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Accurate Method of the Magnetic Field Measurement of Quadrupole Magnets

M. Kumada, I. Sakai, H. Someya and H. Sasaki

Abstract

We present an accurate method of the magnetic field measurement of the quadrupole magnet. The method of obtaining the information of the field gradient and the effective focussing length is given. A new scheme to obtain the information of the skew field components is also proposed. The relative accuracy of the measurement was 1×10^{-4} or less.

1. Introduction

In order to construct a good quadrupole magnet with high homogenous field distribution, the accuracy of the measurement has to be better than the required homogeneity, or at least comparable to it. Although the question of how high accuracy is required is interesting, we shall not discuss it in this paper. We assume the required relative accuracy of the field gradient is near or less than 1×10^{-4}.

One of the most well known methods of measuring the field of the quadrupole magnet is the use of a so-called "harmonic coil", which essentially picks up all the harmonic components inside the given volume where the coil is rotated by 2π radian. The separation of the individual harmonic components, from fundamental to higher harmonics including the phase, is usually performed by Fourier analysis of the integrated voltage of the "pick up" coil.

It is easy to see that the accuracy of this method is determined by the resolution of the most dominant component of the voltage, which is the quadrupole component in this case. Thus, to achieve the accuracy of 1×10^{-4} is very difficult by this method. To overcome this difficulty, one of the authors devised[1),2),3)] a method of suppressing the unwanted fundamental component. Essentially, the suppression of the fundamental component is done by providing an additional coil. In practice, however, it is still difficult to suppress the fundamental component by the additional coil alone. This is because a small alignment error can produce an appreciable amount of spurious fundamental component in a quadrupole magnet. This difficulty is overcome by adding back-up coils to suppress the unwanted signals.

Thus by reducing the measurement of the quadrupole component to that of the deviation from it, the relative accuracy may be much improved. In fact it is possible to achieve relative accuracy of 1 x 10^{-4} in this way. It seems, however, this idea has not yet acquired general familiarity. This may be due to the limited circulation of the information on this subject. The purpose of this paper is to communicate this idea as widely as possible to those who may have need in the future to construct a quadrupole magnet.

In the following, we shall present a method of measuring the magnetic field of a conventional iron core quadrupole magnets[6)]. In ref. 6) and 7), only the normal components are measured. In this note, we shall show that the skew components may be also measurable with a small modification of this method. The method of the measurement of superconducting quadrupole magnets is given in refs. 1), 2), and 3). The difference in method arises from the difference in shape of the useful beam aperture: an elliptic shape for the conventional and a round shape for the superconducting magnet.

In order to pick up static field, a relative motion between the pick up coil and the magnet is required. This relative motion is done either by rotation (harmonics coil) or by translation (pendulum flux-meter)[6)]. One of the advantages of the latter method is its easiness of the suppression of the unwanted fundamental component. In the case of the pendulum fluxmeter, the output signal is free from spurious fundamental component, whereas in the case of the harmonics coil suppression of the spurious one was the key point idea. In the case of harmonics coil one cannot locate the coil beyond the bore radius. This is not the case with the pendulum fluxmeter. In the elliptically shaped aperture like most of the conventional magnet, the motion of translation becomes another advantage. This is simply because at large transverse coordinate x, the amplitude of the higher harmonics is much larger than that at small x and the resultant increase of the signal helps to improve the accuracy of determining the harmonics.

In the following we shall only discuss the method of measuring the relative quantity and not that of the absolute. This is due to the complicated behavior of the hysteresis effect in the iron in addition to the difficulty of determining the absolute quantity. It is often the case that the accuracy of the absolute quantity is not so much required as that of the relative one.

2. Field gradient

In a DC quadrupole magnet , the rectilinear motion induces the voltage on the coil (single coil), whose time-integral is proportional to the first derivative of the magnetic field with respect to the coordinate x or y. If the output of the two coils (twin coil) is connected differentially, the time-integral of the output voltage of the twin coil due to the rectilinear motion is proportional to the second derivative. In practice, the output voltage is also proportional to other quantities such as the rate of change in position, the effective area A of each coil, the distance between the coils δx, and the displacement of the twin coil Δx. Elimination of the dependence of the rate of change in position on the output is done by the integration of the output voltage with respect to time[6]. The voltage integration is done by a voltage to frequency converter (VFC). Errors resulting from the dependence of the effective area of the coils are corrected by interchanging the position of each coil and by the normalization[6]. Let N be the number of the integrated pulses, then after permutation the sum of the outputs can be written as

$$N(x)^{twin} = \frac{f_0}{e_0}(A_1 + A_2)\frac{\partial^2 B(x)}{\partial x^2}\delta x \Delta x \tag{1}$$

where superscript "twin" denotes the integrated output of the twin coil, e_0 is the input full scale voltage of the VFC, δx is the distance between the coils and f_0 is the corresponding output frequency of the VFC. We take the right-handed coordinate, where s-axis is along the central orbit lying in the midplane (the x-s plane) and y-axis is perpendicular to the midplane. Note that N does not depend on the rate of change in position but depends on the sum of the two effective areas, A_1 and A_2, of the coils.

