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Sum Rules and Systematics for Baryon Magnetic Moments*

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ABSTRACT

The new experimental values of hyperon magnetic moments are compared with sum rules predicted from general quark models. Three difficulties encountered are not easily explained by simple models. The isovector contributions of nonstrange quarks to hyperon moments are smaller than the corresponding contribution to nucleon moments, indicating either appreciable configuration mixing in hyperon wave functions and absent in nucleons or an additional isovector contribution beyond that of valence quarks; e.g. from a pion cloud. The large magnitude of the Ξ^- moment may indicate that the strange quark contribution to the Ξ moments is considerably larger than the value $\mu(\Lambda)$ predicted by simple models which have otherwise been very successful. The set of controversial values from different experiments of the Σ^- moment include a value very close to $-(1/2)\mu(\Sigma^+)$ which would indicate that strange quarks do not contribute at all to the Σ moments.

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The values predicted by the simple broken-$SU(3)$ model for magnetic moments of baryons, [1] using the quark mass values obtained from hadron masses [2, 3] are in rough agreement with experiment. But recent more precise measurements of these moments [4, 5, 6] reveal unexplained discrepancies at the 15% level, which were not yet noted in the 1982 textbook data [7]. We investigate these discrepancies by examining a number of old and new sum rules and noting the underlying assumptions and the comparison with experiment.

The Coleman-Glashow formula for the magnetic moments,

$$\mu(p) - \mu(n) = 4.70 n.m. = \mu(\Sigma^+) - \mu(\Sigma^-) + \mu(\Xi^-) - \mu(\Xi^o) \tag{1}$$

was originally derived from SU(3) symmetry but has a much more general derivation. A similar but different sum rule has been obtained by Sachs [8].

$$3\{\mu(p) + \mu(n)\} = 2.64 n.m. = \mu(\Sigma^+) - \mu(\Sigma^-) - \mu(\Xi^-) + \mu(\Xi^o). \tag{2}$$

Table 1 lists the accepted values for baryon moments in the fall of 1981 [3, 9, 10] and world averages taken in the summers of 1982 [11] and 1983. The latter uses the new recently published values with greatly reduced errors for the Σ^+ and Σ^- moments [4, 5, 6]. Also listed are the results obtained from the three sets of data for the right hand sides of eqs. (1) and (2) and for tests of other sum rules defined below.

Despite some controversy between values of moments obtained from different experiments, all three sets of data give values for the right hand side of the Coleman-Glashow sum rule (1) in significant disagreement with the predicted value of 4.7. But the experimental test of the Sachs sum rule (2) is confused. The first two sets of data give 2.72 ± 0.2 and 2.78 ± 0.18 in good agreement with the predicted value of 2.64. The third set gives 2.94 ± 0.06 in strong disagreement.

The main difference between the three sets of data is the disagreement over the value of the Σ^- moment which varies between -0.89 n.m. [11] in the 1981 value and -1.11 n.m. [6] in the new data. These new values give serious disagreement not only with the Sachs sum rule, but also with other relations discussed below. The resolution of this controversy over the Σ^- moment is thus of considerable interest.

For our subsequent analysis we note that a convenient parametrization to describe the magnetic moment of a baryon with two quarks of flavor a and one quark of flavor b denoted by β_{aab} is

$$\mu(\beta_{aab}) = 2A\mu_a + B\mu_b \tag{3a}$$

where A and B are parameters that depend upon the wave functions of the baryons and μ_f denotes the magnetic moment of a quark of flavor f. This parametrization is valid in a large class of models, including relativistic bag models as well as the nonrelativistic quark model with arbitrary configuration mixing.

In the nonrelativistic quark model, the parameters A and B are expectation values of functions of the quark angular momenta,

$$2A = \langle (2/g_a)L_{za} + 2S_{za} \rangle \tag{3b}$$

$$B = \langle (2/g_b)L_{zb} + 2S_{zb} \rangle \tag{3c}$$

where L_a and L_b are the total orbital angular momenta, S_a and S_b are the total spin angular momenta J_a and J_b are the total angular momenta and g_a and g_b are the g-factors of the two quarks of flavor a and the quark of flavor b respectively, and the expectation value is taken in the state of maximum $J_z = J_{za} + J_{zb}$. The parameters are normalized so that $A = B = 1$ describes the spin 3/2 state with all spins parallel and no orbital angular momentum.

We first consider only the general parametrization (3a), without the relations (3b) and (3c) which hold only in the nonrelativistic quark model. The standard broken-SU(3) symmetry limit relevant to all models uses SU(3)-symmetric baryon wave functions; i.e. the values of the parameters A and B and the three quark moments μ_f are taken to be the same for all the six baryons, but otherwise not determined a priori and fit to the experimental moments. This breaks SU(3) only in the values of the quark moments. In any given model the values of A and B are determined by the baryon wave function. When SU(3) is broken also in the baryon wave functions different values of A and B can occur for baryons in different isospin multiplets, but isospin symmetry requires the same values within any multiplet.

