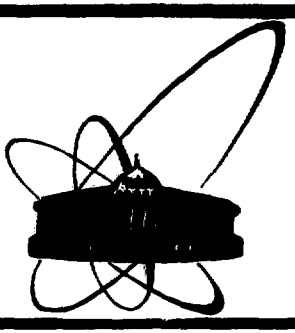

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SITUATION WITH COLLECTIVE TWO-PHONON STATES IN DEFORMED NUCLEI

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INTRODUCTION

Many two-phonon collective states have been observed in the spherical doubly even nuclei. In 1966-1970 there appeared experimental indications to the two-phonon states in ^{154}Gd, ^{168}Er and ^{240}Pu. It was mentioned in ref. [4] that the lowest two-phonon states in deformed nuclei are difficult to observe in comparison with the spherical ones since they lie in the energy region with many rotational bands on the two-quasiparticle and one-phonon states.

In recent years the situation with the two-phonon collective states in deformed nuclei became more complicated. New experiments have not confirmed the existence of the two-phonon states in ^{154}Gd and ^{168}Er. Based on the analysis of the experimental data it has been concluded in ref. [5] that the two-phonon states are absent in the deformed nuclei.

According to the generally accepted treatment (see refs. [6,7]) the one-, two- and three-phonon states should exist in the doubly even spherical and deformed nuclei. The influence of the Pauli principle on the excited states in the two-phonon components of the wave functions has been studied in refs. [8,9,10]. In ref. [9] it has been concluded that the collective two-phonon states should not exist in the deformed doubly even nuclei. It should be noted that A.Bohr and B.Mottelson [11] try to uphold the existence of the two-phonon states in deformed nuclei.

The problem of two-phonon collective states became more acute in two cases. The first one concerns the two-phonon octupole states in the Ra, Th and U isotopes. In doubly even Ra, Th and U isotopes there are low-lying states with $I^{\pi}K = 1^{-}0$. Starting from paper [12] these states are treated as the octupole vibrational states ($\lambda\mu i = 301$). Based on the experimental study of spectra the authors of papers [13,14] state that in ^{224,226}Ra and ^{226,228}Th there are no 0^{+} states with energies close to the twice energies of the $I^{\pi}K = 1^{-}0$ states and containing large octupole two-phonon components {301, 301}. Due to non-observation of the two-phonon octupole states the authors of paper [14] have doubts about the interpretation of the first $K^{\pi} = 0^{-}$ states as one-phonon states. In ref. [15] a modified macroscopic-microscopic method has been used to calculate the potential energy

of nuclei in the Ra region. A stable octupole deformation has been obtained for a number of nuclei. The lowest $K^{\pi}=0^{-}$ states are interpreted as being associated with this deformation. In ref. [16] the levels $K^{\pi}=0^{-}$, $I=1,3,5$ in doubly even isotopes of Rn, Ra, Th and U have been analysed. These levels demonstrate features of strong Coriolis coupling. The existence of strong Coriolis coupling effects is typical only for rotational bands based on the one-phonon octupole states, but not for bands based on $I^{\pi}K=1^{-}0$ states with stable octupole deformation. The α-, β- and γ-transitions to or from these $I^{\pi}K=1^{-}0$ states have been analysed in ref. [16]. It was shown that there is no additional retardation of these transitions; this indicates that there is no significant difference in the shape of the ground and $1^{-}0$ states. It was also concluded that the $I^{\pi}K=1^{-}0$ states have no stable octupole deformation and are usual octupole vibrational states in the Ra, Th and U isotopes. Thus the experimental non-observation of the two-phonon octupole 0^{+} states is in agreement with the conclusions of paper [9].

The second case concerns ^{168}Er. This nucleus is studied most thoroughly experimentally [17] and therefore it is used to test the description of the low-lying states in different models. A further study of the levels of ^{168}Er may turn to be deciding for elucidating the situation with the two-phonon collective states in deformed nuclei.

The interacting boson model has been used [18] to describe the states with positive parity in ^{168}Er. It was shown that the model reproduces correctly the $K^{\pi}=0^{+}$ and 2^{+} rotational bands below the gap and their decaying properties. It has been stated [11] that an attempt to fit the spectrum of ^{168}Er on the basis of the interacting boson model leads to major disagreements at almost every point at which it is possible to confront the model with experiment. It was pointed out [11] that this nucleus is important for the analysis of γ-vibrations, ($\lambda\mu=221$). The two-phonon 0^{+} states of the type $\{221, 221\}$ in ^{168}Er with an energy below 2 MeV are non-observed experimentally. The level with $K^{\pi}=4^{+}$ and energy of 2.03 MeV is the lowest candidate for the two-phonon state of type $\{221, 221\}$. If so, there is a strong anharmonicity of γ-vibrations. According to ref. [11] a strong anharmonicity of γ-vibrations may imply a potential surface with a minimum for $\gamma\neq 0$. The γ-vibrations in ^{168}Er are analysed in ref. [19] in a macroscopic and in a microscopic model with special emphasis on anharmonicities of the two-phonon states. It should be noted, it is necessary to prove experimentally that the $K^{\pi}=4^{+}$ 2.03 MeV state in ^{168}Er is the two-phonon

one. This can be achieved by the Coulomb excitation by heavy ions. At present the experimental data on ^{168}Er do not contradict the conclusion of paper [9] about the absence of the two-phonon collective states in deformed nuclei.

