ISBN 0-7727-7010-7

PUBLICATION 80-06

UNIVERSITY OF TORONTO

Department of Civil Engineering

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II. HYDROLOGIC EVALUATION OF DISPOSAL SITES

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ABSTRACT

The hydrologic evaluation of mine tailings disposal sites after they are abandoned is considered in relation to their potential environmental impact on a long term basis. There is a direct relation between the amounts and types of water leaving a disposal site and the severity of the potential damage to the environment. The evaluation of the relative distribution of the precipitation reaching the ground into evaporation, runoff and infiltration is obtained for a selected site and type of tailings material whose characteristics and physical properties were determined in the soils laboratory. A conceptual model of the hydrologic processes involved and the corresponding mathematical model were developed to simulate the physical system. A computer program was written to solve the set of equations forming the mathematical model, considering the physical properties of the tailings and the rainfall data selected. The results indicate that the relative distribution of the precipitation depends on the surface and upper layer of the tailings and that the position of the groundwater table is governed by the flow through the bottom of the profile considered. The slope of the surface of the mass of tailings was found to be one of the principal factors affecting the relative distribution of precipitation and, therefore, the potential pollution of the environment.

MINE TAILINGS DISPOSAL:
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INTRODUCTION

With increasing population growth and increasing material requirements of modern life, questions concerning the adequacy of the methods of disposal of wastes, refuse and tailing materials have arisen. Mineral tailing disposal sites, including landfilling or tailing ponds are usually selected and designed considering the abandonment procedure and sometimes also the reclamation procedure. The evaluation of the long term effects on the environment of the abandonment procedure can be performed to a large extent by studying the behaviour of the whole system of disposal in time and its response to hydrologic variables. Much of the environmental impact of mine tailing disposal sites is related to the amount and quality of water leaving the site. Water passing through the tailings carries with it various dissolved and suspended materials which are often deleterious and can have undesirable chemical and biological characteristics. This water, which is commonly called leachate, may become a source of contamination of surface and/or groundwaters. The volumes of leachate leaving the site are directly related to the severity of potential environmental pollution for a given type of tailings.

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The distribution of the total amount of precipitation over the area of the site into runoff, infiltration, and evapotranspiration has a marked effect on the volumes of leachate and seepage leaving the site. It is, therefore, important to analyze and quantify the modes and amounts of water leaving the site.

The objectives of this paper are, consequently, to evaluate the long term hydrologic effects of mine tailing disposal sites. In order to evaluate the quantities involved in each of these hydrologic processes, it is necessary to analyze the effect of different factors such as the slope of the surface of the site, the type of cover, the physical properties of the tailings material and to analyze how they are affected by a sequence of precipitation and dry periods during an appropriate period of time. Furthermore, there exist interrelations between hydrologic processes such as runoff, evaporation and infiltration with some of the properties of the soil such as permeability, moisture content and hydraulic conductivity. These interrelations are time dependent and therefore, for the evaluation of the process as a whole, it is necessary to simulate the hydrologic phenomena over a period of time. The problem can be established as the study of the vertical movement of water in a porous media subjected to certain boundary conditions.

MATHEMATICAL MODEL

In hydrology, as in other fields of applied physics, one of the most powerful methods of investigating complex phenomena is conceptual modelling. The hydrologic processes of infiltration, evaporation, flow of water through the soil and its subsequent movements in the mass of soil are interrelated. The variables governing these processes are in general interrelated and time-dependent; therefore, the evaluation of the processes, including the quantification of the variables, can be done by modelling the combined hydrologic processes taking place in the system. The characteristics of the processes, with variables distributed and interrelated, lead to the concept of a non-linear distributed-parameter system model. The conceptual model considers the physical continuity of the hydrologic system from the infiltration-evaporation process taking place in the surface of the soil, to the final movement of water in the saturated region, passing through the unsaturated region.

The conceptual model can be established in general by the study of the vertical movement of water in a porous medium subjected to certain boundary conditions in the surface and bottom of the profile. These boundary conditions express the different processes such as infiltration, evaporation, ponded water in the surface of the soil and the upward or downward movement of water in the basal boundary.

The mathematical model results from the application of physical principles to the hydrologic processes involved in the conceptual model. The flow of water through both saturated and unsaturated regions is a continuous system governed by the same general principle of motion.

This principle of motion may be applied to the flow system combined with the principle of mass conservation and the thermodynamic equilibrium to obtain the mathematical equations required by the model. The continuous system of flow indicates the existence of a variable which governs the flow in both regions. This variable is the total potential, which is defined as the energy per unit of mass of water. This energy of the water may be divided into potential energy and kinetic energy. The potential energy, neglecting the temperature gradient, corresponds to the gravitational potential energy plus the pressure head. The kinetic energy is negligible for the small velocities of the flow of water through the soil, especially in this case where the velocities are in the range of a few meters per year.

According to infiltration theory, the equation governing the verticle movement of water in an homogeneous porous medium for saturated and unsaturated unsteady conditions, is

[1] $$C(\Psi)\,\frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial z}\left[K(\Psi)\,\left(\frac{\partial \Psi}{\partial z} + 1\right)\right]$$

In this equation, the specific moisture capacity, $C(\Psi)$, and the hydraulic conductivity, $K(\Psi)$, are single-valued functions of pressure head, Ψ, which varies through depth, z, and time, t. This equation leads to a simpler expression for the saturated flow case where the specific moisture capacity is zero and the hydraulic conductivity is constant for homogeneous soil and equal to the saturated permeability, K_{sat}, and may be written

[2] $$\frac{\partial}{\partial z}\left[K_{sat}\,\frac{\partial \Psi}{\partial z} + 1\right] = 0$$

Hence

[3] $$\frac{\partial 2\Psi}{\partial z2} = 0$$

which is known as the Laplace equation and is valid in the saturated region of the system.

Equations [1] and [2] are the basic equations that constitute the mathematical model, representing the flow of water through the soil in both unsaturated and saturated regions, respectively. The mathematical model represents the vertical movement of water in a soil profile selected in the mine tailings site, assuming that the flow takes place through a vertical column of soil with unit cross-section area. This column is divided into 19 sections by 20 nodes separated 10 cm each. The nodes are numbered from the bottom to the surface of the soil. The bottom node or bottom boundary is located in the saturated region of the soil. The upper node is located coincident with the surface of the soil and corresponds to the surface boundary of the model. The vertical co-ordinate, z, is oriented positive downwards. A schematic diagram of the model is shown in Figure 1.

BOUNDARY CONDITIONS

The mathematical model requires the relations governing the flow through the soil and the mathematical relations representing the processes that take place at both the upper and lower boundaries. In order to obtain these relations, it is necessary to determine the physical phenomena governing the flow of water in and out of the profile, through the corresponding boundaries.

Surface Boundary

The physical processes occurring in the surface and upper layer of the soil include precipitation, evapotranspiration, infiltration, runoff or overland flow and snow or freezing conditions. All of these processes affect directly or indirectly the flow of water through the soil. Both the input and the output of water to the model can be described mainly by the processes of infiltration and evaporation from soil, respectively.

The first includes the effects of precipitation, evaporation from water, overland flow and ponded water, and the second includes the effects of evapotranspiration. Both processes, infiltration and evaporation from soil are closely related to soil surface characteristics, soil properties and moisture content of the soil.

The infiltration process is one aspect of the general theory of fluid movement in a porous medium. The mathematical relation governing infiltration may be expressed in terms of the same variables affecting the flow through the soil with the addition of restrictions arising from the application of boundary conditions.

The infiltration theory has been developed from Darcy's Law and a continuity statement which have lead to theoretical equations relating the infiltration rate to soil parameters such as hydraulic conductivity, soil sorptivity, the initial moisture content of the profile, the saturated moisture content, and a series of factors which are functions of these parameters. The empirical equations, derived from field observations and laboratory research, give values of the infiltration rate close to the observed values and they may be used to evaluate infiltration with the same degree of approximation as the other variables; furthermore, the theoretical equations lead to expressions similar to those of the empirical equations, when higher degree terms of the theoretical solutions are neglected (Philip 1957).