Normalization is done by the sum of the outputs N^{single} of the two single coils. N^{single} is written as

$$N(x)^{single} = \frac{f_0}{e_0}(A_1 + A_2)\frac{\partial B(x)}{\partial x}\Delta x \ . \tag{2}$$

From eqs. (1) and (2), we obtain

$$\frac{\partial^2 B(x)/\partial x^2}{\partial B(0)/\partial x} = \frac{1}{\delta x}\cdot\frac{N(x)^{twin}}{N(0)^{single}} \ . \tag{3}$$

As we are directly observing the small deviation itself whose magnitude is in the order of 1 % or less of the fundamental component, considerable relaxation of a mechanical and electrical tolerances are expected. Indeed, both electrical and mechanical accuracy of 1 %, which leads to a relative accuracy of 0.01 %, is easily realized. Figure 1(a) shows the typical example[7]. The solid line shows the data obtained from eq. (3) and the dotted line shows the integrated one.

3. Integrated field gradient

In many cases the length of the quadrupole magnet is not long compared with the betatron oscillation wavelength. The homogeneous region in the field gradient in the longitudinal direction is shorter than the magnet core length. Figure 2 shows a typical example of the longitudinal distribution of the field gradient in the quadrupole magnet of BSF beam line[7] where the bore radius is 57 mm and the length of the core is 300 mm. From this figure it can be seen that the homogeneous region is roughly half of the effective length ℓ_G:

$$\frac{L(1\%)}{\ell_G} \simeq \frac{18(\mathrm{cm})}{34(\mathrm{cm})} = 0.53,$$

where L(1%) denotes the length of the region where the field gradient drops down by 1% of the central field gradient. Thus half of the length of the magnet is occupied by the fringe field. Now short magnet length suggests the validity of the thin lens approximation, or δ function approximation. Actually the effect of the presence of higher harmonics on resonances or diffusion may be investigated by δ function approximation in the computer simulation or phase space mapping[8]. In any case, information of the radial distribution of the integrated field gradient is of significant importance.

Accuracy of the effective length is maintained by taking the ratio of the outputs of two coils, namely, a long coil and a reference coil. The latter is embedded in the former coil. The key point of this measurement is the simultaneous motion of the coils. Due to the simultaneous motion, the output ratio of the two coils is insensitive to the fluctuation of the displacement Δx, drift and ripples of the power supply and other common mode noises. Ripples which are not able to be removed from the simultaneous motion are further suppressed by providing an additional stationary coil.

The radial distribution of the effective length may be obtained in the following way. In terms of the number of the integrated pulses N, the effective length ℓ_G is written as

$$\ell_G(x) = \frac{\int_{-\infty}^{\infty} \frac{\partial B(x,s)}{\partial x} ds}{\frac{\partial B(x,0)}{\partial x}}$$

$$= \frac{A}{W} \cdot \frac{N_L}{N_{ref}} \tag{4}$$

where W and A denote the width of the long coil and the effective area of the reference coil, N_L and N_{ref} are the number of the integrated pulses of the long coil and the reference coil. The length of the long coil is determined from the longitudinal field distribution given by the point measurement.

Similar to the discussion in the section of "field gradient", the system of twin long coil and Fourier analysis is also possible for the integrated field gradient. Equations corresponding to the eqs. (1), (2) and (3) are, respectively,

$$N_L(x)^{twin} = \frac{f_0}{e_0}(w_1 + w_2)\int_{-\infty}^{\infty} \frac{\partial B^2(x,s)}{\partial x^2} ds\delta x\Delta x, \tag{5}$$

$$N_L(0)^{single} = \frac{f_0}{e_0}(w_1 + w_2)\int_{-\infty}^{\infty} \frac{\partial B(0,s)}{\partial x} ds\Delta x, \tag{6}$$

$$\int_{-\infty}^{\infty} \frac{\partial^2 B(x,s)}{\partial x^2} ds \Big/ \int_{-\infty}^{\infty} \frac{\partial B(0,s)}{\partial x} ds = \frac{1}{\delta x} \cdot \frac{N_L(x)^{twin}}{N_L(0)^{single}} \tag{7}$$

where subscript L denotes "long coil". The radial distribution of the integrated field gradient is obtained by integrating eq. (5) with respect to x.