In view of the contradictions concerning the two-phonon states in doubly even deformed nuclei, a further theoretical investigation is needed. In the present paper the two-phonon states are studied by using the introduced in ref. [20] phonon operators depending on the sign of the angular momentum projection into the nuclear symmetry axis. A secular equation to determine the energies of nonrotational states is obtained, in which the Pauli principle is taken into account in the two-phonon components of the wave functions. The centroid energies of the two-phonon states are calculated for many deformed nuclei. The position of the three-phonon poles is calculated to elucidate the fragmentation of two-phonon states. The situation with ^{168}Er and doubly even Th and U isotopes is analysed.

1. THE MODEL, BASIC EQUATIONS

The formulae of the quasiparticle-phonon nuclear model (see refs. [21,22]) for doubly even deformed nuclei taking into account the Pauli principle have been obtained in ref. [8]. A case for the isoscalar and isovector multipole-multipole forces was considered in this paper. It has been shown in ref. [23] that the isovector part of the multipole-multipole forces slightly influences the excited states of doubly even nuclei with an energy less than 3 MeV. Therefore, we take into account only the isoscalar part of the multipole-multipole forces.

Taking into account the RPA secular equations, the Hamiltonian of the quasiparticle-phonon nuclear model is

$$H_M = H_\nu + H_{\nu q}\,, \tag{1}$$

$$H_\nu = \sum_{q\sigma} \epsilon(q)\, a^+_{q\sigma}\, a_{q\sigma} - \frac{1}{4} \sum_{\substack{g=\lambda\mu i \\ g'=\lambda\mu i'}} \frac{1}{\kappa_0^{\lambda\mu}\sqrt{Y_g Y_{g'}}} \sum_{\sigma} Q^+_{g\sigma} Q_{g'\sigma}\,, \tag{2}$$

$$H_{\nu q} = -\frac{\sqrt{2}}{4} \sum_{g\sigma} \sum_{qq'} \frac{v^{(-)}_{qq'}}{\sqrt{Y_g}} \{(Q^+_{g\sigma} + Q_{g-\sigma})(f^{\,g}(qq')B(qq';\mu-\sigma) + f^{\,g}(qq')\bar{B}(qq';\mu-\sigma) + \text{h.c.}\}\,. \tag{3}$$

We use a new definition of the phonon operator [20]

$$Q^{+}_{g\sigma} = \frac{1}{2} \sum_{qq'} \{ \psi^{g}_{qq'} A^{+}(qq'; \mu\sigma) - \phi^{g}_{qq'} A(qq', \mu-\sigma) + \bar{\psi}^{g}_{qq'} \bar{A}^{+}(qq'; \mu\sigma) - \bar{\phi}^{g}_{qq'} \bar{A}(qq'; \mu-\sigma) \}, \quad (4)$$

which depends explicitly on the sign of the angular momentum projection into the symmetry axis $\sigma = \pm 1$.

$$A^{+}(qq'; \mu\sigma) = \sum_{\sigma'} \delta_{\sigma'(K-K'),\sigma\mu} a^{+}_{q\sigma'} a^{+}_{q'-\sigma'}, \quad (5)$$

$$\bar{A}^{+}(qq'; \mu\sigma) = \sum_{\sigma'} \delta_{\sigma'(K+K'),\sigma\mu} a^{+}_{q'\sigma'} a^{+}_{q\sigma'},$$

$$B(qq'; \mu\sigma) = \sum_{\sigma'} \delta_{\sigma'(K-K'),\sigma\mu} a^{+}_{q\sigma'} a_{q'\sigma'}, \quad (6)$$

$$\bar{B}(qq'; \mu\sigma) = \sum_{\sigma'} \delta_{\sigma'(K+K'),\sigma\mu} a^{+}_{q\sigma'} a_{q'-\sigma'}.$$

Here $a^{+}_{q\sigma}$ is the quasiparticle creation operator, $q\sigma$ are the quantum numbers of the single-particle states; $g = \lambda\mu i$, $\kappa_0^{(\lambda\mu)}$ is the constant of isoscalar multipole forces, i is the root number of the secular equation for one-phonon states, always $K \geq 0$, $\mu \geq 0$; $\epsilon(q)$ is the quasiparticle energy, $u^{(\pm)}_{qq'} = u_q v_{q'} \pm u_{q'} v_q$, $v^{(\pm)}_{qq'} = u_q u_{q'} \pm v_q v_{q'}$; u_q and v_q, the canonical Bogolubov transformation coefficients; $f^{g}(qq')$ and $\bar{f}^{g}(qq')$ are the single-particle matrix elements. The explicit form of Y_g and the other notation are given in refs.[20,21].