In order to describe the process of infiltration, two of the most common empirical equations are considered:

Horton's (1940) equation

[4] $$f = f_c + (f_o - f_c)\, e^{-kt}$$

and Holtan's (1961) equation

$$[5] \qquad f = a\,(S-F)^n + f_c$$

where f is the infiltration rate at time t, f_c is the final constant infiltration rate, f_o is the initial infiltration capacity, S is the storage potential of the soil expressed as a volumetric difference between saturation and wilting point, F is the accumulated infiltration volume and k, a, and n are constants for a particular soil in a given condition. These two equations are based on the reduction of the infiltration rate as a result of the increasing moisture content of a control layer in the upper part of the soil. In the Holtan eouation [5], the above idea is expressed in the difference between the storage potential, S, as a limit value of the infiltration, and the accumulated infiltration, F. In the Horton equation [4], the idea of proportionality is implicit. The infiltration rate difference $(f-f_c)$ decreases with time.

The evaluation of infiltration for different relative values of rainfall intensity may be performed by applying both equations. Figure 2(a) shows the shape of the infiltration curve and the rainfall intensities, i, for different possibilities, which are, in the first time interval $i<f$, in the second time interval $i>f$ and, in the third time interval, the infiltration capacity is greater than the rainfall intensity at the beginning of the interval $(2 \rightarrow t^*)$ but smaller at the end $(t^* \rightarrow 3)$. This situation and the assumption of the Horton equation that is valid when there is available water, lead to the combined use of the Horton and Holtan equations. The first is used directly in the cases where $i>f$ and is complemented by the Holtan equation in the cases where $i<f$ and when moisture variations in the upper layer of the soil modify the initial value, f_o, of the Horton equation.

Using the concept of storage potential or volumetric difference between the moisture content of the soil at saturation and at the wilting point (15 bar), it is possible to determine, for a given soil, the effects of increasing mass infiltration, which reduces the infiltration capacity, and the recovery of the infiltration capacity when the moisture content of the upper layer of the soil is reduced by the flow of water toward deeper layers or by evaporation from soil. Both processes have similar effects on the recovery of the infiltration capacity.

Figure 2(b) shows the effects of wet and dry periods. The infiltration capacity, originally at point A, which corresponds to a dry condition of soil, decreases with increasing moisture content in the soil to reach the point B where the rainfall stops and the moisture content of the soil decreases due to the combined effects of gravity flow toward deeper layers and evaporation to the atmosphere. This decrease in moisture content of the soil increases the infiltration capacity to point C where a new period of rainfall starts, decreasing the infiltration capacity to point D, where the rainfall stops. Again, the infiltration capacity increases at a decreasing rate. In order to evaluate the recovery of the infiltration capacity during the dry period between two consecutive rainfall events, the model determines the variation of moisture content in the upper layer of the soil, and, using the Holtan equation, evaluates the recovery of infiltration capacity.

To determine the variation of moisture content of the upper layer of the soil, evaporation, infiltration and percolation are considered. The evaporation is evaluated from atmospheric and hydrometeorologic conditions as potential evapotranspiration, which is in general higher than the hydraulic conductivity values of the fine tailing materials in normal abandonment conditions; therefore, the evaporation process can be considered as a constant value for the total period of simulation. This value represents a potential evaporation. The actual evaporation is calculated separately for ponded conditions and for evaporation from soil. The percolation is obtained from the application of the model to the complete system of flow.

To complete the evaluation of the processes occurring in the surface of the soil, the model must consider the effects of the slope, roughness and size of the surface of the tailing disposal site. This can be accomplished by analyzing the overland flow process and calculating the average depth of flow corresponding to a given rainfall intensity, slope, roughness characteristic and size of the tailing surface. The time period involved in these calculations can be selected considering the availability of rainfall intensity data, the general accuracy of the model and the sensitivity of the variables with time. An appropriate time step for this surface process is judged to be one hour, which is a relatively short time step for the deeper region processes, but gives acceptable values for the more rapid variations of the surface processes.

The equation relating the velocity of flow on a surface (sheet flow) to the slope, roughness, and depth of flow is the Manning equation. Considering the size of an average mine tailing surface and the selected time step, the average depth of flow can be evaluated with the slope as a parameter, for a given roughness coefficient. In this work, the slopes considered range between 0.1% and 6% corresponding to two different approaches in mine tailings disposal site design: a traditional approach, with slopes near 0.1% after abandonment and a more recent approach with final slopes of about 6% (Robinsky 1975). Both values were considered as lower and upper limits for the range of slope variation.

A balance of water can be applied to the surface of the soil for each time step considering the rainfall as an input and the excess, being overland flow, evaporation and infiltration, as outputs. Assuming that there is no evaporation from the soil surface during the rainfall event, the average depth of precipitation during the time step is available for infiltration and overland flow. During the dry periods, the ponded water is available for evaporation and infiltration. The amount of evaporation and infiltration depends on the potential evaporation and infiltration capacity.

The infiltration is calculated from its final value of the previous time step and the rainfall intensity value of the time step being analyzed. The initial value of the infiltration capacity corresponds to the final value of the previous step or the value given by the Holtan equation, which considers the variation in moisture content in the soil. In general, there are three cases of relative values for rainfall intensity and infiltration rate (see Figure 2(a)):

(A)	$i<f$	(Time $0 \rightarrow 1$)
(B)	$i>f$	(Time $1 \rightarrow 2$)
(C)	f varies from $f>i$ to $f<i$	(Time $2 \rightarrow 3$)

The mass infiltration in these three cases can be evaluated as follows:

Case (A), $f>i$, therefore the total infiltrated water is

[6] $$F_{0-1} = i_1 \, (t_1 - t_0)$$

Case (B), $f<i$, assuming that the portion of the f-curve in the interval is calculated from the previous condition, then

[7] $$F_{1-2} = \int_{t_i}^{t_2} f(t)dt$$

[8] $$= \int_{t_1}^{t_2} [f_c + (f_o - f_c)\, e^{-kt}]\ dt$$

[9] $$= f_c\,(t_2 - t_1) + \frac{f_c - f_c}{k}\; e^{-kt_1} - e^{-kt_2}$$

Case (C), f varies from $f>i$ to $f<i$, passing through a value $f = i$ at a time t^*. Assuming the validity of the Horton equation in the interval $(t_2 - t_3)$, determine t^* as follows:

[10] $$i = f_c + (f_o - f_c)\, e^{-kt^*}$$

hence

[11] $$t^* = -\frac{1}{k} \ln \left(\frac{i - f_c}{f_o - f_c} \right)$$

Therefore

[12] $$F_{2-t^*} = i \left[-\frac{1}{k} \ln \left(\frac{i - f_c}{f_o - f_c} \right) - t_2 \right]$$

and

[13] $$F_{t^*-3} = f_c\,(t_3 - t^*) + \frac{f_o - f_c}{k} \left(e^{-kt^*} - e^{-kt_3} \right)$$

hence

[14] $$F_{2-3} = F_{2-t^*} + F_{t^*-3}$$

or

[15] $$F_{2-3} = i \left[-\frac{1}{k} \ln \left(\frac{i - f_c}{f_o - f_c} \right) - t_2 + \frac{f_o - f_c}{k} \left(\frac{i - f_c}{f_o - f_c} - e^{-kt_3} \right) \right]$$

The mass infiltration calculated and the percolation during the time step considered are used to evaluate the initial infiltration capacity for the next time step, by applying the Holtan equation. The relation between infiltration rates for each hour and the initial rate are obtained from typical infiltration curves.

The mathematical relation that considers the effects of infiltration and evaporation may be obtained by applying the definition of specific moisture capacity.

[16] $$C(\Psi) = \frac{\partial\theta}{\partial\Psi}$$

where θ is the moisture content of the soil, to the upper layer of soil where the infiltration or evaporation takes place. Then from equation [1]:

[17] $$\frac{\partial}{\partial z}\left[K(\Psi)\left(\frac{\partial\Psi}{\partial z}+1\right)\right] = \frac{\partial\theta}{\partial t}$$

Considering the depth Δz of the upper layer and for a time step equal to unity

[18] $$K(\Psi)\left(\frac{\Delta\Psi}{\Delta z}+1\right) = Q$$

Therefore

[19] $$\frac{\Delta\Psi}{\Delta z} = \frac{Q}{K(\Psi)} - 1$$

or

[20] $$\Psi_{JJ} - \Psi_{JJ-1} = \left(\frac{Q}{K(\Psi)} - 1\right)\left(z_{JJ} - z_{JJ-1}\right)$$

This equation relates the two consecutive values of pressure head in one layer (JJ) and in the layer above (JJ-1) to the respective depths, and the evaporation or infiltration Q ($Q>0$ represents infiltration and $Q<0$ represents evaporation).

