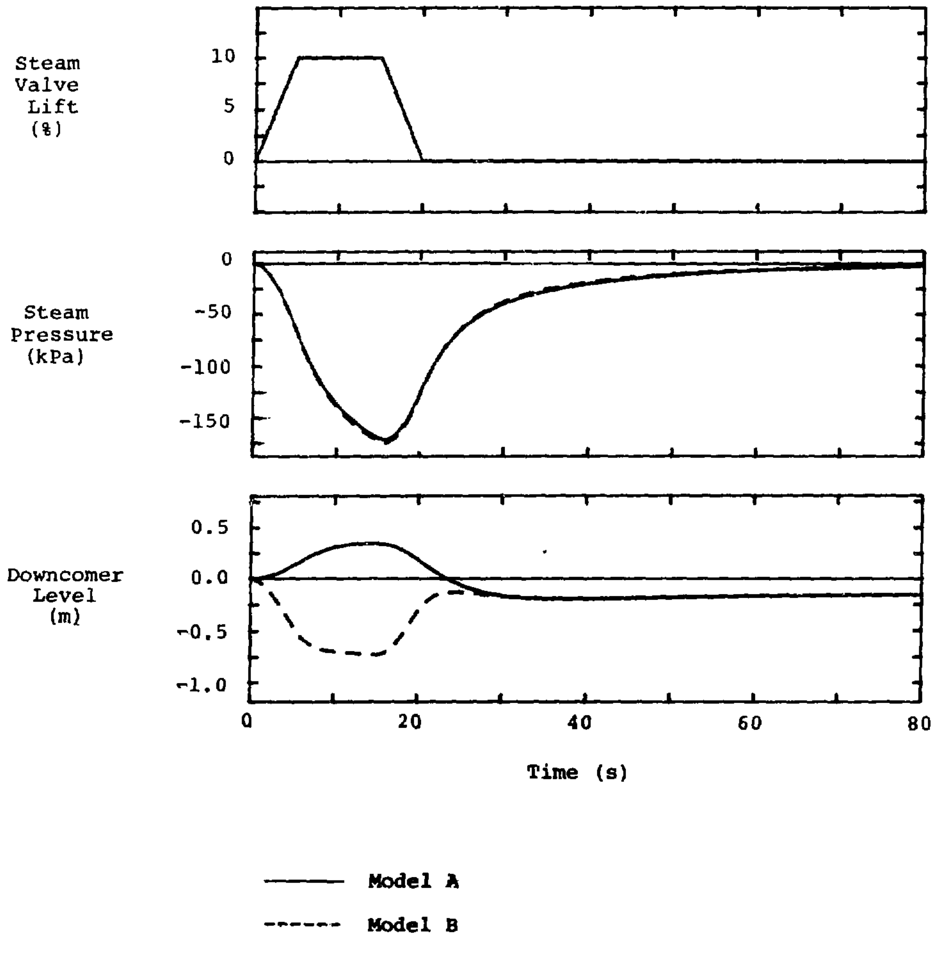

FIGURE 3 **RESPONSES OF TWO 4th-ORDER REDUCED MODELS OBTAINED BY THE FIRST (SKIRA & DEHOFF) METHOD**

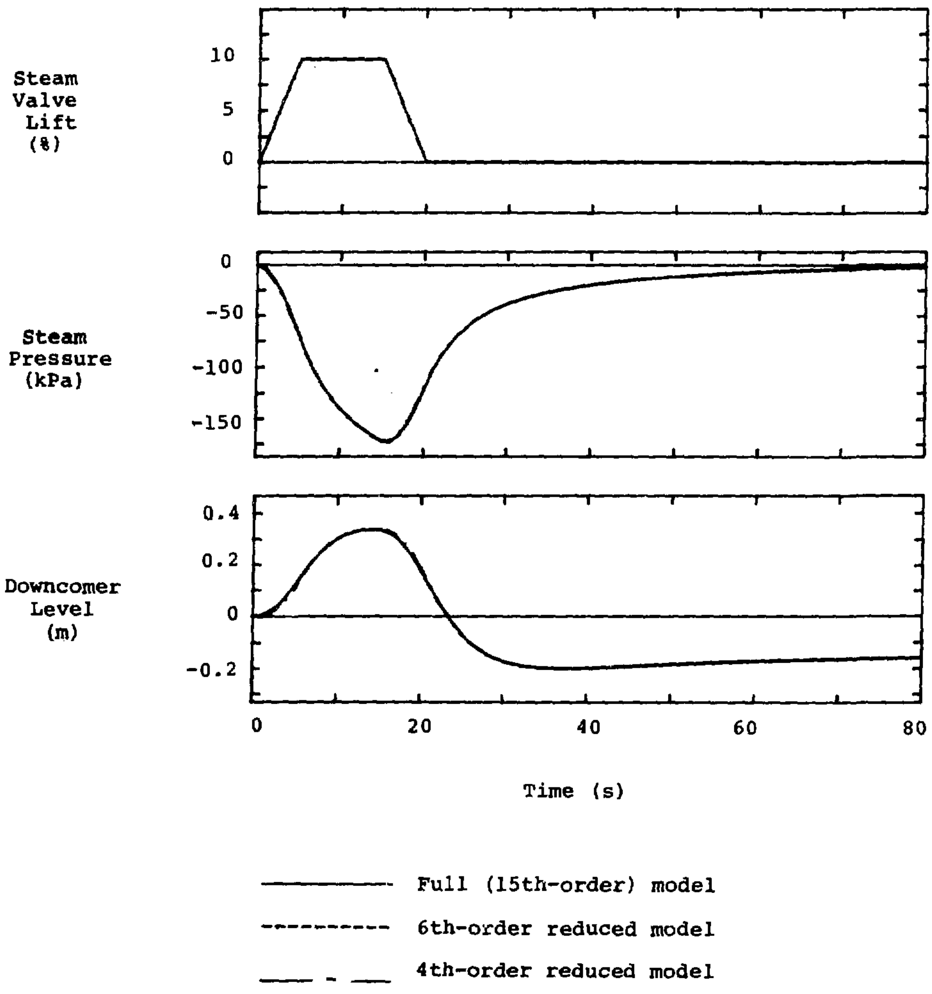

FIGURE 4 RESPONSES OF MODELS, REDUCED BY THE SECOND (DAVISON) METHOD, TO A PULSE IN STEAM VALVE LIFT

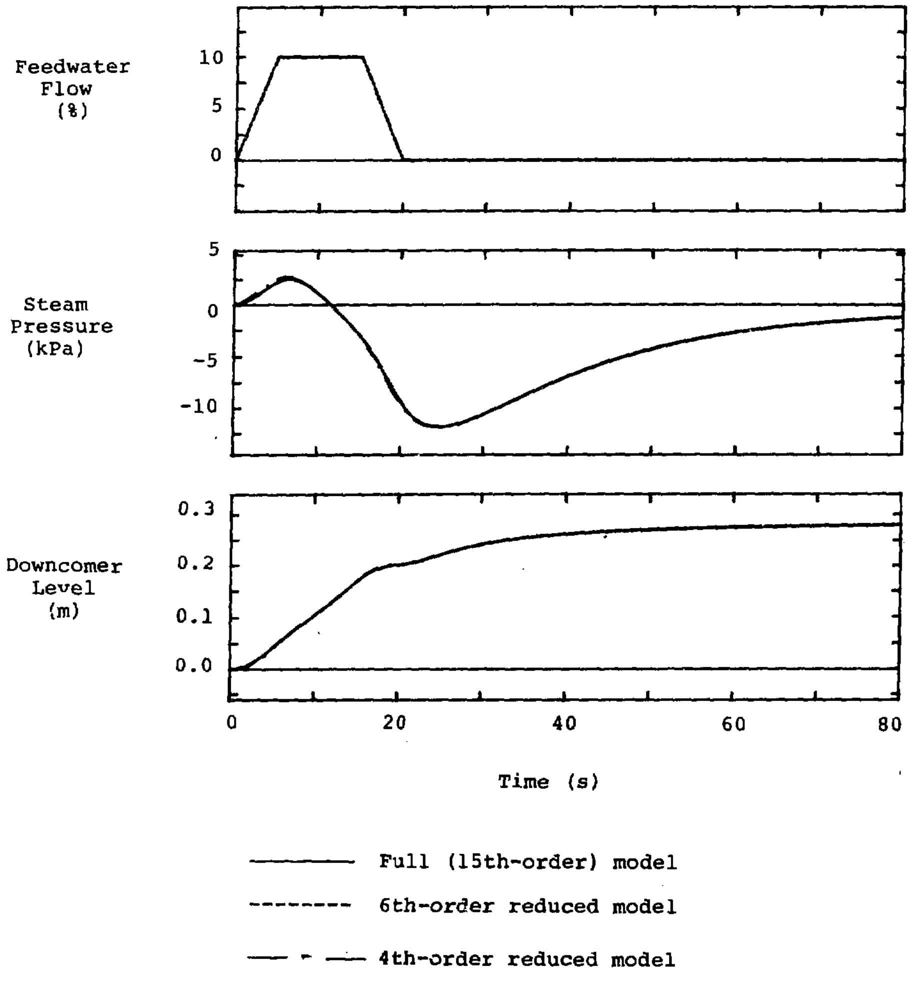

FIGURE 5 RESPONSES OF MODELS, REDUCED BY THE SECOND (DAVISON) METHOD, TO A PULSE IN FEEDWATER FLOW

Both reduced models give good approximations of the complete model output response. It can be noted that the state vectors of the reduced models are good approximations of the retained state variables in the complete model.

4.4 Discussion of Results

From Section 4.2 it can be seen that the first reduction technique does not lead to a satisfactory result. It generates a reduced model by keeping the slow dynamics of the original model and adding a corrective input function such that the steady state is correct. However, this extra input excites the slow modes differently, leading to an incorrect dynamic response which depends on the state variables retained.

The second reduction technique also keeps the slow dynamics of the original model, but it does not introduce any corrective input, so the states approximate the dynamics of the true plant, but they have incorrect