The excited nonrotational state wave function of a doubly even deformed nucleus is

$$\Psi_n(K^{\pi}_0 \sigma_0) = \{ \sum_{i_0} R^{n}_{i_0} Q^{+}_{g_0\sigma_0} + \sum_{g_1\sigma_1 \, g_2\sigma_2} \frac{\sqrt{1+\delta_{g_1 g_2}}}{2} \delta_{\sigma_1\mu_1+\sigma_2\mu_2,\sigma_0 K_0} P^{n}_{g_1 g_2} Q^{+}_{g_1\sigma_1} Q^{+}_{g_2\sigma_2} \} \Psi_0, \quad (7)$$

where Ψ_0 is the ground state wave function, $n = 1, 2, 3, \ldots$ are the numbers of the states with given K^{π}_0. We use the exact commutation relations for phonons, given in ref.[20] and calculate

$$\sum_{\sigma\sigma_2 \, \sigma'\sigma'_2} \delta_{\sigma'\mu+\sigma'_2\mu_2,\sigma_0 K_0} \delta_{\sigma\mu+\sigma_2\mu_2,\sigma_0 K_0} \langle \Psi_0 | Q_{g'_2\sigma'_2} Q_{g'\sigma'} Q^{+}_{g\sigma} Q^{+}_{g_2\sigma_2} | \Psi_0 \rangle = \quad (8)$$

$$= (\delta_{\mu+\mu_2, K_0} + \delta_{|\mu-\mu_2|, K_0})(\delta_{gg'} \delta_{g_2 g'_2} + \delta_{gg'_2} \delta_{g_2 g'}) + K^{K_0}(g'_2 g' | g g_2).$$

The form of the function $K^{K_0}(g_2'g'|gg_2)$ is presented in ref.[20]. It somewhat differs from that given in refs.[8-10]. It has been shown in ref.[8] that $K^{K_0}(g_2'g'|gg_2)$ are small if $g_2'g' \neq gg_2$. Therefore, we shall retain the diagonal $K^{K_0}(g_2g|gg_2)$ and quasidiagonal $K^{K_0}(g_2g'|gg_2)$ with $g' = \lambda\mu i'$ functions. Below we give the formula for the case $K_0 = \mu + \mu_2$ ($\mu \neq 0$, $\mu_2 \neq 0$)

$$
\begin{aligned}
K^{K_0}(g_2 g'|gg_2) = \\
= -\delta_{\mu+\mu_2,K_0} \sum_{q_1q_2q_3q_4} \{\psi^{g'}_{q_1q_3}\psi^{g}_{q_1q_2}\psi^{gg_2}_{q_4q_2}\psi^{gg_2}_{q_4q_3} \times \\
\times [\delta_{K_1-K_3,\mu}\delta_{K_1-K_2,\mu}\delta_{K_4-K_3,\mu_2}\delta_{K_4-K_2,\mu_2} + \\
+ \delta_{K_3-K_1,\mu}\delta_{K_2-K_1,\mu}\delta_{K_3-K_4,\mu_2}\delta_{K_2-K_4,\mu_2} + \\
+ \delta_{K_3-K_1,\mu}\delta_{K_2-K_1,\mu}\delta_{K_4+K_2,\mu_2}\delta_{K_4+K_3,\mu_2} + \\
+ \delta_{K_1+K_3,\mu}\delta_{K_1+K_2,\mu}\delta_{K_3-K_4,\mu_2}\delta_{K_2-K_4,\mu_2} + \\
+ \delta_{K_1+K_3,\mu}\delta_{K_1+K_2,\mu}\delta_{K_3+K_4,\mu_2}\delta_{K_2+K_4,\mu_2} + \\
+ \delta_{K_1+K_3,\mu}\delta_{K_1-K_2,\mu}\delta_{K_4+K_3,\mu_2}\delta_{K_4-K_2,\mu_2} + \\
+ \delta_{K_1-K_3,\mu}\delta_{K_1+K_2,\mu}\delta_{K_4-K_3,\mu_2}\delta_{K_4+K_2,\mu_2}] - \\
- \phi^{g'}_{q_1q_3}\phi^{g}_{q_1q_2}\phi^{gg_2}_{q_4q_2}\phi^{gg_2}_{q_4q_3} \times \qquad (9) \\
\times [\delta_{K_3-K_1,\mu}\delta_{K_2-K_1,\mu}\delta_{K_3-K_4,\mu_2}\delta_{K_2-K_4,\mu_2} + \\
+ \delta_{K_1-K_3,\mu}\delta_{K_1-K_2,\mu}\delta_{K_4-K_3,\mu_2}\delta_{K_4-K_2,\mu_2} + \\
+ \delta_{K_3-K_1,\mu}\delta_{K_2-K_1,\mu}\delta_{K_4+K_3,\mu_2}\delta_{K_4+K_2,\mu_2} + \\
+ \delta_{K_3+K_1,\mu}\delta_{K_2+K_1,\mu}\delta_{K_3-K_4,\mu_2}\delta_{K_2-K_4,\mu_2} + \\
+ \delta_{K_3+K_1,\mu}\delta_{K_2+K_1,\mu}\delta_{K_3+K_4,\mu_2}\delta_{K_2+K_4,\mu_2} + \\
+ \delta_{K_1+K_3,\mu}\delta_{K_1-K_2,\mu}\delta_{K_4+K_3,\mu_2}\delta_{K_4-K_2,\mu_2} + \\
+ \delta_{K_1-K_3,\mu}\delta_{K_1+K_2,\mu}\delta_{K_4-K_3,\mu_2}\delta_{K_4+K_2,\mu_2}]\},
\end{aligned}
$$