It should be noted that eq. (7) involves two independent coils (twin coil) and four independent measuring sequences. One obtains the normal harmonic components from above procedure. Rotation of the twin coil by 90° around each of its axis gives the skew harmonics. For the reduction of number of data taking the provision of the additional twin coil to pick up skew components might be preferable.

The radial distribution of the integrated field gradient is obtained either from the combination of eqs. (3) and (4) or from that of eqs. (5), (6) and (7). In the former case the measuring sequence is simpler than in the latter, but the tolerance of the signal to noise ratio is tighter than the latter. Figure 1(b) shows the typical data[7] obtained by the former process.

4. <u>Harmonic coefficients</u>

Coefficients of the harmonic components are obtained by the Taylor expansion of the data from eq. (3). In fact, it turns out that the information on the median (horizontal) plane or the vertical plane is sufficient to construct the field distribution on the transverse plane. In the following, we shall show how this is done.

Let $\Phi(r,\theta)$ be the scalar potential on the transverse plane, where r and θ are polar coordinates, then we have

$$\Phi(r,\theta) = -\sum_{n=1}^{\infty} \frac{r^n}{n}(b_n \sin n\theta + a_n \cos n\theta) \tag{8}$$

where b_n and a_n are usually called the normal and skew coefficients. In most cases the skew components are small compared to the normal components.

Rewriting the scalar potential in cartesian coordinates, the vertical and horizontal magnetic induction B_y, B_x are obtained as

$$
\begin{aligned}
B_y &= -\frac{\partial\Phi(x,y)}{\partial y} \\
&= b_1 \\
&\quad +b_2x \qquad\qquad -a_2y \\
&\quad +b_3(x^2-y^2) \qquad\qquad -2a_3xy \\
&\quad +b_4(x^2-3y^2)x \qquad\qquad +a_4(y^2-3x^2)y \\
&\quad +b_5(x^4-6x^2y^2+y^4) \qquad\qquad +4a_5(y^2-x^2)xy \\
&\quad +b_6(x^4-10x^2y^2+5y^4)x \qquad\qquad +a_6(-y^4+10x^2y^2-5x^4)y \\
&\qquad \vdots \qquad\qquad\qquad \vdots
\end{aligned} \tag{9}
$$

and

$$
\begin{aligned}
B_x &= -\frac{\partial\Phi(x,y)}{\partial x} \\
&= \qquad\qquad\qquad a_1 \\
&\quad +b_2y \qquad\qquad +a_2x \\
&\quad +2b_3xy \qquad\qquad +a_3(x^2-y^2) \\
&\quad +b_4(3x^2-y^2)y \qquad\qquad +a_4(x^2-3y^2)x \\
&\quad -4b_5(y^2-x^2)xy \qquad\qquad +a_5(x^4-6x^2y^2+y^4) \\
&\quad +b_6(5x^4-10x^2y^2+y^4)y \qquad\qquad +a_6(x^4-10x^2y^2+5y^4)x \\
&\qquad \vdots \qquad\qquad\qquad \vdots \quad .
\end{aligned} \tag{10}
$$

On the median plane where $y = 0$, eqs. (9) and (10) are reduced to a very simple form written as

$$B_y = \sum_{n=1}^{\infty} b_n x^{n-1} \tag{11}$$

and

$$B_x = \sum_{n=1}^{\infty} a_n x^{n-1} \quad , \tag{12}$$

whereas on the vertical plane they are written as

$$\begin{aligned} B_y &= b_1 - b_3 y^2 + b_5 y^4 - \cdots - a_2 y + a_4 y^3 - a_6 y^5 + \cdots \\ &= b_1 - a_2 y - b_3 y^2 + a_4 y^3 + b_5 y^4 - a_6 y^5 - \cdots \end{aligned} \tag{13}$$

and

$$\begin{aligned} B_x &= a_1 - a_3 y^2 + a_5 y^4 - \cdots + b_2 y - b_4 y^3 + b_6 y^5 - \cdots \\ &= a_1 + b_2 y - a_3 y^2 - b_4 y^3 + a_5 y^4 + b_6 y^5 - \cdots \quad . \end{aligned} \tag{14}$$

It is seen that on the vertical plane, normal coefficient b_n and skew coefficient a_n appear alternatively in the Taylor expansion of B_y, whereas on the median (horizontal) plane they are decoupled. In either case, a set of the normal and skew coefficient is obtained by the Taylor expansion of the two orthogonal field components, B_y and B_x. This is equivalent to the Fourier analysis.