Bottom Boundary

According to the model, the bottom boundary is located in the saturated region where the flow is governed by the Laplace equation. The following mathematical relation representing the bottom boundary condition can be obtained from equation [2] in which the hydraulic conductivity is a constant equal to the saturated permeability K_{sat}:

[21] $$K_{sat}\left(\frac{\partial\Psi}{z}+1\right) = R$$

Therefore

[22] $$\Psi_2 = \Psi_1\left(\frac{R}{K_{sat}} - 1\right)(z_2 - z_1)$$

where R is the flow through the bottom boundary. The flow may be from the model system downward, $R>0$, or from the groundwater system upward entering the model system through the bottom boundary, $R<0$. These flows can be considered as recharge or discharge of the groundwater system (Freeze, 1967).

METHOD OF SOLUTION

The mathematical model described in the previous section includes two differential equations of flow and two equations of boundary conditions. The partial differential equation governing the unsaturated flow is of the non-linear type (Equation [1]) in which the coefficients $C(\Psi)$ and $K(\Psi)$ are dependent on the variable. The solution of equations of this type can be performed by numerical methods using a high-speed digital computer.

The method of solution applied is the finite difference method, in which the partial differential equations and the corresponding boundary condition equations are replaced with equivalent algebraic equations. This replacement is made of discrete points in the continuous domain substituting the derivatives with finite difference expressions connecting the values of the variables considered in adjacent points. These finite difference equations can be established in each internal point considered leading to a set of equations relating the values of the variables through the complete profile. This can be represented in a two dimensional plane (Z,T) in which the ordinate is the vertical distance from the bottom of the soil profile to the surface, and the abscissa is the time. Figure 3 shows these co-ordinates and the nomenclature adopted.

The differential equation governing the flow in the unsaturated region (Equation [1]) is replaced with a finite difference equation as follows:

$$[23] \qquad C\left(\Psi_J^{n+\frac{1}{2}}\right)\left(\Psi_J^{n+\frac{1}{2}} - \Psi_J^{n}\right)\frac{\Delta Z}{\Delta T} = K\left(\Psi_{J+\frac{1}{2}}^{n+\frac{1}{2}}\right)\left[1 + \frac{1}{2\Delta Z}\left(\Psi_{J+1}^{n}+\Psi_{J+1}^{n+1}-\Psi_{J}^{n}-\Psi_{J}^{n+1}\right)\right]$$

$$- K\left(\Psi_{J-\frac{1}{2}}^{n+\frac{1}{2}}\right)\left[1 + \frac{1}{2\Delta Z}\left(\Psi_{J}^{n}+\Psi_{J}^{n+1}-\Psi_{J-1}^{n}-\Psi_{J-1}^{n+1}\right)\right]$$

in which the values of the indexes of $\pm\frac{1}{2}$ indicates the points half way between the nodes, therefore the point $n+\frac{1}{2}$ is in between the nodes n and $n+1$. Rearranging the above equation

$$[24] \quad \left[\frac{-1}{2\Delta Z} K\left(\Psi_{J+\frac{1}{2}}^{n+\frac{1}{2}}\right)\right]\Psi_{J+1}^{n+1} + \left[C\left(\Psi_J^{n+\frac{1}{2}}\right)\frac{\Delta Z}{\Delta T} + \frac{K\left(\Psi_{J+\frac{1}{2}}^{n+\frac{1}{2}}\right)}{2\Delta Z} + \frac{K\left(\Psi_{J-\frac{1}{2}}^{n+\frac{1}{2}}\right)}{2\Delta Z}\right]\Psi_J^{n+1} - \left[\frac{K\left(\Psi_{J-\frac{1}{2}}^{n+\frac{1}{2}}\right)}{2\Delta Z}\right]\Psi_{J-1}^{n+1} =$$

$$= \left[\frac{K\left(\Psi_{J+\frac{1}{2}}^{n+\frac{1}{2}}\right)}{2\Delta Z}\right]\Psi_{J+1}^{n} + \left[C\left(\Psi_J^{n+\frac{1}{2}}\right)\frac{\Delta Z}{\Delta T} - \frac{K\left(\Psi_{J+\frac{1}{2}}^{n+\frac{1}{2}}\right)}{2\Delta Z} - \frac{K\left(\Psi_{J-\frac{1}{2}}^{n+\frac{1}{2}}\right)}{2\Delta Z}\right]\Psi_J^{n} + \left[\frac{K\left(\Psi_{J-\frac{1}{2}}^{n+\frac{1}{2}}\right)}{2\Delta Z}\right]\Psi_{J-1}^{n} +$$

$$+ K\left(\Psi_{J+\frac{1}{2}}^{n+\frac{1}{2}}\right) - K\left(\Psi_{J-\frac{1}{2}}^{n+\frac{1}{2}}\right)$$

This difference equation includes the values of the functions $C(\Psi)$ and $K(\Psi)$ at intermediate points of the grid, such as $\Psi_{J\pm\frac{1}{2}}^{n+\frac{1}{2}}$.

The values of the variable in the intermediate points are determined by extrapolation from previously known values as follows:

$$[25] \qquad \Psi_{J+\frac{1}{2}}^{n+\frac{1}{2}} \simeq \frac{3}{4}\left(\Psi_{J+1}^{n} + \Psi_{J}^{n}\right) - \frac{1}{4}\left(\Psi_{J+1}^{n-1} + \Psi_{J}^{n-1}\right)$$

$$[26] \qquad \Psi_{J-1}^{n+\frac{1}{2}} \simeq \frac{3}{4}\left(\Psi_{J}^{n} + \Psi_{J-1}^{n}\right) - \frac{1}{4}\left(\Psi_{J}^{n-1} + \Psi_{J-1}^{n-1}\right)$$

and

[27] $$\Psi_J^{n+\frac{1}{2}} \approx 2\,\Psi_J^n - \frac{1}{2}(\Psi_J^n + \Psi_J^{n-1})$$

The differential equation governing the flow in the saturated region (Equation [2]) is replaced with a finite difference equation as follows:

[28] $$-\Psi_{J+1}^{n+1} + 2\,\Psi_J^{n+1} - \Psi_{J-1}^{n+1} = 0$$

and the boundary conditions (Equations [20] and [22]) are replaced with the corresponding finite difference equations:

[29] $$\Psi_{JJ}^{n+1} - \Psi_{JJ-1}^{n+1} = \frac{\Delta Z\ Q}{K(\Psi_{JJ-\frac{1}{2}}^{n+1})}$$ (Surface Boundary)

and

[30] $$\Psi_2^{n+1} - \Psi_1^{n+1} = \frac{Z\ R}{K_{sat}}$$ (Bottom Boundary)

The value of the pressure head $\Psi_{JJ-\frac{1}{2}}^{n+1}$ is obtained by extrapolation.

The system of simultaneous equations formed by the finite difference equations may be written as follows for each internal node:

[31] $$-A_J\ \Psi_{J+1}^{n+1} + D_J\ \Psi_J^{n+1} - C_{J-1}^{n+1} = B_J$$

where

[32] $$A_J = \frac{K(\Psi_{J+\frac{1}{2}}^{n+\frac{1}{2}})}{2\Delta Z}$$

[33] $$D_J = \frac{\Delta Z}{\Delta T}\ C(\Psi_J^{n+\frac{1}{2}}) + \frac{1}{2\Delta Z}\left[K(\Psi_{J-\frac{1}{2}}^{n+\frac{1}{2}}) + K(\Psi_{J+\frac{1}{2}}^{n+\frac{1}{2}})\right]$$

[34] $$C_J = \frac{K(\Psi_{J-\frac{1}{2}}^{n+\frac{1}{2}})}{2\Delta Z}$$

and

[35] $$B_J = A_J\,\Psi^n_{J+1} + \left[D_J - 2(A_J + C_J)\right]\Psi^n_J + C_J\,\Psi^n_{J-1} + 2\Delta Z\,(A_J - C_J)$$

In the saturated region, the finite difference equation can be written as follows:

[36] $$-\Psi^{n+1}_{J+1} + 2\,\Psi^{n+1}_J - \Psi^{n+1}_{J-1} = 0$$

The above equation represents a linear variation of pressure head with depth in the saturated zone.