In the cases $K_0=\mu-\mu_2$, $K_0=0$, $\mu=\mu_2$, $\sigma_1=-\sigma_2$ and $K_0\neq 0$, $\mu=0$ the functions $K^{K_0}(g_2g'|gg_2)$ differ from (9) by the Kronecker symbols. For the case $K_0=0$, $\mu=\mu_2=0$ the function $K^{K_0}(g_2g'|gg_2)$ coincides with that in ref./9/. The normalization condition in the diagonal for $K^{K_0}(g_2g|gg_2)$ approximation is

$$1=\langle\Psi_n(K_0^{\pi}\sigma_0)|\Psi_n(K_0^{\pi}\sigma_0)\rangle= \\ =\sum_{i_0}(R^n_{i_0})^2+\sum_{g_1g_2}\frac{1}{2}(1+\delta_{g_1g_2})(P^n_{g_1g_2})^2\{1+\frac{1}{2}K^{K_0}(g_2g_1|g_1g_2)\}. \tag{10}$$

If the Pauli principle is violated maximally, then $K^{K_0}(g_2^0g_1^0|g_1^0g_2^0)=2$, thus the component $g_1^0g_2^0$ is excluded from the wave function (7).

Now we calculate the average value H_M over the state (7), the variational principle is used to determine the equations for the excitation energies η_n and functions $R^n_{i_0}$ and $P^n_{g_1g_2}$

$$(\omega_{g_0}-\eta_n)R^n_{i_0}-\sum_{g_1\geq g_2}(1+\delta_{g_1g_2})^{-1/2}U_{g_1g_2}(g_0)P^n_{g_1g_2}\{1+\frac{1}{2}K^{K_0}(g_2g_1|g_1g_2)\}=0, \tag{11}$$

$$(\omega_{g_1}+\omega_{g_2}+\Delta\omega_{g_1g_2}-\eta_n)P^n_{g_1g_2}-(1+\delta_{g_1g_2})^{-1/2}\sum_{i_0'}U_{g_1g_2}(g_0')R^n_{i_0'}=0. \tag{12}$$

Here ω_g are the one-phonon energies

$$\Delta\omega_{g_1g_2}=-\frac{1}{4(1+\delta_{g_1g_2})}\sum_{i_3}\{\frac{K(g_2\lambda_1\mu_1i_3|g_1g_2)}{\kappa^{(\lambda_1\mu_1)}\sqrt{Y_{g_1}Y_{\lambda_1\mu_1i_3}}}+\frac{K(g_1\lambda_2\mu_2i_3|g_2g_1)}{\kappa^{(\lambda_2\mu_2)}\sqrt{Y_{g_2}Y_{\lambda_2\mu_2i_3}}}\}, \tag{13}$$

$$U_{g_1g_2}(g_0)=\frac{1}{2\sqrt{2}}\sum_{\sigma_1\sigma_2}\sigma_1\mu_1+\sigma_2\mu_2,\sigma_0K_0\langle\Psi_0|Q_{g_0\sigma_0}H_{vq}Q^+_{g_1\sigma_1}Q^+_{g_2\sigma_2}|\Psi_0\rangle. \tag{14}$$