steady-state values. However, the error is small since the contribution of the fast modes to the steady state is usually less than that of the slow modes. On the other hand, the fast terms dropped are reintroduced in the output as a direct input-output coupling, so the output has the correct steady-state value. Finally, the amount of reduction will be determined by the desired accuracy of the reduced model.

5. CONCLUSIONS

Given a high-order, linear model of a system to be controlled, modal reduction is a rapid and efficient tool to create a low-order model, convenient both for improved understanding and for control design. This order-reduction approach allows the designer to solve the control problem on a low-order model, by concentrating on the major dynamics effects and disregarding small effects and interactions. However, the designer remains responsible for performing a suitable analysis of the physical properties of the system.

Order reduction does not eliminate the analysis task, but it permits separation of physical analysis from the control design. These operations are combined when one attempts to design a controller using the high-order model of the plant. However, this approach does not provide a true separation of the tasks. Indeed, the controller designed with the reduced model does not necessarily perform as expected on the complete model or on the actual plant. So each controller proposed with the low-order model must be validated on the complete model. Only such a trial procedure or previous experience can establish the acceptability of a reduced model.

An application of order reduction for controller design can be found in reference [8], where optimal control is applied to the steam generator model.

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APPENDIX A - STEP RESPONSE SIMPLIFICATION

State Variables

1.	T_{pi}	Primary Water Input Plenum Temperature
2.	T_{pi}	First Primary Water Lump Temperature
3.	T_{p2}	Second Primary Water Lump Temperature
4.	T_{p3}	Third Primary Water Lump Temperature
5.	T_{p4}	Fourth Primary Water Lump Temperature
6.	T_{po}	Primary Output Plenum Temperature
7.	T_{m1}	First Tube Metal Lump Temperature
8.	T_{m2}	Second Tube Metal Lump Temperature
9.	T_{m3}	Third Tube Metal Lump Temperature
10.	T_{m4}	Fourth Tube Metal Lump Temperature
11.	L_d	Downcomer Level
12.	L_{s1}	Subcooled Length
13.	P_s	Steam Pressure
14.	x_e	Boiling Section Exit Quality
15.	T_d	Downcomer Temperature

Control Variables

1. $\frac{\delta C_L}{C_L}$ Steam Valve Lift

2. $\frac{\delta W_{Fi}}{W_{Fi}}$ Feedwater Flow

Output Variables

Outputs are P_s and L_d.