The finite difference equations for the boundary conditions are Equations [29] and [30]. The solution of the system of simultaneous equations may be expressed as $\Psi(Z,T)$, in which the variables Z and T are discrete. The system of simultaneous equations constitute a tridiagonal system and can be solved by elimination with an appropriate algorithm using a digital computer (Conte 1972).

EXPERIMENTAL DATA

In order to solve the equations governing the flow through the profile, it is necessary to evaluate the hydraulic conductivity values and the specific moisture capacity values for each node and intermediate points required by the finite difference equations, and the pressure versus moisture content curve, which is used to express the pressure head solution to a moisture content solution of the system.

These functional relations between moisture content and hydraulic conductivity versus pressure head can be obtained only experimentally. The specific moisture capacity curve can be derived from the moisture content versus pressure curve by geometric or numerical methods. Other properties of the soil that are required directly or indirectly to solve the problem are the saturated permeability, K_{sat}; the air pressure entry of the soil, which is the required pressure head or suction to allow air entry the soil surface, and the initial moisture conditions of the profile (Gonzalez and Adams 1980).

The soil surface characteristics such as roughness coefficient, slope, and amount and type of vegetation are necessary to determine the surface processes which affect the boundary conditions. Rainfall data and potential evaporation are also required to solve the system. The precipitation data used is rainfall data from the Malton Weather Station of the Atmospheric Environmental Service of Environment Canada, which was recorded on an hourly basis and measured in hundredths of an inch (0.254 mm).

COMPUTER PROGRAM

In order to apply the mathematical model to simulate the hydrologic processes taking place in the mine tailing disposal site, the set of mathematical equations defining the model must be solved with the use of a high speed digital computer for which a computer program is required. The computer program was written in Fortran IV language for the IBM 370/165 computer of the University of Toronto Computer - Centre.

The schematic flow chart presented in Figure 4 indicates the general features of the main program. Beginning with the reading of all the rainfall data, functional relations for both wetting and drying processes, the parameters defining the surface characteristics, the initial values of the variables, and the parameters defining the profile, the program determines the depth of depression storage, above which the surface water is considered to run off. The initial moisture content of the soil profile is then calculated.

The developed mathematical model considers the hydraulic conductivity, the specific moisture capacity and the moisture content as single-value functions of pressure head which is true only for separated processes of wetting or drying due to the hysteresis involved; therefore, in order to use the correct curve corresponding to the process that is taking place in a particular node, the program determines the type of process occurring and selects the proper functional relation for the unsaturated case of flow. Thus, the program determines the type of flow (saturated or unsaturated) and the type of process (wetting or drying) for all nodes of the profile in each time step.

At the surface boundary, the program analyzes the processes of rainfall, potential evaporation, infiltration and runoff on an hourly basis determining the total available water on the surface, the evaporation from water, the evaporation from soil, the runoff and the infiltration. The water balance on the surface is performed on an hourly basis and the values of the variables are determined for 6 hour periods. The coefficients of the surface boundary condition equation are calculated from the above balance for time steps of 6 hours. After the matrix of coefficients for the mathematical equations is completed, the program solves the simultaneous equations by means of a subroutine.

The main program uses a subroutine named TRID, which uses the method of elimination for tridiagonal systems. The solution of the system is a vector whose elements are the pressure head in each node of the profile. The subroutine INTERP is used to determine the values of the functional relations for the values of pressure head required and to interpolate the values of moisture content. The subroutine INFIL analyzes the rate of infiltration corresponding to the surface conditions and the moisture content of the upper layer of the soil integrating the infiltration curve with the limitations of availability of water and moisture content of the soil, determining the amount of infiltration, ponded water or evaporation. The subroutine READY gives the hourly rainfall data from the original set of data which was read as a set of non zero values with the corresponding location.

APPLICATION

The model was applied with the data collected and the functional relations determined in the laboratory to simulate the hydrologic processes in a tailing disposal site with a 2500 ft (762 m) radius and a surface roughness coefficient (Manning n) of 0.025 which represents average conditions.

For a simulation period of 37.5 days, calculations were performed on an hourly basis for the surface and upper soil layer processes and on a 6 hour basis for the rest of the soil profile. This somewhat arbitrary separation of the time increment was made because the physical process of infiltration which takes place on the surface and upper layer of the soil is more sensitive to time variation and rainfall intensity variations than the lower parts of the profile, and because there is a substantial reduction in the computer time required.

The functional relations were entered into the computer as tables of values for both wetting and drying processes. The evaporation was considered to be the yearly average for the Southern Ontario region as 25 in/yr (635 mm/yr). This value is well above the maximum rate at which a soil of the type of the tailings can sustain even under initially saturated conditions; therefore, it was unnecessary to provide the program with a function representing variable evaporation. The flow through the bottom of the profile considered was selected to be in a range with a maximum value smaller than the saturated permeability, which is in direct relationship with the groundwater flow.

Mention is made regarding the slope of the tailings surface because of the two different approaches in the disposal site design; the traditional approach, with slopes near 0.1%, and a newer approach with final slopes of about 6%, after abandonment. Both values were considered as the lower and upper limits for the range of slope variations.

Some remarks are also made concerning limitations of the program. According to the theory of linear differential equations, the general solution of a J^{th} order differential equation will contain J arbitrary constants. These constants can be evaluated from the N boundary conditions in terms of the dependent variables and their derivatives at certain points or boundaries. The numerical solution of the boundary value problems can be accomplished either by iterative methods or can be reduced to the solution of a set of N simultaneous linear equations in N unknowns. In this case, the computer program was written to solve a system of N simultaneous equations in which the number of equations depends on the size of the depth step. The selection of the size of the step is a compromise between the accuracy of the solution and the computer time used. This balance can be supplemented by including the limitations to the accuracy of the solution introduced by the several assumptions made in the development of the mathematical model. The linear interpolation of values of the functional relations and the laboratory determination of them, make it appropriate for this case to select the size of the depth steps of about 20 cm and time steps of 1 hour for the surface processes and 6 hours for the rest of the profile.

RESULTS

The mathematical model was run to simulate the hydrologic processes occurring in a mine tailing disposal site under a given pattern of hydrometeorological conditions and to evaluate the effects of various parameters on the system of flow through both saturated and unsaturated conditions, and on the relative distribution of precipitation into runoff, infiltration and evapotranspiration. Several preliminary runs were made to determine the appropriate depth of the profile to analyze the corresponding depth steps, the time increment, the length of simulation and the sensitivity of the model to various parameters. The initial conditions for the simulation corresponds to a saturated soil, which is the situation most likely to occur when the mine tailing site is abandoned. The preliminary runs demonstrated that simulation periods of slightly over a month yielded consistent results. Longer simulation periods did not materially change the relations extracted from the results.

The relation between infiltration and the slope of the surface of the site is illustrated in Figure 5. Infiltration increases with decreasing slope, from its minimum for the 6% slope to its maximum at the 0.1% slope. The flow through the bottom of the profile, Q, has a negligible effect on this relation. The infiltration process is governed by surface and upper layer conditions.

The relation between runoff and slope of the surface is presented in Figure 5. It was found that the runoff increases with increasing slope from the zero value at a slope near 0.1% to the maximum at the 6% slope, which is the upper limit of the slope range considered. The values of the flow through the bottom have a negligible effect on this relation. The effects of slope on both infiltration and runoff are presented together in order to visualize that these processes are strongly related.

The relation between the evaporation from ponded water, which takes place when there is available water on the surface of the soil, and the slope, is shown in Figure 6. The evaporation from ponded water was found to increase with decreasing slope, due to the increasing evaporation opportunity when slope decreases (the combined effects of infiltration, which increases, and runoff, which decreases, with decreasing slope).