As the knowledge of the dominant terms of the harmonic coefficients is useful, we will now consider the field distribution of the quadrupole magnet which has the following symmetry in the scalar potential Φ.

First we consider the case,

(i) $\Phi(\theta + \frac{\pi}{2}) = -\Phi(\theta)$

This condition requires the following equation,

$$\frac{n}{2} = (2m + 1),$$

where $n = 2,6,10,14,18,\cdots$ and $m = 0,1,2,\cdots$.

Thus on the median plane, the field gradients normalized by the fundamental are,

$$\frac{\partial B_y(x)}{\partial x}/\frac{\partial B_y(0)}{\partial x} = 1 + 5\frac{b_6}{b_2}x^4 + 9\frac{b_{10}}{b_2}x^8 + 13\frac{b_{14}}{b_2}x^{12} + \cdots, \qquad (15)$$

and

$$\frac{\partial B_x(x)}{\partial x}/\frac{\partial B_y(0)}{\partial x} = \frac{a_2}{b_2} + 5\frac{a_6}{b_2}x^4 + 9\frac{a_{10}}{b_2}x^8 + 13\frac{a_{14}}{b_2}x^{12} + \cdots. \qquad (16)$$

The next case considered is,

(ii) $\Phi(\theta) = -\Phi(\pi - \theta)$.

In this case we have

$$n = 2m, \text{ or } n = (2m + 1)$$

where $m = 1,2,3,\cdots$.

Thus, on the median plane, we obtain

$$\frac{\partial B_y(x)}{\partial x}/\frac{\partial B_y(0)}{\partial x} = 1 + 3\frac{b_4}{b_2}x^2 + 5\frac{b_6}{b_2}x^4 + 7\frac{b_8}{b_2}x^6 + \cdots, \qquad (17)$$

or

$$\frac{\partial B_x(x)}{\partial x} / \frac{\partial B_y(0)}{\partial x} = \frac{a_3}{b_2} + 4\frac{a_5}{b_2}x^3 + 6\frac{a_7}{b_2}x^5 + 8\frac{a_9}{b_2}x^7 + \cdots. \qquad (18)$$

The eqs. (17) and (18) do not hold simultaneously if the symmetry is perfect.

In the ideal normal magnet, the skew component is zero, and the significant equations are eqs. (15) and (17).

It may be worthy of mention that in the quadrupole magnet which has a symmetric potential of the case (i). the dominant terms next to the fundamental one are, downwardly, 12th pole, 20th pole, 28th pole, ···. These terms are called "allowed terms", whereas other terms such as normal sextupole, normal octupole, normal decapole, etc., and all other skew terms are called "inhibited terms".

Due to the presence of the errors, inhibited terms exist and one can obtain both the normal and skew components by Taylor expansion from eqs. (3), (9) and (10). Vertical component B_y and horizontal component B_x are obtained by rotating the axis of each coil by 90 degrees. On the median plane the above process could be also applied to the integrated field gradient.

5. Hardware

We have shown that except the information of the absolute field strength and the effective length, the information of the nonlinear deviation is directly obtained from the twin coil. This leads to the considerable relaxation of the mechanical and electrical tolerances. Tolerances of 1 %, for example, may be sufficient to achieve an accuracy of the field gradient of 1×10^{-4} or less. Due to this advantage, hardwares, both mechanical and electrical, need not be designed accurately more than necessary. Typical examples of the distribution of the field gradient and the effective length are shown in Fig. 1(a) and (b)[7]. In Figs. 3, 4 and 5, we show the system designed for the magnetic field measurement of the BSF beam lines[9]. We called this system "Pendulum fluxmeter". Figure 3 is a schematic diagram showing how this pendulum is located inside the quadrupole magnet. Figure 4 shows how the reference coil, long coil, VFC and counter are assembled. They are connected to CAMAC stand alone automatic measurement system. Figure 5 shows how the rectilinear motion is performed. The horizontal displacement is done by the pulleys and a wire belt. Note that the axis of the coil is fixed and always directed in the vertical direction, in spite of the circular motion of the coil. We have proved that this system has worked satisfactorily for determining the normal harmonic coefficients[6]. The modification for picking up the skew components may be easily done but has yet to be experimentally verified.