A consistent inclusion of the Pauli principle leads to the appearance of the factor $(1+\frac{1}{2}K^{K_0}(g_2g_1|g_1g_2))$ in (11) and the shift of the two-phonon poles $\Delta\omega_{g_1g_2}$ in (12). If the Pauli principle is violated maximally, then owing to the factor $(1+\frac{1}{2}K(g_2g_1|g_1g_2))$ the corresponding terms are excluded automatically in (11). The shift of the poles has been investigated in refs./8,9/. It is the larger, the larger is $|K(g_2g_1'|g_1g_2)|$ and the less are $Y_{g_1}Y_{g_1'}$ and $Y_{g_2}Y_{g_2'}$. The stronger is collectivization of the one-phonon states g_1 and g_2, the larger is the shift $\Delta\omega_{g_1g_2}$. The inclusion of the Pauli principle leads to corrections to the RPA and they turn out to be the larger, the stronger is collectivization of the one-phonon state. At $K^{K_0}(g_2,g'|g,g_2)\approx 0$ =0 all the formulae become those from refs./7,21/.

2. CENTROID ENERGIES OF THE TWO-PHONON STATES

The single-particle energies and wave functions of the Saxon-Woods potential with the parameters of ref.[22] have been used in the calculation. The single-particle levels with principal quantum numbers $N=4 \div 9$ for neutrons and $N=3 \div 9$ for protons in the energy interval from -30 to +5 MeV have been taken into account. The pairing constants G_N and G_Z are determined from the experimental data on pairing energies. We calculate the states with energies less than 4 MeV; therefore, the phonon space is restricted. Ten one-phonon roots have been used for each multipolarity with $\lambda\mu = 20, 22, 30, 31, 32$ and 44. The energies and wave functions of one-phonon states have been calculated by the RPA method and the blocking effect has been taken into account for the first two-quasiparticle poles (see ref.[7]). The constants of the multipole-multipole isoscalar interaction $\kappa_0^{(\lambda)}$ have been defined by the energies of the first states with $K^\pi = 0^+, 2^+, 0^-, 1^-, 2^-$. The calculated B(E2)- and B(E3)-values turned out to be close to the experimental ones (see ref.[10]).

It should be noted that the RPA calculations with an effective charge of 0,2 MeV give B(E2)-values which are in agreement with experiment (see refs.[24,25]). There is no such a large difference between the calculated B(E2) values and the experimental data as in ref.[19]. In 1965 it has been pointed out [26] that the bloking effect should be taken into account for the first neutron and proton poles. It is important for the calculation of the $B(E\lambda)$-values. The calculated $B(E\lambda)$-values and energies also become close to the experimental ones if anharmonicity is taken into account. So, in solving equations (11) and (12), in order to obtain for the first $K^\pi=2^+$ state in ^{168}Er the experimental energy value equal to 0,821 MeV, the constant $\kappa_0^{(22)}$ should be taken such that the energy of the one-phonon state $\omega_{221}=1.04$ MeV and $B(E2)_{s.p.u.}=4.6$.

As a result of the inclusion of the Pauli principle in the two-phonon components of the wave function (7), the energies of collective two-phonon states in doubly even deformed nuclei increased by 1-3 MeV and reached the excitation region of 3-5 MeV. At 3-5 MeV the two-phonon collective states in deformed nuclei are fragmented over many levels. Thus, the calculations with phonons depending on the sign of the projection K confirm the conclusions of paper [9].

The results of calculation for the centroid energies of the two-phonon states of type $|\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2|$ with and without the Pauli principle are given in table 1. It is

Table 1

Centroid energies of the two-phonon states in deformed nuclei

Nucleus	K^{π}	Two-phonon configuration		Energies, MeV calculated	
		$\lambda_1\mu_1 i_1$	$\lambda_2\mu_2 i_2$	with the Pauli principle	without the Pauli principle
^{158}Gd	0^+	201	201	6.0	3.0
		221	221	4.0	2.6
		301	301	3.6	2.5
		441	441	4.0	4.0
		202	202	4.0	3.9
	2^+	201	221	3.7	2.8
		221	441	3.8	3.3
		301	321	3.8	3.0
	4^+	201	441	3.4	3.3
		221	221	4.5	2.7
	0^-	201	301	3.5	2.7
		221	321	4.7	3.1
^{160}Dy	0^+	201	201	5.0	2.6
		221	221	4.0	2.2
		301	301	3.8	3.0
	2^+	201	221	4.0	2.5
		301	321	3.0	2.8
	4^+	221	221	4.5	1.3
^{232}U	0^+	201	201	6.0	2.3
		221	221	4.0	2.1
		301	301	3.7	1.3
		321	321	2.3	2.2
		441	441	2.0	2.0
	2^+	201	221	4.4	1.8
		221	441	2.1	2.0
		301	321	2.4	1.7
	4^+	201	441	5.2	2.0
		221	221	4.1	2.0
		321	321	3.9	2.2

seen from the table that the inclusion of the Pauli principle increases the energies of the collective two-phonon states by 1-3 MeV, and they are, as a rule, above 3 MeV. If both the phonons forming the two-phonon state are weakly collectivized, the effect of the Pauli principle is small and the energies of these two-phonon states are close to the sum of energies of phonons. Thus, the centroid energies of the two-phonon states composed of weakly collectivized phonons, for instance, corresponding to the second roots of the secular equations $|202, 202|$ turn out to be less than those composed of the first roots of the secular equation $|201, 201|$. This is exemplified in table 1 for the $K^{\pi}=0^{+}$ state in ^{158}Gd.