We assume isospin symmetry and define the parameters A_N, A_Σ, A_Ξ, B_N, B_Σ, and B_Ξ for each multiplet. We can also use isospin symmetry to apply the parametrization (3a) to the Λ

$$\mu(\Lambda) = 2A_\Lambda \mu_{no} + B_\Lambda \mu_s \tag{4a}$$

where μ_{no} denotes the isoscalar component in the magnetic moments of nonstrange quarks,

$$\mu_{no} = (1/2)(\mu_u + \mu_d) \tag{4b}$$

If the eight A and B parameters and the three quark moments μ_u, μ_d and μ_s are all adjusted freely to fit the seven baryon moments, the model has no predictive power. Many models assume that the quark moments are proportional to their electric charges and assume isospin symmetry to obtain $\mu_u = -2\mu_d$. It is also common to assume that $\mu(\Lambda) = \mu_s$ and thus obtain the value of μ_s from experiment. We can describe these assumptions and their violations by defining two parameters ϵ and δ by the relations

$$(\mu_u/\mu_d) = -(2+\epsilon) \tag{4c}$$

$$\mu(\Lambda) = \mu_s + \delta \tag{4d}$$

Some models [3] have introduced an anomalous isoscalar component which destroys the proportionality between quark moments and charges. In conventional models with no such anomalous isoscalar component $\epsilon = 0$. In many models where the nonstrange quarks in the Λ are coupled to spin zero and the entire moment comes from the spin of the strange quark. $A_\Lambda = 0$, $B_\Lambda = 1$ and $\delta = 0$.

It is convenient to introduce fictitious baryons denoted by β_f whose constituent quarks all have the magnetic moment of a quark of flavor f and have moments given by eq. (3a) with $\mu_a = \mu_b = \mu_f$.

$$\mu(\beta_f) = (2A+B)\mu_f \tag{4e}$$

The conventional nonrelativistic broken-SU(3) static quark model [1] sets $\epsilon = \delta = 0$ to give $\mu(\Lambda) = \mu_s$ and chooses the the values $A = 2/3$ and $B = -1/3$ for the other baryons as given by s-wave $SU(6)$ wave functions with a and b spins coupled antiparallel to give spin 1/2. This leaves μ_u and μ_s as two free parameters determined by fitting experiment. The model with no free parameters [2,3] determines the quark moments μ_u and μ_s by assuming them to be Dirac moments and obtains values for the two quark masses m_u and m_s by various fits to hadron masses. Both simple models disagree with experiment. The question is whether a better significant fit to the data is obtainable with a credible model having a sufficient number of constraints on the eleven parameters.

We now substitute the expression (3a) into the two sum rules (1) and (2) and replace the equality in (1) by the inequality expressing the direction of the disagreement with experiment, to see which constraints on the parameters are imposed by the experimental data. This gives

$$2A_N - B_N > 2A_\Sigma - B_\Xi \tag{5a}$$

for the Coleman-Glashow sum rule (1) and

$$3\frac{\mu_u + \mu_d}{\mu_u - \mu_d}(2A_N + B_N) = \left(1 + \frac{2\epsilon}{3+\epsilon}\right)(2A_N + B_N) = 2A_\Sigma + B_\Xi \tag{5b}$$

for the Sachs sum rule (2). Although both sum rules are obtainable from the full SU(3) symmetry; i.e. all A's equal and all B's equal, only the ***nonstrange*** contributions to the Σ and Ξ contribute. The contributions from the strange quarks cancel out completely if isospin is a good symmetry, since both sum rules (1) and (2) depend only on the ***isovector*** contributions to hyperon magnetic moments. They are therefore not affected by the usual SU(3) symmetry breaking mechanism; namely the change in the magnetic moment of the strange quark due to its larger mass.

A single constraint on these parameters (5a) is sufficient to derive the Coleman-Glashow sum rule (1). The disagreement with experiment indicates that the nonstrange contributions are smaller in hyperons than in the nucleon. A different single constraint on these parameters (5b) is needed to derive the Sachs sum rule (2), together with the proportionality of the quark moments to their charges; i.e. $\epsilon = 0$. The Sachs sum rule (2) is thus sensitive to an anomalous isoscalar component in the magnetic moments of nonstrange quarks, while the Coleman-Glashow sum rule (1) is not.