6. Conclusion

We have shown that normal harmonic coefficients can be determined very accurately. In the conventional quadrupole magnet, where the useful aperture can be designed to be larger than the bore radius, our method is superior to that of the harmonics coil method. We have also pointed out that our method is capable of being extended so that the skew harmonic coefficients are also obtained. The authors hope that the method and the idea behind it acquire more familiarity and this method does help in designing conventional quadrupole magnets[9].

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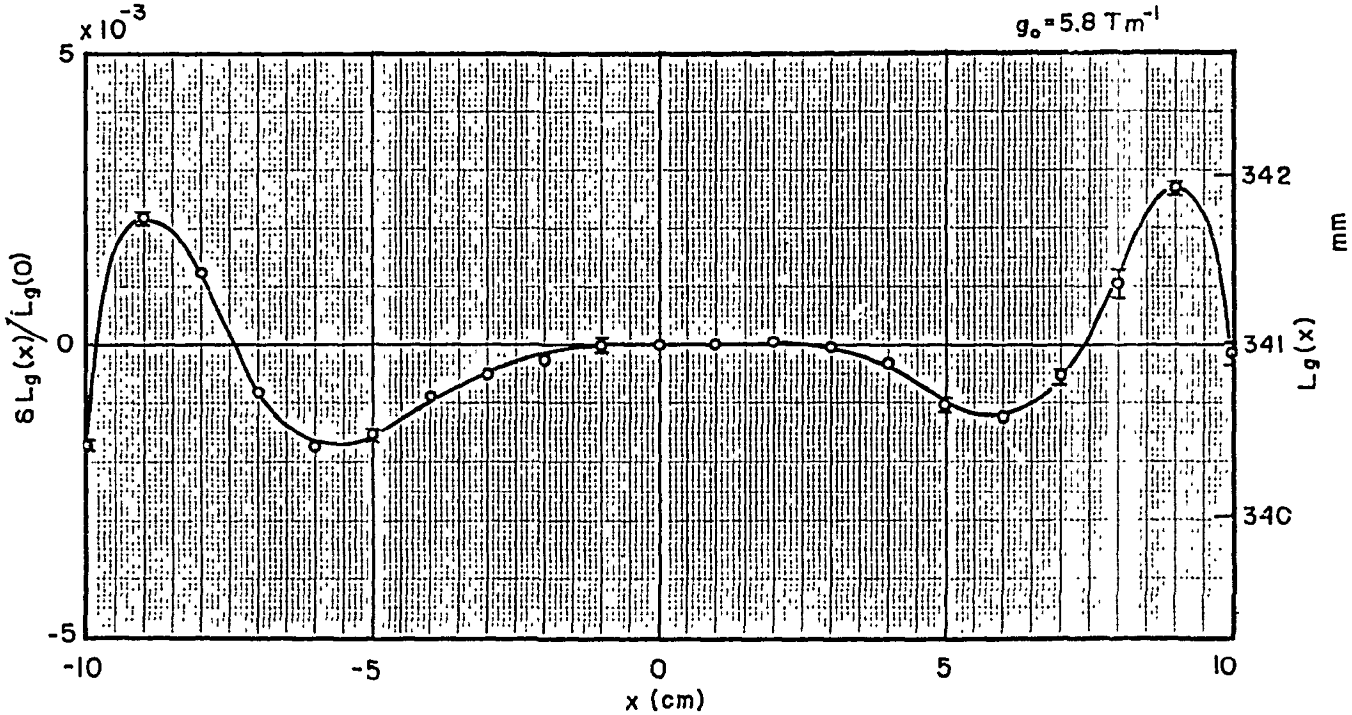

Fig.1b Radial dependence of the effective focussing length of the quadrupole magnet.