The relation between the evaporation from soil, which takes place when there is no ponded water on the surface of the soil, and the slope is shown, for four different values of flow through the bottom, in Figure 6. The values of Q are those indicated. It is clear that the slope of the soil surface has very little effect on the evaporation from soil which decreases with increasing flow through the bottom. The flow through the bottom, as it will be shown later, governs the position of the groundwater table. The variations in the position of the groundwater table affect the hydraulic conductivity of the soil in the unsaturated region and in the surface of the soil and correspondingly influence the upward movement of water towards the evaporating region.

The relation between moisture variation, which is the decrease in water content in the soil profile, and the slope is shown in Figure 7 for the values of Q indicated previously. It was found that the moisture variation increases with the slope. This effect is small compared to the effect of the flow through the bottom on the moisture variation.

The relation between the position of the groundwater table and the flow through the bottom is shown in Figure 8 for different values of the slope of the surface. It was found that the depth of the groundwater table increases with increasing flow through the bottom for all the considered slope values. In other words, the position of the groundwater table is governed by the flow through the bottom. The curves for different slopes are markedly parallel and close to each other.

Based on the results of the hydrological evaluation, a long-term water balance for a disposal site can be made for different values of the surface slope. Taking the general pattern of rainfall as the average conditions for a period of time, for example 10 years, the amounts of infiltration and runoff may be evaluated for that period considering their relative distribution to be constant.

The relative distribution of the infiltration and runoff is shown in Table 1 for various values of slope.

SUMMARY AND CONCLUSIONS

The environmental impact of mine tailing disposal sites after they are abandoned is closely related to the amounts of water leaving the site and to their relative distribution. According to the results obtained from the above simulation model, the relative distribution of the water leaving the site depends on the distribution of precipitation reaching the ground, which can be considered the main source of water to the system. The following are some general but quantitative observations.

The precipitation reaching the ground may be divided into runoff, infiltration and evaporation. The relative distribution of each of these depends on the characteristics of the surface of the site and on the properties of the tailings, especially the upper layer, thus, for surface slopes greater than 1%, the amount of total rainfall which infiltrates is approximately 50% or less. For a surface slope of 6%, the amount of infiltrated water is only around 25% of the total precipitation. The slope of the surface also affects the evaporation from water. Increasing the slope decreases the amount of evaporation from water, thus, for a surface slope of 0.1% the evaporation from ponded water on the soil is of the order of 1.37 cm for the period of time considered, and for a surface slope of 6% the resulting evaporation from ponded water is 0.37 cm, for the same period of time.

The slope of the surface has a small influence on the evaporation from soil. The values obtained are 4.9 cm of evaporation from soil for a slope surface of 0.1%, and 5.1 cm of evaporation from soil for a surface slope of 6%. The position of the groundwater table has a small effect on this relative distribution. This effect decreases with deeper levels of the groundwater table. The upward movement of water from the saturated region to the aeration zone becomes extremely slow, limiting the evaporation rates. After dry periods, in which the groundwater table is likely to descend, the evaporation from soil is small or negligible. Runoff and infiltration are the most important factors affecting the potential pollution of the environment, the latter being most difficult to analyze.

The general pattern of flow toward the groundwater region and the limitations of the upward movement of water and subsequent evaporation, make it more important, from the point of view of environmental protection, to control the infiltration by increasing the runoff, which is easier to collect and treat, if necessary, before delivery to receiving water bodies.

According to the results of the simulations, the slope of the surface of the site is one of the most important factors governing the relative distribution of the precipitation reaching the ground. Therefore, the infiltration may be controlled to an extent by increasing the slope of the surface of the site. In this case, the runoff increases from a value of around zero for a surface slope of 0.1% to 3.4 cm for a surface slope of 6%, and the infiltration decreases from a value of 3.6 cm for a surface slope of 0.1% to 1.2 cm of infiltrated water for a surface slope of 6%. These two combined effects result in a general net effect of diminishing the potential pollution of the environment. Thus, increasing the slope of the site surface from 0.1% to 6% results in an approximate two-thirds reduction of the potential pollution of the groundwater system.

The relations obtained were final values for simulations of the behaviour of the flow system through the profile of soil selected and for the complete period of time analyzed. Longer simulation runs were found not to vary substantially the relations between the variables.

ACKNOWLEDGEMENTS

The work upon which this paper is based was supported in part by funds provided by the Natural Sciences and Engineering Research Council of Canada and the University of Toronto. The authors wish to express their thanks to Dr. E. I. Robinsky of the Department of Civil Engineering, University of Toronto, for his valuable comments on the manuscript.

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Table 1. Relative distribution of infiltration and runoff as a percentage of total rainfall.