Both sum rules follow from the assumption that the contribution to the baryon moment of the two u quarks in the proton is the same as that of the two u quarks in the Σ^+, and similarly for the d quark in the proton and the d quark in the Ξ^-. The left hand sides of the two sum rules (1) and (2) depend respectively on the isovector and isoscalar contributions to the nucleon moments. But the Coleman-Glashow relation (1) depends on the *sum* of the absolute magnitudes of the isovector contributions from nonstrange quarks in the Σ and Ξ, while the Sachs sum rule (2) depends upon the *difference* between these contributions and the additional assumption that $\epsilon = 0$ in eq. (4c).

A relation for the *ratio* of these two contributions is obtained by combining the two sum rules (1) and (2),

$$\frac{\mu(\Sigma^+) - \mu(\Sigma^-)}{\mu(\Xi^-) - \mu(\Xi^o)} = -\frac{2A_\Sigma}{B_\Xi} = 3.56\left(\frac{1+0.76\epsilon}{1+1.85\epsilon}\right) \tag{5c}$$

where we have used the relation (5b) to generalize the Sachs sum rule to apply to the case $\epsilon \neq 0$. The experimental values of the left hand side are all greater than the right hand side for reasonable values of ϵ, and also greater than the the value 4 predicted by using the broken-$SU(3)$ prediction $-3/2$ for $\mu(p)/\mu(n)$ instead of the experimental value -1.46. This indicates that the nonstrange contribution is quenched even more in the Ξ than in the Σ.

The disagreement with experiment of the Coleman-Glashow sum rule (5a) places strong restrictions on any modification of the simple model. To explain why the isovector contributions from nonstrange quarks in the Σ and Ξ hyperons are *smaller* than the corresponding contributions to the nucleon moments it is necessary to assume either a configuration mixing or other quenching mechanism on nonstrange quarks which affects hyperons more than nucleons, or an additional isovector contribution which does not come from the valence quarks; e.g. a sea of quark-antiquark pairs or a pion cloud. Such "quenching" of the nonstrange contributions has been noted in previous analyses of the data [12]. If the Sachs sum rule *agrees* with experiment, as indicated by the earlier data, this quenching should not affect the *difference* between the isovector nonstrange contributions to the Σ and Ξ moments. But if the new data are correct and the Sachs sum rule is also in disagreement with experiment, the differences are also affected.

These discrepancies are not easily explained without destroying the excellent simple predictions of the nucleon and Λ moments from hadron masses and no free parameters [2,3]. Introducing "fine tuning" effects to fit the data with new free parameters having no obvious physical meaning tends to lose the connections in the simple model between the baryon moments and other properties of hadrons; e.g. quark masses determined completely from hadron masses and radiative decays of vector mesons described as magnetic dipole transitions with the same quark magnetic moments [3].

The nonstrange moments in the hyperons could be quenched without affecting the good predictions of the nucleon and Λ moments by introducing configuration mixing only in the Σ and Ξ baryons. Such mixing could explain the disagreement with the Coleman-Glashow sum rule, and have only a small effect on the Sachs sum rule which has been shown to be insensitive to configuration mixing [8]. This point is discussed further below in eq. (15c).

Mixing with an SU(3) D-wave decuplet, by analogy with the d-wave admixture in the deuteron, has been suggested to fix discrepancies in other properties of the the Σ and Ξ wave functions without harming the good results for the nucleon and Λ [13]. Such decuplet mixing is forbidden by isospin invariance for the nucleon and Λ but allowed for the Σ and Ξ. However, this approach has not led to useful results. The states in the decuplet with $L = 2; S = 3/2$ which would be admixed by a tensor force into the hyperon wave functions turn out to have zero magnetic moment. This is shown below in eq. (5d). Thus such admixtures simply reduce the magnitudes of all magnetic moments by a factor $(1 - x^2)$ where x is the amplitude for the admixed decuplet wave function. This would appear as a quenching of both the strange and nonstrange quark contributions to the hyperon moments by the same factor, which is in disagreement with experiment.

We now obtain additional sum rules from the simple assumption that the baryon magnetic moments are given by eq. (3a) and that the parameters A, B and μ_f are the same for the nucleon, Σ and Ξ, but with no further assumptions.