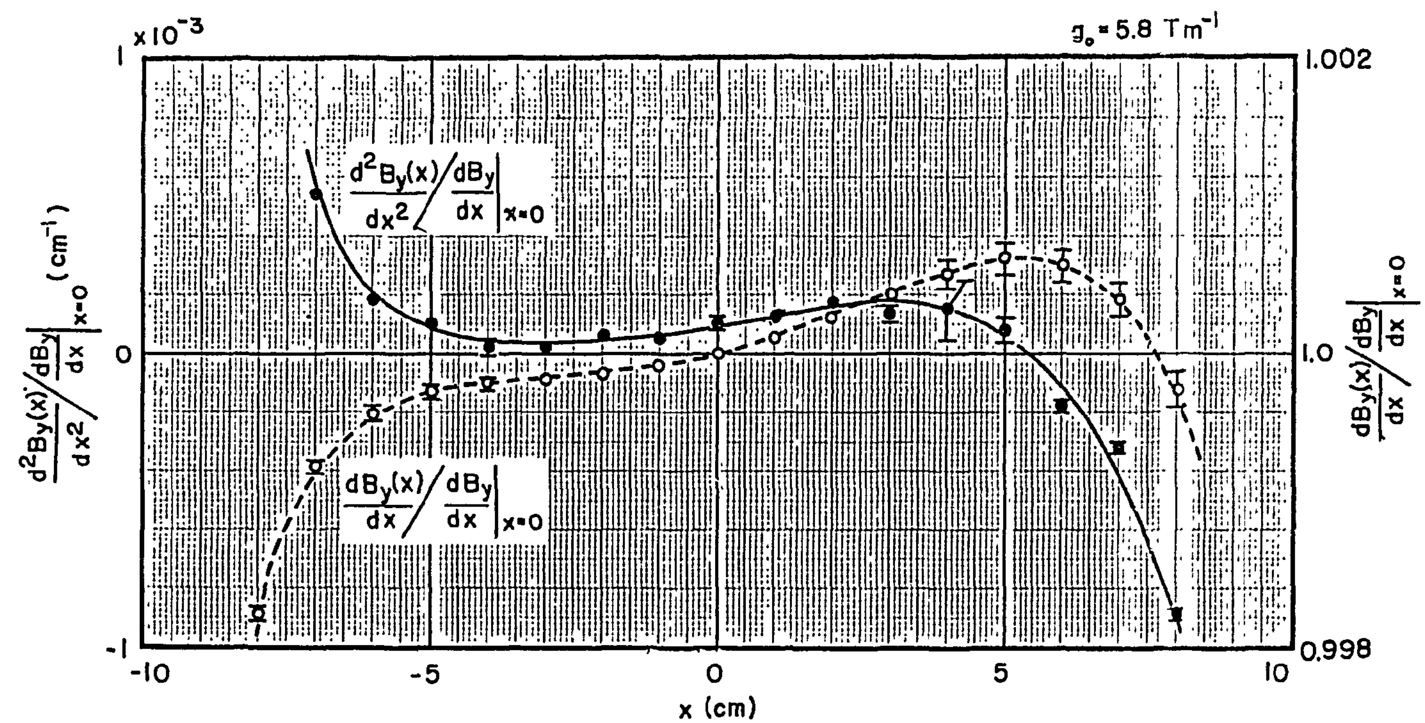

Fig.1a Magnetic field distribution of the quadrupole magnet
Solid and dotted curves show the second and the first derivative respectively.

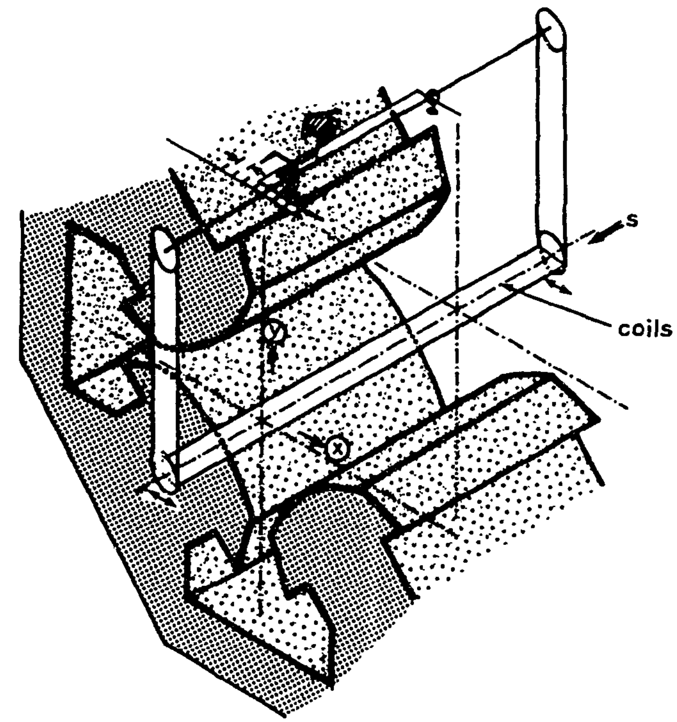

Fig.3 Pendulum flux meter and quadrupole magnet.