Slope (%)	Infiltration (%)	Runoff (%)
0.1	72	0
1.0	54	25
3.0	33	55
6.0	24	68

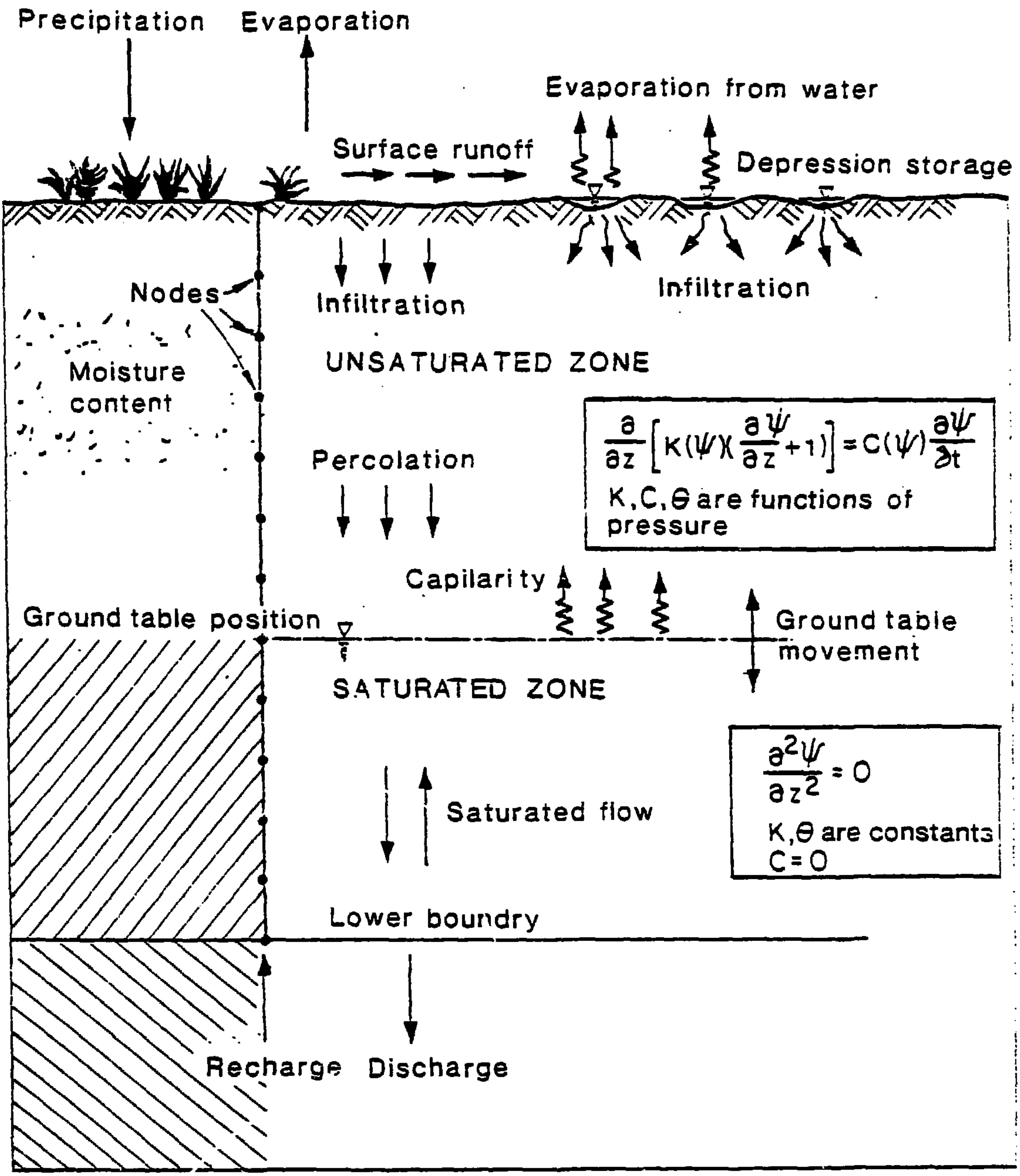


Figure 1. Schematic Diagram of the Model

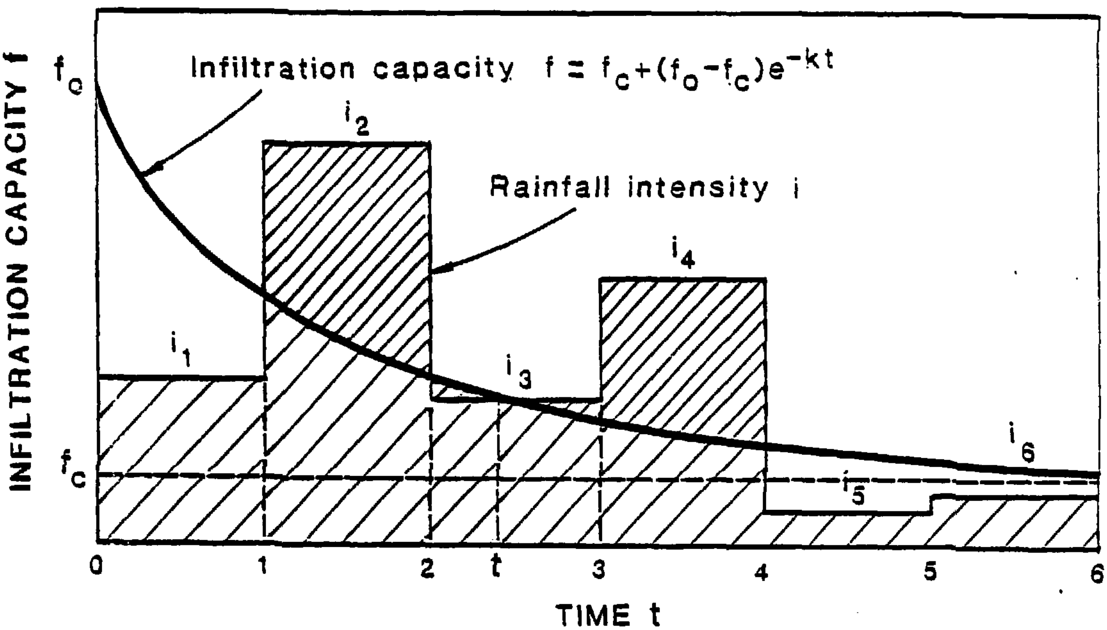


Figure 2. (a) Infiltration Capacity Curve and Rainfall Intensities

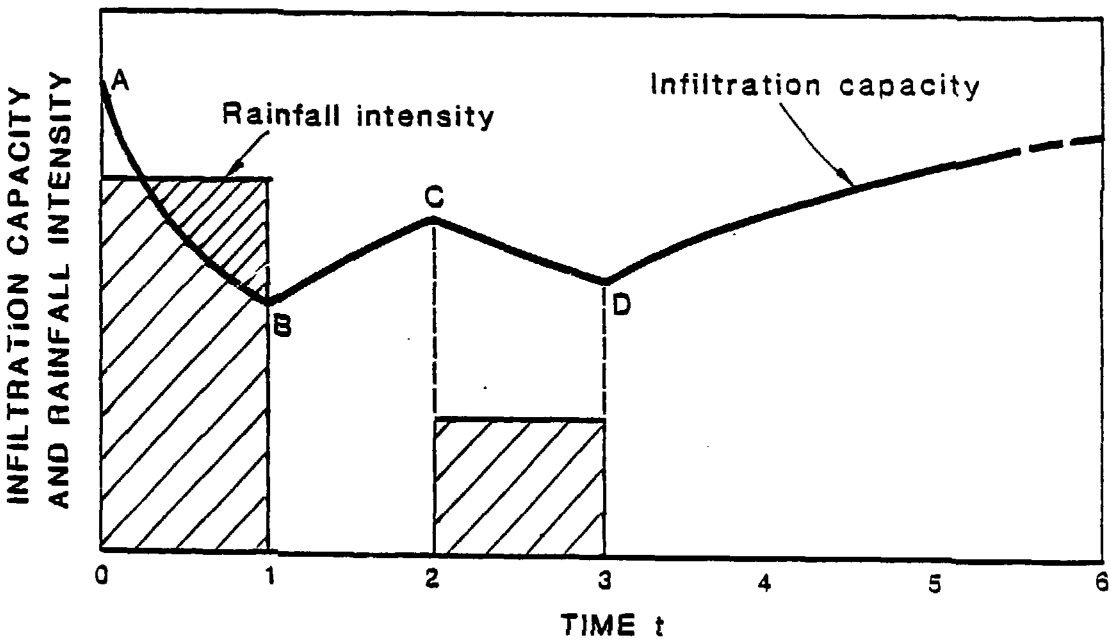


Figure 2. (b) Infiltration Capacity and Rainfall Intensity Relation in the Surface and Upper Layer of the Soil Profile.