The Coleman-Glashow sum rule can be rewritten

$$\mu(p) - \mu(\Sigma^+) - \mu(\Xi^-) = -\mu(\beta_s) = \mu(n) - \mu(\Sigma^-) - \mu(\Xi^o) \tag{6a}$$

In many models $\mu(\beta_s) = \mu_s$. This is discussed below in eq. (15c). Then if $\delta = 0$

$$\mu(\beta_s) = (2A_\Xi + B_\Sigma)\mu_s = \mu(\Lambda) = 0.61 n.m. \tag{6b}$$

and

$$\mu(p) - \mu(\Sigma^+) - \mu(\Xi^-) = 0.61 n.m. = \mu(n) - \mu(\Sigma^-) - \mu(\Xi^o) \tag{6c}$$

The experimental disagreement of the relation (1) now appears on both sides of the relation (6c) as shown in Table 1. All sets of data give values much larger than 0.61 for the left hand side and and much smaller for the right hand side. Introducing these experimental inequalities into the relation (6c) and substituting the parametrization (3a) gives

$$2(A_N - A_\Sigma)\mu_u + (B_N - B_\Xi)\mu_d - (2A_\Xi + B_\Sigma)\mu_s > 0.61 >$$

$$> 2(A_N - A_\Sigma)\mu_d + (B_N - B_\Xi)\mu_u - (2A_\Xi + B_\Sigma)\mu_s \tag{6d}$$

Since A and μ_u are positive while B and μ_d are negative, the inequalities (6d) require that either or both of the parameters A_Σ and B_Ξ must be smaller in magnitude than the corresponding nucleon parameters A_N and B_N. Since A_Σ and B_Ξ define the nonstrange quark contributions to the hyperon moments, we see that both inequalities (6d) require quenching the nonstrange contributions to hyperon moments. Quantitative conclusions are difficult because of the experimental error and the discrepancies between the sets of data.

An additional sum rule relates the isoscalar component of the nucleon moment to hyperon moments,

$$\mu(p)+\mu(n)=0.88 n.m. = \frac{\mu(\Sigma^+)^2-\mu(\Sigma^-)^2+\mu(\Xi^-)^2-\mu(\Xi^o)^2}{\mu(\Sigma^+)-\mu(\Sigma^-)+\mu(\Xi^-)-\mu(\Xi^o)} \tag{7a}$$

This can be rewritten

$$\{\mu(\Sigma^+)-\frac{\mu(p)+\mu(n)}{2}\}^2-\{\mu(\Sigma^-)-\frac{\mu(p)+\mu(n)}{2}\}^2+$$

$$+\{\mu(\Xi^-)-\frac{\mu(p)+\mu(n)}{2}\}^2-\{\mu(\Xi^o)-\frac{\mu(p)+\mu(n)}{2}\}^2=0 \tag{7b}$$

Combining eqs. (1) and (7a) gives

$$\frac{\mu(p)-\mu(n)}{\mu(p)+\mu(n)}=5.34=\frac{\{\mu(\Sigma^+)-\mu(\Sigma^-)+\mu(\Xi^-)-\mu(\Xi^o)\}^2}{\mu(\Sigma^+)^2-\mu(\Sigma^-)^2+\mu(\Xi^-)^2-\mu(\Xi^o)^2} \tag{7c}$$

The experimental situation is confused, since the three sets of data all show agreement with a discrepancy of less than two standard deviations for eqs. (7a), (7b) and (7c). This is inconsistent, since the product of the two sum rules (7a) and (7c) is just the Coleman-Glashow sum rule (1) which disagrees with experiment. Agreement with the sum rules (7a) and (7b) implies no isoscalar quenching. Agreement with the sum rule (7c) implies equal isoscalar and isovector quenching. Thus the question of whether nonstrange quenching affects the isoscalar contributions as well as the isovector contributions is still open.

The implications of the relation (7c) as a test of models are clarified by substituting eq. (3a).

$$\left(\frac{2A_N-B_N}{2A_N+B_N}\right)=\left(\frac{2A_\Sigma-B_\Xi}{2A_\Sigma+B_\Xi+\{A_\Sigma B_\Sigma)-A_\Xi B_\Xi\}\{(\mu_s/(\mu_u+\mu_d)\}}\right) \tag{7d}$$

This relation is seen to be still valid if the nonstrange contributions to the nucleon moments are enhanced relative to the corresponding contributions in the hyperons, defined by the parameters A_Σ and B_Ξ, but keep the same (B/A) ratio, while the products $A_\Sigma B_\Sigma$ and $A_\Xi B_\Xi$ retain the equality of SU(3) symmetry.

Another isoscalar sum rule tests the strange quark assumption (6b) without explicitly assuming that the parameters A and B are equal for all baryons,

$$\mu(p)+\mu(n)=0.88 n.m. = \mu(\Sigma^+)+\mu(\Sigma^-)+\mu(\Xi^-)+\mu(\Xi^o)-2\mu(\beta_s) \tag{8a}$$

If the Λ moment is entirely due to the strange quark, eq.(6b) holds and

$$\mu(p)+\mu(n)=0.88 n.m. = \mu(\Sigma^+)+\mu(\Sigma^-)+\mu(\Xi^-)+\mu(\Xi^o)-2\mu(\Lambda) \tag{8b}$$

Substituting eq. (3a) gives

$$2A_N+B_N=2A_\Sigma+B_\Xi+\{2(2A_\Xi+B_\Sigma)\mu_s-2\mu(\beta_s)\}/(\mu_u+\mu_d) \tag{8c}$$

This sum rule thus follows only from eq. (6b) and the assumptions that the isoscalar contribution to the baryon moment of the two u quarks in the proton is the same as that of the two u quarks in the Σ^+, and similarly for the d quark in the proton and the d quark in the Ξ^-.