APPENDIX B

Steam Generator: 15th-Order Model

MATRIX STA   4th-Order Model A

1	11	selected variables
2	13	
3	15	
4	14	
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

MATRIX A

1	-4.955399E-02	1.017674E-03	-1.065533E-03	-15.9053
2	-0.127730	-0.196224	0.457600	-0.515532
3	1.842600E-03	9.095818E-03	-7.852214E-02	-5.83674
4	-1.293423E-03	-3.568750E-04	1.351991E-03	-0.297603

MATRIX B

1	1.34254	0.603640
2	-49.9745	2.56381
3	0.535324	-1.10197
4	-4.444273E-02	1.257635E-02

MATRIX C

1	0.000000	1.00000	0.000000	0.000000
2	1.00000	0.000000	0.000000	0.000000

MATRIX D

1	0.000000	0.000000
2	0.000000	0.000000

MATRIX STA **4th-Order Model B**

1	11	Selected Variables
2	13	
3	15	
4	12	
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

MATRIX A

1	-2.57232	-1.45616	9.03275	31.6944
2	-0.209499	-0.243455	0.750408	1.02729
3	-0.923930	-0.525641	3.23659	11.6308
4	7.330399E-02	3.658591E-02	-0.254548	-1.04272

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MATRIX B

1	-11.3736	24.9[illegible]14
2	-50.3867	3.35[illegible]96
3	-4.13107	7.84[illegible]85
4	-1.90270	-0.328[illegible]03

MATRIX C

1	0.000000	1.00000	0.000000	0.000000
2	1.00000	0.000000	0.000000	0.000000

MATRIX D

1	0.000000	0.000000
2	0.000000	0.000000

APPENDIX C

Steam Generator: Reduced Models

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MATRIX A

1	-5.273889E-02	-2.588933E-03	6.690002E-03	-15.8820	3.431171E-02	1.787385E-02
2	-0.143179	-0.292805	0.331834	-0.257904	-8.873889E-02	0.886702
3	1.893152E-03	8.411587E-03	-8.017659E-02	-5.83576	-2.936987E-03	7.216703E-03
4	4.588883E-04	6.086152E-04	-5.019256E-03	-0.308565	-2.216543E-02	4.720208E-04
5	5.521344E-02	2.691299E-02	-0.188164	0.112637	-0.812936	-7.853503E-03
6	-0.108177	8.893026E-03	0.548799	-0.588279	1.43946	-0.847157

MATRIX B

1	1.27260	0.671767
2	-62.1596	1.92120
3	0.518352	-1.08481
4	-2.997255E-02	-1.261253E-03
5	-1.58919	-5.855814E-02
6	4.93999	1.62860

MATRIX C

1	0.000000	1.00000	0.000000	0.000000	0.000000	0.000000
2	1.00000	0.000000	0.000000	0.000000	0.000000	0.000000

MATRIX D

1	0.412953	0.246832
2	-0.132388	-3.630428E-02

MATRIX A

1	-4.955399E-02	1.017674E-03	-1.065533E-03	-15.9053
2	-0.127730	-0.196224	0.457600	-0.515532
3	1.842600E-03	9.095818E-03	-7.852214E-02	-5.83674
4	-1.293423E-03	-3.568760E-04	1.351991E-03	-0.297603

MATRIX B

1	2.00704	1.03445
2	-46.9032	2.32530
3	0.645932	-0.927857
4	-2.618605E-02	2.011275E-02

MATRIX C

1	0.000000	1.00000	0.000000	0.000000
2	1.00000	0.000000	0.000000	0.000000

MATRIX D

1	-15.5555	1.11459
2	-1.26531	-0.360377

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