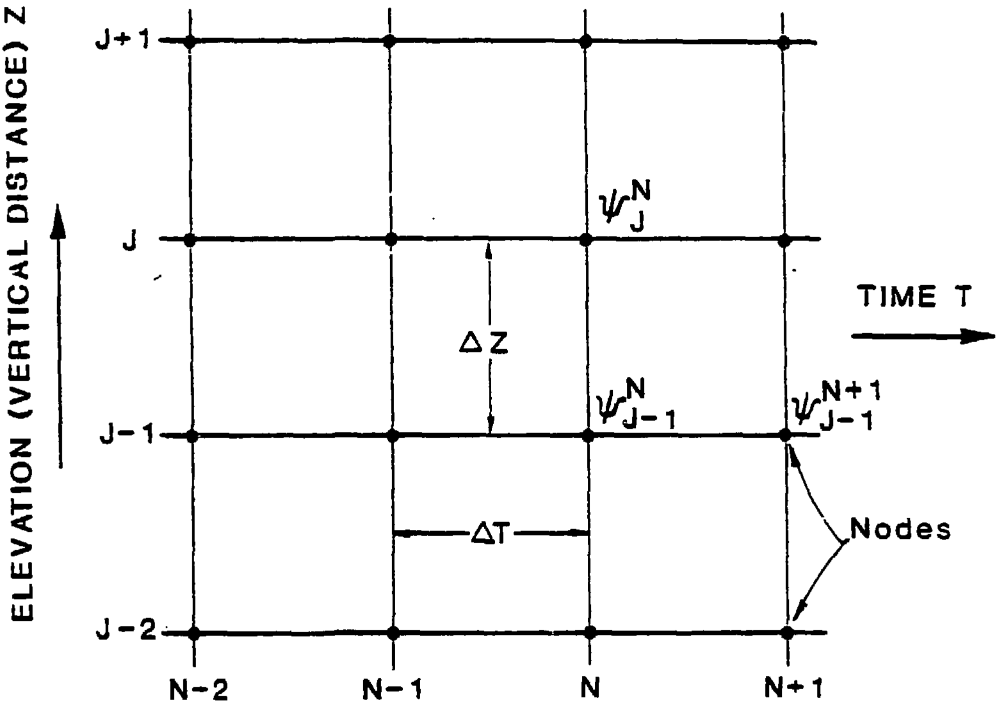
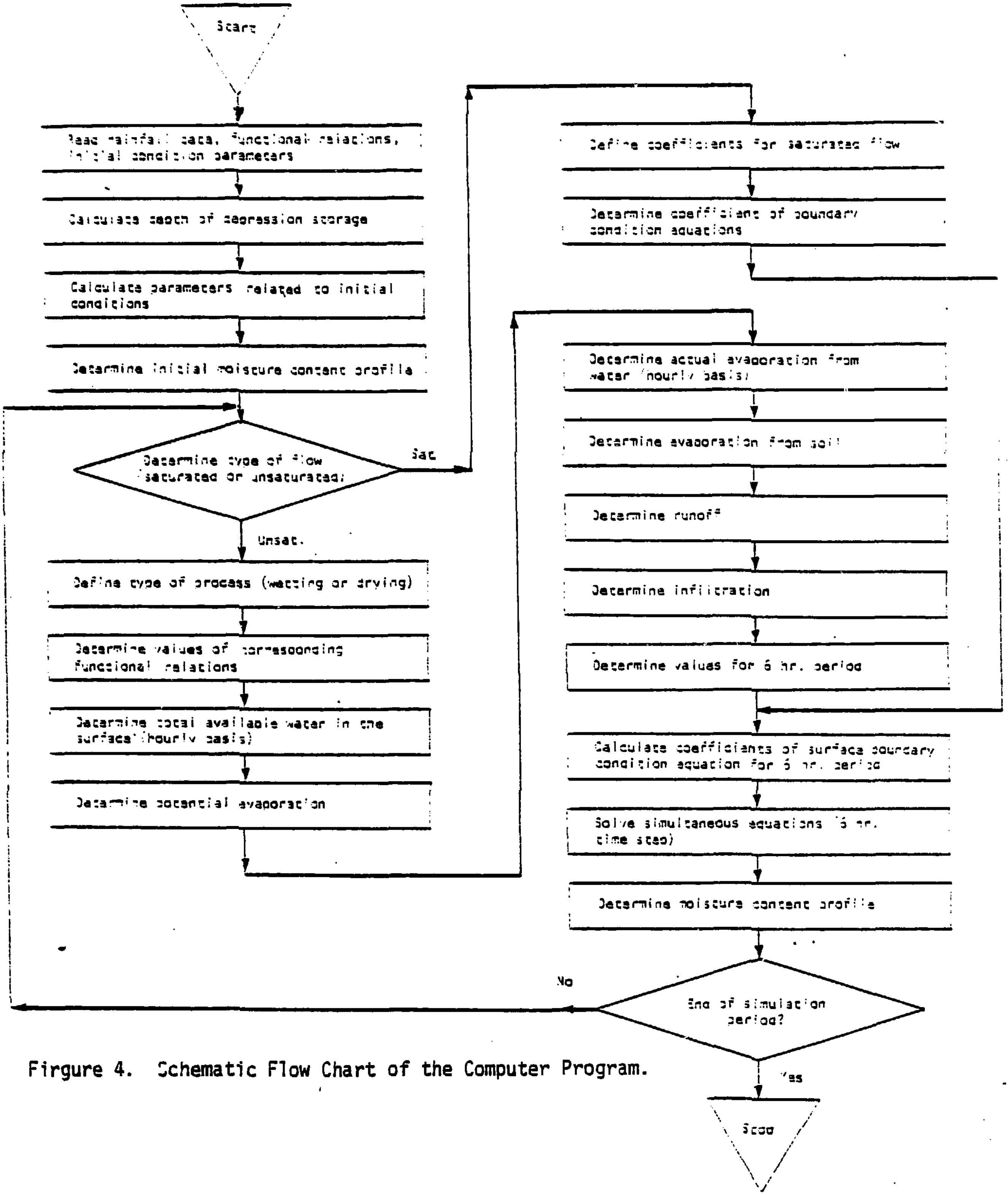


Figure 3. Two Dimensional Net Representing the Space and Time Coordinates (ψ_J^N represents the Pressure Head at Node J in the Time Step N).



Firgure 4. Schematic Flow Chart of the Computer Program.