The results of calculations with phonons depending on the sign of the angular momentum projection differ from the calculations of refs.[9,10] by that the shift of the two-phonon pole for the configuration $|\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2|$ depends on the value of $K_0=\mu_1\pm\mu_2$. In most cases this difference is not large, though for some weakly collectivized phonons it turns out to be considerable. This is shown in table 1. The comparison of the centroid energies for ^{160}Dy, given in table 1, with the results obtained in[9] shows that the centroid energies of the two-phonon states calculated taking into account the Pauli principle are similar in both the cases.

Our basic result about the shift of the centroid energies of the lowest collective two-phonon states to the energy region of 3-5 MeV concerns all doubly even deformed nuclei and is independent of the choice of the model parameter. The shift of the centroid energies of the two-phonon states is the larger, the stronger the collectivization of their phonons. The stronger is collectivized a phonon, the larger the corrections to the RPA due to the ground state correlations[27], the stronger the shifts of the two-phonon poles. In $^{162,164}Dy$ and $^{164,166,168}Er$ the γ-vibrational states are strongly collectivized and are close to the region of applicability of the RPA. Therefore, the shifts of the two-phonon poles of type $|221, 221|$ turned out to be somewhat overestimated. In the case of ^{232}U, which is in the vicinity of the deformed nuclei region, a certain overestimation of the shifts of two-phonon states also occurs, especially for the configuration $|301, 301|$. If the shifts of two-phonon poles or centroid energies are considerably larger than 2 MeV, the RPA cannot be used for the description of the corresponding one-phonon states.

3. ON THE FRAGMENTATION OF COLLECTIVE TWO-PHONON STATES

Now we turn to the fragmentation of collective two-phonon states in doubly even deformed nuclei. We do not calculate the fragmentation of two-phonon states. To calculate it one should, first, include three-phonon components into the wave function (7), and second, take into account a large number of one-phonon states up to $10^2 - 10^3$. In our calculations the two-phonon states turned out to be fragmented when one-phonon poles occur near their energies. For instance, in ^{158}Gd the two-phonon state with $K^\pi = 0^-$ |201, 301| is fragmented over the following levels: 1.11 MeV - 0.6%, 1.73 MeV - 2.9%, 3.53 MeV - 66.4%, 3.7 MeV - 17.3%, 3.72 MeV - 2.2%. The state with $K^\pi = 2^+$|201, 221| is fragmented analogously. It should be noted that we have used a small phonon space. With increasing number of one-phonon states the fragmentation of two-phonon states should become stronger.

The contribution of two-phonon components to the low-lying states with the dominating one-phonon component will be (5-20)%. The components consisting of one collective and the other noncollective phonons have, as a rule, the largest value. For instance, in ^{158}Gd the contribution of |201, 205| to the first $K^\pi = 0^+$ state with an energy of 1.1 MeV is 8.5%, of |201, 443| to the first state with an energy of 1.2 MeV is 11.6% and so on. The contribution of the components with two collective phonons is not large. For instance, in ^{158}Gd the contribution of |221, 221| to the third $K^\pi = 0^+$ state with an energy of 1.9 MeV is 2.8%; of |201, 201|, 2.0%; of |201, 221| to the third $K^\pi = 2^+$ state with an energy of 2.1 MeV is 7.4%, the contribution of |221, 221| to the first $K^\pi = 4^+$ state is 1.1%, to the second one with an energy of 1.6 MeV is 1.3% and so on. The contribution of the components, consisting of two collective phonons, to the states with the excitation energy up to 2 MeV does not exceed 10%.