The experimental test of this sum rule is open to question. The first two sets of data give 0.66 ± 0.2 and 0.66 ± 0.18 for the right hand side of eq. (8b) which is not in disagreement with 0.88, the new data with the value of the Σ^- moment as -1.11 n.m. give 0.55 ± 0.06,. This is within two standard deviations from the other data, but is in strong disagreement with 0.88 and suggests either a quenching of the isoscalar nonstrange contribution or an enhancement of the strange contribution in the Σ and Ξ relative to the corresponding contributions in the nucleon or Λ respectively.

An alternative sum rule which also tests the strange quark assumption (6b) eliminates the nucleon contributions and the effects of nonstrange quenching in hyperons relative to nucleons, but makes the additional assumption that the u and d contributions to the hyperon moments are in the ratio -2 of the quark charges. The nonstrange contributions are eliminated by using the appropriate linear combinations to give

$$\mu(\Sigma^+) + 2\mu(\Sigma^-) + 2\mu(\Xi^-) + \mu(\Xi^o) = 3\mu(\beta_s) \tag{9a}$$

The assumption (6b) then gives

$$\mu(\Sigma^+) + 2\mu(\Sigma^-) + 2\mu(\Xi^-) + \mu(\Xi^o) = 3\mu(\Lambda) = -1.8 \tag{9b}$$

Substituting eq. (3a) exhibits the two basic assumptions underlying the sum rule,

$$(\mu_u + 2\mu_d)(2A_\Sigma + B_\Xi) + 3\mu_s(2A_\Xi + B_\Sigma) = 3\mu(\beta_s) \tag{9c}$$

Here again the first two sets of data give -2.2 ± 0.3 which and -2.3 ± 0.3 which are within two standard deviations of -1.8, but the new data give -2.5 ± 0.1 which is in strong disagreement and consistent with the explanation of the similar disagreement for the sum rule (8) as due to enhancement of the strange contributions in the Σ and Ξ relative to the corresponding contributions in the Λ.

Another sum rule relating the strange quark contributions in the Σ and Ξ hyperons to the Λ moment is obtained from the assumption (6b) by noting that

$$\mu(\Sigma^-) + \mu(\Xi^-) = \mu(\beta_s) + \mu(\beta_d) = \{1 + \frac{\mu(\beta_d)}{\mu(\beta_s)}\}\mu(\Lambda) \tag{9d}$$

The fictitious baryons β_f are of particular interest because their magnetic moments are much less sensitive to small configuration mixing effects than baryons containing two different types of consitutents with different values of quark moments, as shown below in eq. (15c).

The left hand side is -1.64 ± 0.15 and -1.70 ± 0.15 with the first two sets of data and -1.80 ± 0.05 with the new data. Using the experimental value of $\mu(\Lambda)$ to calculate $\{\mu(\beta_d)/\mu(\beta_s)\}$ from eq. (9d) gives $\{\mu(\beta_d)/\mu(\beta_s)\} = 1.68 \pm 0.24$ and 1.77 ± 0.24 with the first two sets of data. This is within two standard deviations of reasonable values. The standard broken SU(6) fit to $\mu(\Lambda)$ uses $\{\mu(\beta_d)/\mu(\beta_s)\} = 1.5$. A quenching factor might reduce this to 1.3 which is still only two standard deviations from the worst prediction. But the new data requires $\{\mu(\beta_d)/\mu(\beta_s)\} = 1.95 \pm 0.08$ which seems highly unreasonable.

Another aspect of this same dilemma is seen in the magnetic moments of the two negative hyperons, which are composed of three quarks all having charge -1/3. These differ from $\mu(\beta_s)$ only because of the difference between the contributions of the d quarks and those of the fictitious s quarks having the same wave function. We can use isospin to compute this difference to obtain

$$\mu(\beta_s) = \mu(\Sigma^-) + \xi\{\mu(\Sigma^+) - \mu(\Sigma^-)\} \tag{10a}$$

$$\mu(\beta_s) = \mu(\Xi^-) + \xi\{\mu(\Xi^o) - \mu(\Xi^-)\} \tag{10b}$$

where

$$\xi = \frac{\mu_s - \mu_d}{\mu_u - \mu_d} = \left(\frac{1}{3+\epsilon}\right)\left(1 - \frac{\mu_s}{\mu_d}\right) \tag{10c}$$

and ϵ is defined by eq. (4c). The parameter $\xi = 1/9$ with $\epsilon = 0$ and the standard $SU(3)$ breaking ratio of $(\mu_s/\mu_d) = (m_d/m_s) = 2/3$.