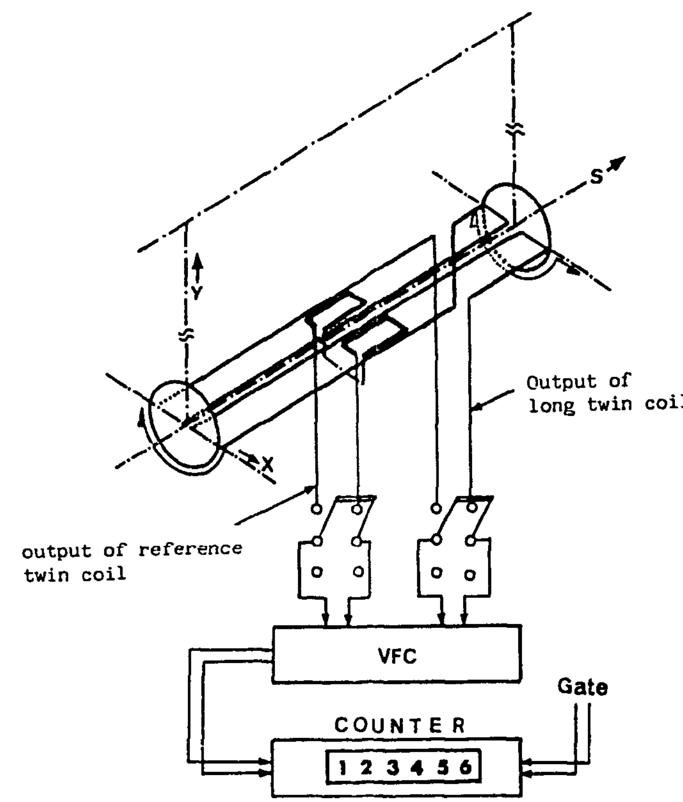

Fig.4 Pendulum flux meter and electric circuit.