For a strong fragmentation of two-phonon states a sufficient number of three-phonon poles is needed at the energies of 4-6 MeV. The inclusion of the Pauli principle in the three-phonon components of the wave function will shift the three-phonon poles. To calculate the energies of the three-phonon poles, the wave function (7) is added by the following terms:

$$\frac{1}{6}\sum_{g_3\sigma_3 g_4\sigma_4 g_5\sigma_5}(1+\delta_{g_3g_4}+\delta_{g_3g_5}+\delta_{g_4g_5}+2\delta_{g_3g_4}\delta_{g_3g_5})^{1/2}\times \tag{15}$$

$$\times\delta_{\sigma_3\mu_3+\sigma_4\mu_4+\sigma_5\mu_5\sigma_0}K_0^{\cdot}F^n_{g_3g_4g_5}Q^+_{g_3\sigma_3}Q^+_{g_4\sigma_4}Q^+_{g_5\sigma_5}\Psi_0.$$

As in ref.[28], using the variational principle we get the secular equation

$$\sum_{g_1'g_2'} P^{n}_{g_1'g_2'}\{(\omega_{g_1}+\omega_{g_2}+\Delta\omega_{g_1g_2}-\eta_n)\delta_{g_1g_1'}\delta_{g_2g_2'}-\sum_{i_0}\frac{U_{g_1g_2}(g_0)U_{g_1'g_2'}(g_0)}{\omega_{g_0}-\eta_n}-$$
$$-\sum_{g_3g_4g_5}\frac{U^{g_3g_4g_5}_{g_1g_2}\,U^{g_3g_4g_5}_{g_1'g_2'}}{\omega_{g_3}+\omega_{g_4}+\omega_{g_5}+\Delta\omega_{g_3g_4g_5}-\eta_n}\}=0, \qquad (16)$$

where $U^{g_3g_4g_5}_{g_1g_2}$ are given in refs.[28]. It is seen from eq. (16) that the fragmentation of two-phonon states is defined by the one-phonon and three-phonon poles and the corresponding functions $U_{g_1g_2}(g_0)$ and $U^{g_3g_4g_5}_{g_1g_2}$. The shift of three-phonon poles due to the Pauli principle in the components of (15) is

$$\Delta\omega_{g_3g_4g_5}=-\frac{1}{4}\sum_{i'}\{\frac{K^{K_0\pm\mu_5}(g_4\lambda_3\mu_3i'|g_3g_4)+K^{K_0\pm\mu_4}(g_5\lambda_3\mu_3i'|g_3g_5)}{\kappa^{(\lambda_3\mu_3)}\sqrt{Y_{g_3}Y_{\lambda_3\mu_3i'}}}+$$
$$+\frac{K^{K_0\pm\mu_5}(g_3\lambda_4\mu_4i'|g_4g_3)+K^{K_0\pm\mu_3}(g_5\lambda_4\mu_4i'|g_4g_5)}{\kappa^{(\lambda_4\mu_4)}\sqrt{Y_{g_4}Y_{\lambda_4\mu_4i'}}}+ \qquad (17)$$
$$+\frac{K^{K_0\pm\mu_4}(g_3\lambda_5\mu_5i'|g_5g_3)+K^{K_0\pm\mu_3}(g_4\lambda_5\mu_5i'|g_5g_4)}{\kappa^{(\lambda_5\mu_5)}\sqrt{Y_{g_5}Y_{\lambda_5\mu_5i'}}}\}.$$

The signs $K_0\pm\mu$ are defined in each case by the signs $\sigma_3,\sigma_4,\sigma_5$ entering in $\sigma_0K_0=\sigma_3\mu_3+\sigma_4\mu_4+\sigma_5\mu_5$.

The shifts of three-phonon poles are calculated for the $K^\pi=0^+$, 2^+ and 4^+ states in ^{158}Gd and ^{168}Er. The shifts vary from 0.1 to 5 MeV and comprise, on the average, 1 MeV. If shifts $\Delta\omega_{g_3g_4g_5}$ are taken into account, the number of three-phonon poles decreases in the interval 4-5 MeV by an order of magnitude and in the interval 5-6 MeV twice, The remaining three-phonon poles with the one-phonon poles are sufficient for the fragmentation of collective two-phonon states, the centroid energies of which are in the region of $3\div5$ MeV.

4. ANALYSIS OF THE TWO-PHONON STATES IN ^{168}Er AND ^{228}Th

Collective two-phonon states with the given K^π or $\lambda\mu$ and the configuration $\{\lambda_1\mu_1i_1,\lambda_2\mu_2i_2\}$ are specified by enhancement of the $E\lambda_1$ transitions to the band of one-phonon state $\{\lambda_2\mu_2i_2\}$, $E\lambda_2$ transitions to the band of one-phonon state $\{\lambda_1\mu_1i_1\}$ and $E\lambda$ hindrance of the transitions to the ground state rotational band of a doubly even nuc-

leus. As a rule, the collective two-phonon states are formed by the first collective phonons with $i_1 = 1$, $i_2 = 1$.

The possible existence of the two-phonon states in deformed nuclei has been analysed in ref.[9]. The conclusion of ref.[5] about the non-observation of two-phonon states in deformed nuclei was confirmed in ref.[9] Since the two-phonon states in ^{168}Er and in the region of the Ra, Th and U isotopes are lively discussed, we shall consider in detail the situation with the two-phonon collective states in ^{168}Er and ^{228}Th.