Experimentally, eq. (10a) gives $\mu(\beta_s) = -0.89 + 3.22\xi$, $-1.01 + 3.34\xi$ and $-1.11 + 3.5\xi$ with the three sets of data. This gives -0.53 ± 0.14, -0.64 ± 0.12 and -0.72 ± 0.04 respectively with $\xi = 1/9$. Here again the new data suggest that $\mu(\beta_s)$ is considerably larger in magnitude than $\mu(\Lambda)$ whereas the old data did not show a significant discrepancy. Eq. (10b) gives $\mu(\beta_s) = -0.7 - 0.55\xi$ with the old data and $-0.75 - 0.5\xi$ with the new data. This gives -0.76 ± 0.06 and -0.81 ± 0.04 respectively with $\xi = 1/9$.

These discrepancies cannot be patched up by adjusting the value of ξ. The terms proportional to ξ have opposite signs in eqs. (10a) and (10b). Thus changing the value of ξ from 1/9 to obtain better agreement in one case would make the disagreement worse for the other. The basic fact that the terms independent of ξ in both (10a) and (10b) are larger in magnitude than $\mu(\Lambda)$ and that the terms proportional to ξ have opposite signs in the two cases force one of the relations to give a value which is larger than $\mu(\Lambda)$.

This disagreement suggests that the contributions of the strange quarks to the magnetic moments of the Σ and/or Ξ hyperons are significantly larger in magnitude than $\mu(\Lambda)$. It is peculiar that the values for $3\mu(\beta_s)$ from eq.(9b) are within two standard deviations of the value $\mu(p)$ predicted by the standard $SU(6)$ model with *no* symmetry breaking, and similarly for the value of the left hand side of eq.(9d) which is very close to the value $\mu(n)$ predicted by the unbroken standard model.

On the other hand, the value of the Σ^- moment as -1.11 n.m. together with the value of the Σ^+ moment as +2.38 n.m. gives

$$(1/3)\{\mu(\Sigma^+) + 2\mu(\Sigma^-)\} = 0.05 \pm 0.03 \tag{11a}$$

Substituting (3a) gives

$$(2/3)(\mu_u + 2\mu_d)A_\Sigma + B_\Sigma = 0.05 \pm 0.03 \tag{11b}$$

But this is just the contribution of the strange quark to the magnetic moments of the Σ under the assumption that $(\mu_u + 2\mu_d) = 0$. That this strange quark contribution should be practically zero is very peculiar. If this relation is confirmed by better data, it indicates one of the three following effects :

1) A very peculiar configuration mixing only in the Σ hyperons.

2). An accidental cancellation of the strange quark contribution by an additional isoscalar contribution like that from setting ϵ in eq. (4c) different from zero.

3) An accidental cancellation by an additional isovector contribution like that from a pion cloud [14, 15, 16] which is just three times as large as the strange quark contribution and opposite in sign.

None of these possibilities seems very likely. It is therefore important to pin down the value of the Σ^- moment.

The above analysis presents recurring evidence for a *quenching* of the nonstrange contributions to the Σ and Ξ moments and an *enhancement* of the strange contributions relative to the corresponding contributions in the nucleon and Λ. We now show that these two features must arise in any model based on eq. (3a) with reasonable values for the parameters which fits the *signs* of the observed moments.

In all models based upon eq. (3a) the quark moments satisfy the inequalities,

$$\mu_u > 0 > \mu_s > \mu_d \tag{12a}$$

$$\mu_u + \mu_d > 0 \tag{12b}$$

The inequality (12a) expresses the common assumptions that the sign of the quark moment is the same as its charge, and that the direction of SU(3) breaking is to reduce the magnitude of μ_s relative to μ_d. The inequality (12b) expresses the common assumption that ϵ is small and nowhere near the value -1 required to violate the inequality.

The signs of the parameters A and B are also the same in all models as in the simple broken-SU(3) model,

$$A_\beta > 0 > B_\beta \tag{12c}$$

for any baryon β. The inequality (12c) fits the simple picture in which the angular momentum of the quark of flavor b is *antiparallel* to the angular momentum of the two other quarks and *antiparallel* to the total angular momentum.

The inequality (12c) also follows from the following systematics in the signs of the observed moments. The sign of the experimental magnetic moment $\mu(\beta_{aab})$ of any baryon containing two quarks of flavor a and one quark of flavor b is always the same as the sign of the moment μ_a of the two identical quarks; i.e. positive for the proton and Σ^+ and negative for the others. Thus, from eq. (3a),

$$\mu(\beta_{aab})/\mu_a = 2A + B(\mu_b/\mu_a) > 0 \tag{13a}$$

Since μ_b/μ_a is positive for the Σ^- and Ξ^- and negative for the Σ^+ and Ξ^o, eq. (13a) requires that $A_\Sigma > 0$ and $A_\Xi > 0$ in agreement with the inequality (12c).