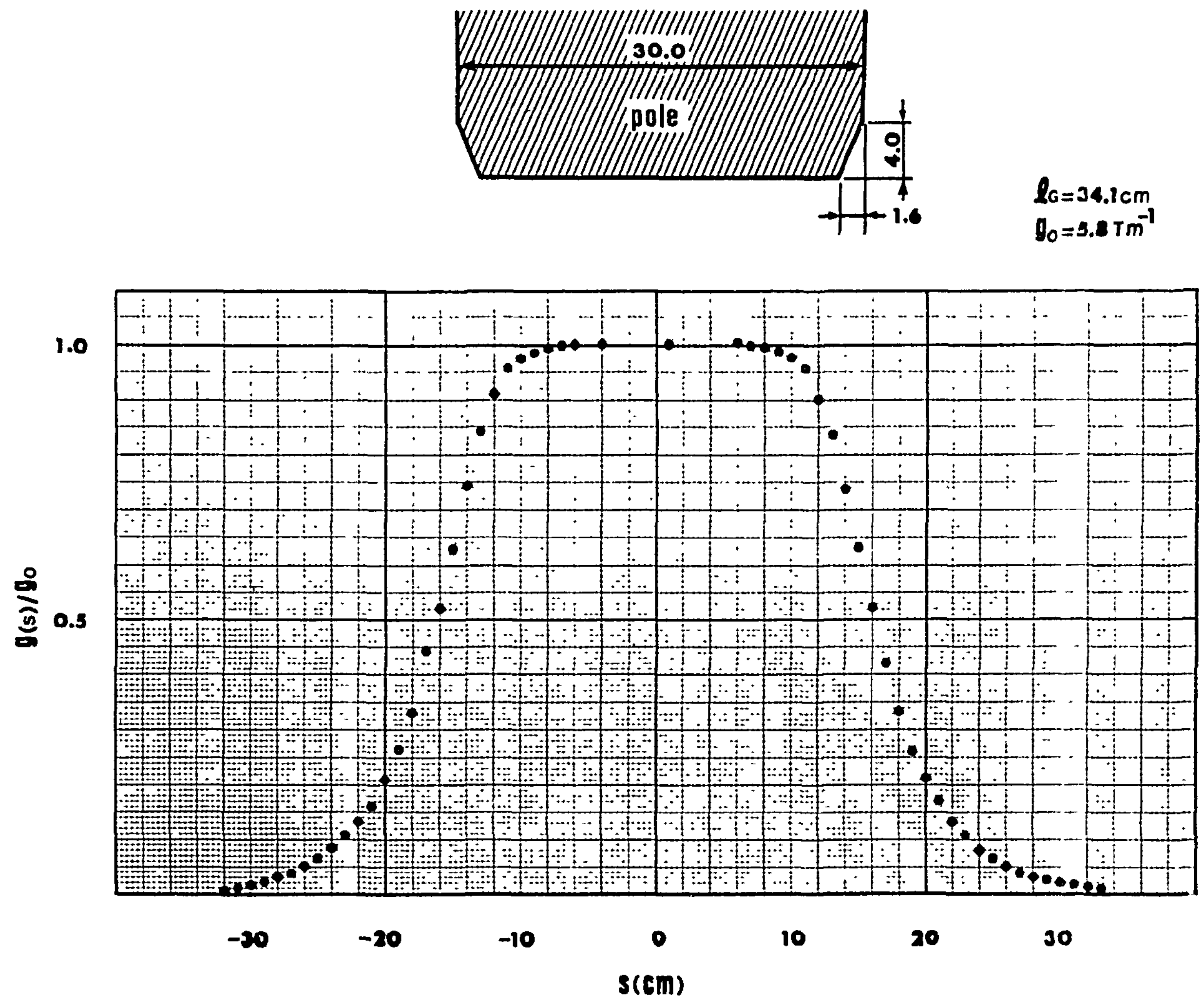

Fig.2 Distribution of the normalized field gradient along the magnetic axis.

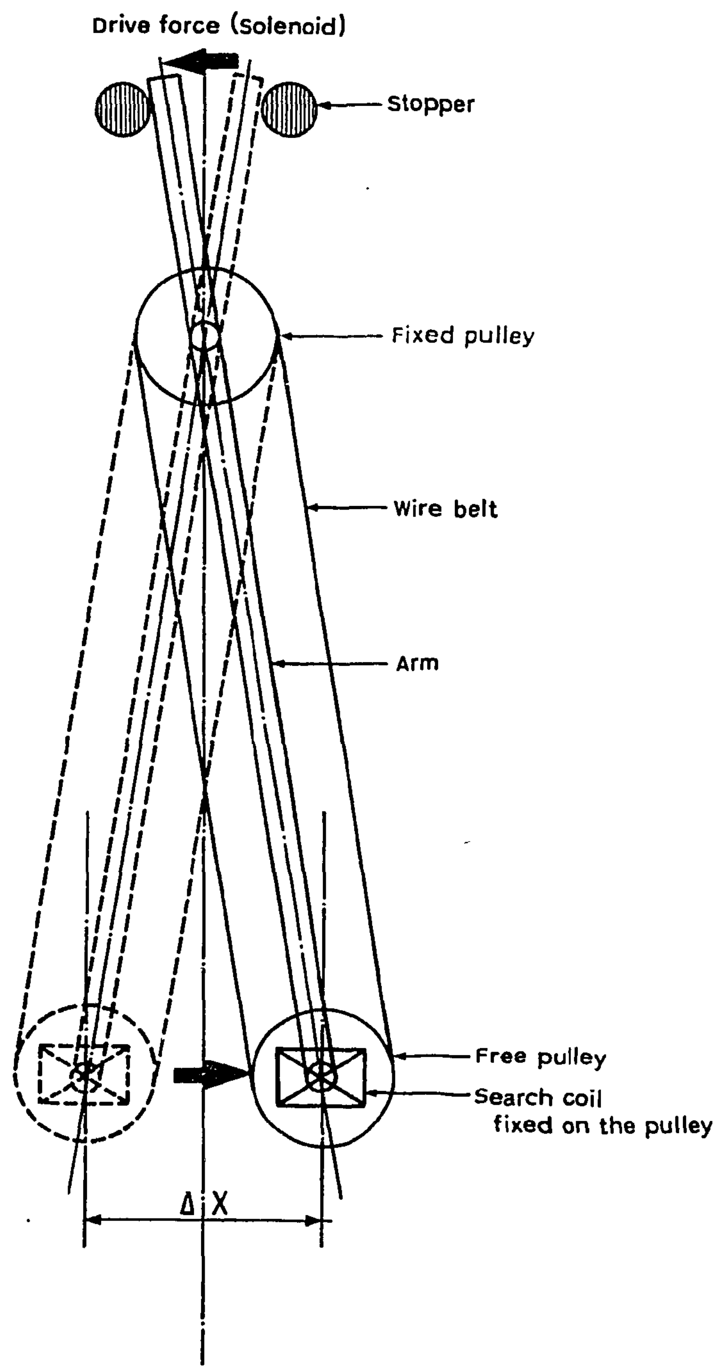

Fig.5 Motion of the search coil on the pendulum.
Direction of the search coil is kept constant through the motion by the pulley.