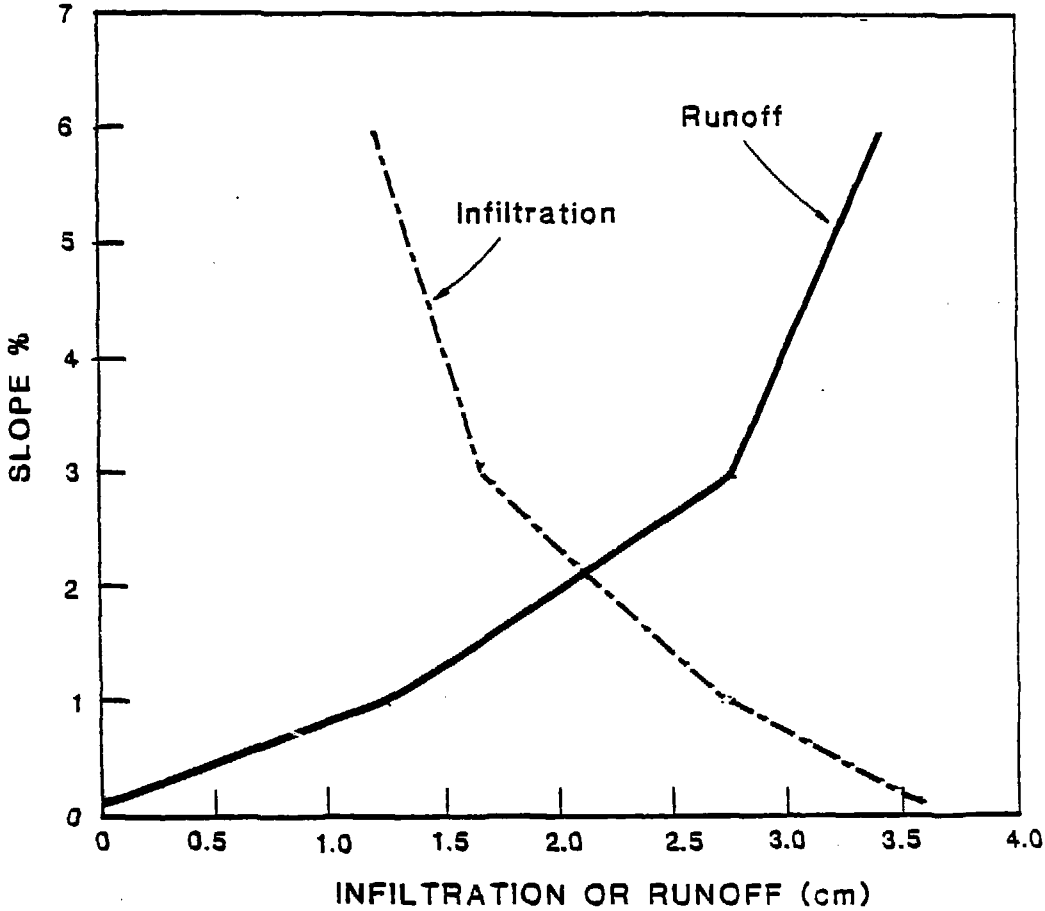


Figure 5. Infiltration vs. Slope and Runoff vs. Slope

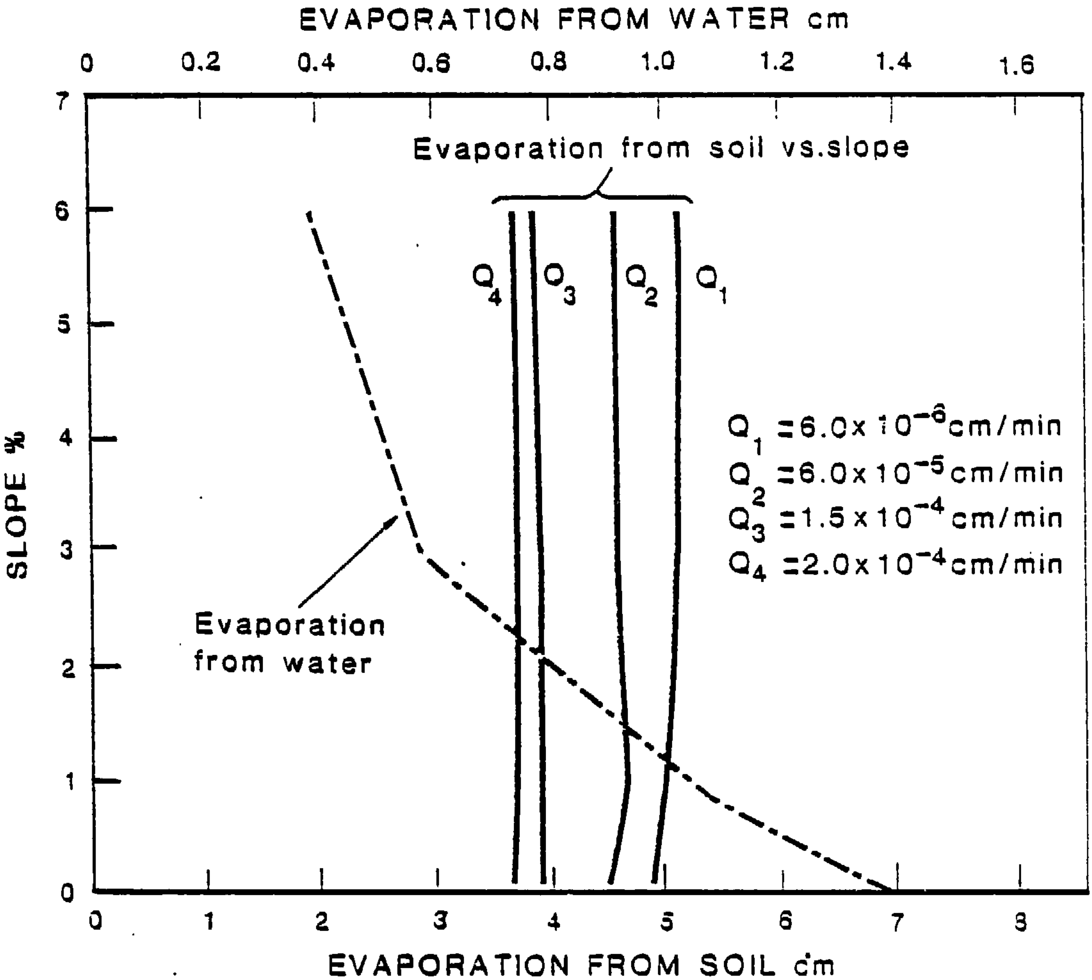


Figure 6. Evaporation from Water vs. Slope and Evaporation from Soil vs. Slope

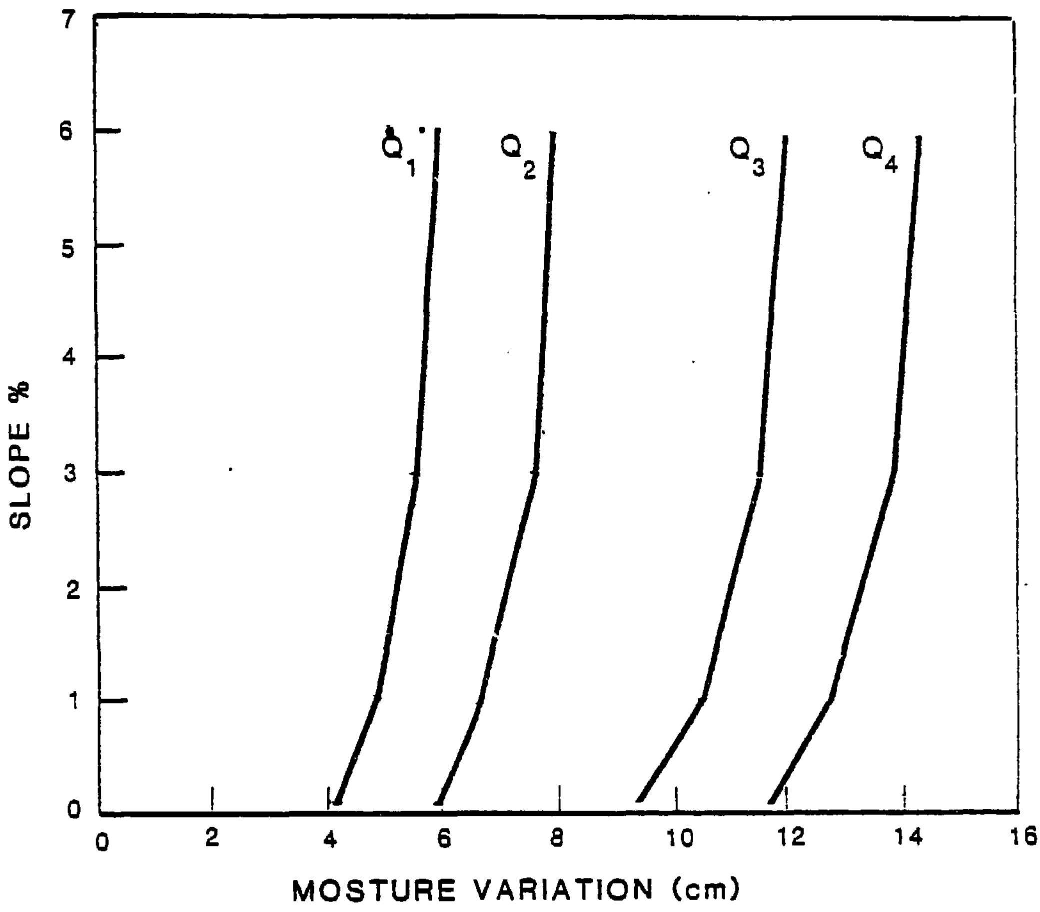


Figure 7. Moisture Variation vs. Slope.

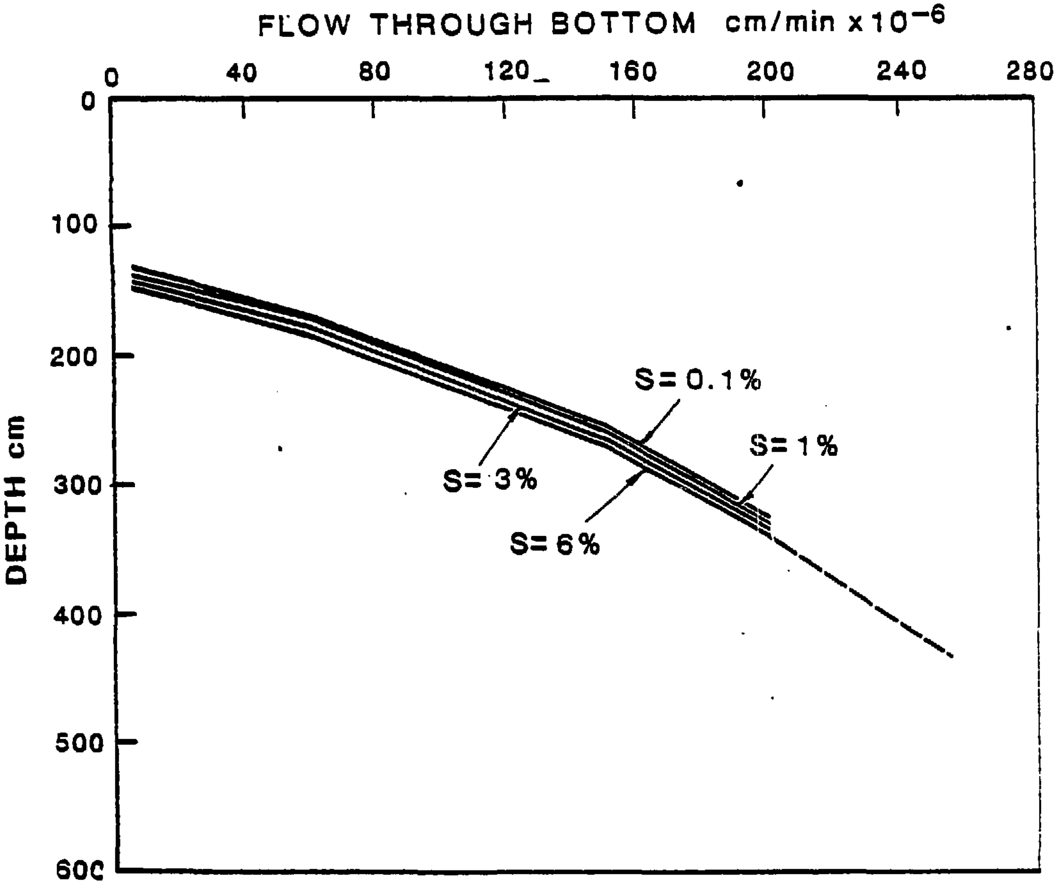


Figure 8. Depth of Ground Water Table vs. Flow through Bottom Boundary.