From eq. (3a), the experimental values of the baryon moments and the inequalities (12a) and (12b),

$$B_\Xi = \frac{\mu(\Xi^o) - \mu(\Xi^-)}{\mu_u - \mu_d} < 0 \tag{13b}$$

$$A_N = \frac{\mu_u\mu(p) - \mu_d\mu(n)}{2(\mu_u^2 - \mu_d^2)} > 0 \tag{13c}$$

The inequality (12c) is seen to hold for all the A parameters and for B_Ξ. There is no direct proof from the data for the signs of B_Σ and B_N; eq. (11) is consistent with $B_\Sigma = 0$ and the data might be fit with $B_N > 0$ by choosing peculiar values for μ_u and μ_d. We disregard these pathological cases and assume that the inequality (12c) also holds for B_Σ and B_N and therefore holds in general.

The application of the inequalities (12) to the expression (3a) shows that:

1. Quenching the contributions of the nonstrange quarks to hyperon moments always *decrease* the absolute magnitude of the moment, except for the Ξ^-. This is exactly the directions of all experimentally observed deviations from the predictions of the simple broken-$SU(3)$ model. Thus these deviations are qualitatively explained by quenching.

2. Inequalities relating the Σ^- and Ξ^- moments to the β_ν moment are obtained by noting that a β_ν wave function can be constructed from either the Σ^- or Ξ^- wave function by changing the d-quarks

in these hyperons into s-quarks and using the inequality (12a).

$$\frac{\mu(\Sigma^-)}{\mu(\beta_s)} > 1 > \frac{\mu(\Xi^-)}{\mu(\beta_s)} \tag{14a}$$

If we assume eq. (6b)

$$\frac{\mu(\Sigma^-)}{\mu(\Lambda)} > 1 > \frac{\mu(\Xi^-)}{\mu(\Lambda)} \tag{14b}$$

The right hand inequality is violated by the experimental data, but only barely at the two standard deviation level. This again suggests that eq.(6b) is not valid and that $\mu(\beta_s)$ is larger in magnitude than $\mu(\Lambda)$ as indicated by eqs. (10). There have been suggestions that electromagnetic corrections to $\mu(\Lambda)$ could invalidate the relations (6b) and introduce a finite value for the parameter δ in eq. (4d). Unfortunately, the calculated electromagnetic correction [17] gives $\mu_s = -0.56$ which is in the wrong direction to explain the discrepancies in eqs.(10) and (14).

We now consider the more restricted nonrelativistic static quark model with Dirac quark moments but arbitrary configuration mixing. Then $g_a = g_b = 2$ and eqs. (3) can be written

$$\mu(\beta_{aab}) = \langle \mu_a(2J_{za} - L_{za}) + \mu_b(2J_{zb} - L_{zb})\rangle \tag{15a}$$

For spin one-half baryons $J_z = 1/2$ and eq.(15a) can be rewritten

$$\mu(\beta_{aab}) = ((4/3)\mu_a - (1/3)\mu_b)(1 - \langle L_z\rangle) +$$

$$+(1/3)(\mu_b - \mu_a)\langle 2J_{za} - L_{za} + 8J_{zb} - 4L_{zb}\rangle \tag{15b}$$

where $L_z = L_{za} + L_{zb}$.

The second term in eq. (15b) vanishes when $\mu_a = \mu_b$. It also vanishes for the s-state wave function used in the simple broken-$SU(3)$ model which has $L_{za} = L_{zb} = 0$, $J_{za} = -4J_{zb}$ and the values $A = 2/3$ and $B = -1/3$ for all baryons. For the case $\mu_a = \mu_b$,which is just the fictitious baryon β_f,

$$\mu(\beta_f) = \mu_f(1 - \langle L_z\rangle) \tag{15c}$$

The value (15c) depends upon the details of the wave function only via the term $\langle L_z\rangle$ which can be expected to be small for any reasonable configuration mixing. Thus relations between observable baryon moments which can be expressed primarily in terms of these fictitious baryon moments β_f are expected to be less sensitive to configuration mixing effects. The Sachs sum rule (2) is seen to have this form. If it agrees with experiment as indicated by the older data, this would suggest that the quenching of nonstrange moments in hyperons indicated by the disagreement with experiment of the relation (1) is a result of configuration mixing.

The second term in eq. (15b) also vanishes for the case of a decuplet baryon wave function which is totally symmetric in the flavors of the three quarks. The moment is given by eq. (15c) with (μ_f) replaced by its mean value for the three quarks,

$$\mu(\beta_{10}) = (1/3)\sum_i (\mu_i)(1 - \langle L_z\rangle) \tag{15d}$$

For the decuplet with $L = 2; S = 3/2$ which would be mixed into the hyperon wave functions by a tensor force, $\langle L_z\rangle = 1$ and the moment (15d) vanishes.

We summarize by drawing the following conclusions:

1. The experimental confusion about the value of the Σ^- moment is serious. If the value of -1.11 n.m. *is confirmed* and eq. (11) *is valid, then the strange quark contribution to the* Σ moments is essentially zero, which is very peculiar.

2. The disagreements with the relations (1) and (6) suggest that the nonstrange quark contributions to nucleon moments are greater than their corresponding contributions to hyperon moments; i.e. that there seems to be some enhancement in the nucleon or quenching in the hyperons.

3. The confusion about the relations (7) leave open the question of whether the *ratio* of isoscalar to isovector nonstrange contributions may be the same in the hyperons as in the nucleon, even though the absolute magnitudes are *different*.

4. If the Sachs sum rule (2) agrees with experiment, the quenching of the nonstrange contributions to the Σ and Ξ hyperons must not affect the difference between the magnitudes of the two isovector contributions.

5. There are disturbing indications, particularly from the value of the Ξ^- moment and the relations (8), (9), (10) and (14), that eq. (6b) does not hold and that the strange quark moments in the Σ and Ξ are *larger* in magnitude than the Λ moment. This suggestion that the Λ moment may have contributions from the nonstrange quarks destroys the beautiful predictions of $\mu(\Lambda)$ from the nucleon moments and hadron masses [2].

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TABLE 1. Experimental Values of Baryon Magnetic Moments and Tests of Sum Rules

Baryon Moment	1981 *Value* *Refs*[3, 9]	1982 *Average* *Refs*[4, 11]	1983 *Data* *Refs*[4, 5]	*Predicted*
$\mu(p)$	2.793 ± 0.000	2.793 ± 0.000	2.793 ± 0.000	*
$\mu(n)$	-1.913 ± 0.000	-1.913 ± 0.000	-1.913 ± 0.000	*
$\mu(\Lambda)$	-0.613 ± 0.005	-0.613 ± 0.005	-0.613 ± 0.005	*
$\mu(\Sigma^+)$	2.33 ± 0.13	2.33 ± 0.13	2.38 ± 0.02	2.68
$\mu(\Sigma^-)$	-0.89 ± 0.14	-1.01 ± 0.12	-1.11 ± 0.004	−1.04
$\mu(\Xi^0)$	-1.25 ± 0.014	-1.25 ± 0.014	-1.25 ± 0.014	−1.43
$\mu(\Xi^-)$	-0.75 ± 0.06	$-0.0.69 \pm 0.04$	-0.69 ± 0.04	−0.50

Sum Rule	1981 *Value*	1982 *Average*	1983 *Data*	*Predicted*
$RHS(1)$	3.7 ± 0.2	3.9 ± 0.2	4.05 ± 0.06	4.7
$RHS(2)$	2.72 ± 0.2	2.78 ± 0.18	2.93 ± 0.06	2.64
$LHS(5c)$	6.44 ± 0.88	5.96 ± 0.55	6.23 ± 0.48	4
$LHS(6c)$	1.21 ± 0.14	1.15 ± 0.14	1.10 ± 0.05	0.61
$RHS(6c)$	0.23 ± 0.14	0.35 ± 0.12	0.45 ± 0.04	0.61
$RHS(7a)$	0.98 ± 0.17	0.85 ± 0.16	0.83 ± 0.04	0.88
$LHS(7b)$	0.36 ± 0.64	-0.1 ± 0.6	-0.2 ± 0.2	0
$RHS(7c)$	3.8 ± 0.7	4.6 ± 0.8	4.9 ± 0.3	5.34
$RHS(8b)$	0.66 ± 0.20	0.60 ± 0.18	0.55 ± 0.06	0.88
$LHS(9b)$	-2.2 ± 0.3	-2.3 ± 0.3	-2.5 ± 0.1	−1.8
$LHS(9d)$	-1.64 ± 0.15	-1.70 ± 0.12	-1.80 ± 0.06	≈ 1.5
$RHS(10a)$	-0.53 ± 0.14	-0.64 ± 0.12	-0.72 ± 0.03	−0.61
$RHS(10b)$	-0.81 ± 0.06	-0.75 ± 0.04	-0.75 ± 0.04	−0.61
$LHS(11)$	0.17 ± 0.10	0.10 ± 0.10	0.05 ± 0.03	0.20