Our calculations have been made with a small one-phonon basis without three-phonon components of the wave functions. Such calculations pretend to a correct description of the nonrotational states (besides 0^+ states) up to the excitation energy of 2 MeV. As was expected, we obtained the states with very overestimated two-phonon components. The results of calculations of ^{168}Er are given in table 2. It is seen from the table that the first quadrupole and octupole one-phonon states are described fairly well. The centroid energies of all collective two-phonon states are above 3 MeV. The contribution of two-phonon components to the states with an energy less than 2 MeV is not large. The analysis of γ-vibrational states in ^{168}Er performed in refs.[11,19] assumes that the $K^\pi = 4^+$ state with an energy of 2.03 MeV is the two-phonon vibrational state. According to our calculations the main part of the 4^+ |221, 221| strength is at the energy of 4.3 MeV. The contribution of the |221, 221| component to the first 4^+ state is 1%, it is not large in the 4^+ state with an energy less than 2.5 MeV. A more accurate calculation of the fragmentation may increase the contribution of the component |221, 221| to the first 4^+ state; however, one can hardly expect it to be higher than 10%.

To discuss the situation with the $K^\pi = 0^+$ |301, 301| two-phonon states in the region of the Ra, Th and U isotopes, we have chosen ^{228}Th, the calculations of which are given in table 3. According to our calculations the centroid energy of the 0^+ |301, 301| state is 8.5 MeV. Such a large shift of 8 MeV cannot be treated seriously. However, it is clear that the centroid energies of this state are larger than 4 MeV, and the state |301, 301| is strongly fragmented. Therefore, the 0^+ states with large components |301, 301| should not exist. The centroid energies of the two-phonon γ-vibrational states in ^{228}Th are equal to 3.5 MeV. It should be noted that the centroid energy of the states composed of one collective and one weakly collective phonon like |201, 223| and |203, 221| are less and their contribution to the first two $K^\pi = 2^+$ states is larger in comparison with the state composed of two collective phonons, for instance |201, 221|.

Table 2

Nonrotational collective states in ^{168}Er

K^{π}	Energy, MeV		Configurations, %
	exp.	calc.	
0^+	1.217	1.2	201 84; 202 1; {221,221} 1;
	1.422	1.6	202 93; 201 2; {221,221} 1;
		1.9	203 96; 201 1;
		3.5	{221,221} 79; {221,222} 2; {221,223} 2;
		3.6	{221,222} 88; {221,221} 2;
		4.2	{201,201} 51; {221,225} 1;
2^+	0.821	0.9	221 87; {201,221} 2; {221,441} 1;
	1.848	1.7	222 98;
	1.930	1.9	223 96;
		2.3	224 92; {201,221} 2;
		2.7	225 82; {201,221} 8; {202,221} 1;
		3.3	{201,221} 82, 225 7; 224 1;
4^+	2.03	1.8	441 83; 443 4; {201,441} 6; {221,221} 1;
		2.4	443 49; 442 43; 441 2; {221,222} 1;
		2.5	442 55; 443 39; 441 3;
		4.2	{202,441} 89; {202,442} 2; {221,221} 1;
		4.3	{221,221} 66; {202,441} 1;
		4.6	{321,321} 78;
0^-	1.786	1.74	301 98;
1^-	1.358	1.3	311 99;
	1.936	1.9	312 99;
2^-	1.569	1.7	321 95; {201,321} 2;
		2.0	322 94; {201,322} 2;
		3.3	{221,301} 92;

Table 3

Nobrotational collective states in ^{228}Th

K^{π}	Energy, MeV		Configurations, %
	exp.	calc.	
0^+	0.830	0.8	201 85; 204 4; {201,204} 4;
		1.41	202 38; 203 28; {201,204} 4; {221,221} 3;
		1.43	202 59; 204 17; {201,204} 3; {221,221} 0.3;
		1.9	{311,311} 85; 303 8; 204 4;
		3.4	{221,221} 37; {201,204} 19; {203,204} 20;
		7.0	{201,201} 50;
		8.5	{301,301} 60;
2^+	0.977	1.0	221 88; {201,223} 4; {203,221} 2;
		1.9	222 64; 223 12; {201,223} 1; {201,221} 0.5;
		1.94	222 29; 223 46; {201,222} 1; {221,441} 2;
		4.3	{201,221} 85; 223 1; {301,324} 1;
		4.6	{301,321} 88; {201,225} 1;
4^+		1.3	441 98;
		1.4	442 79; {201,442} 17;
		2.0	443 93; {201,443} 1; {203,443} 3;
		3.0	{201,442} 78; 442 13; {203,442} 2; {221,221} 1;
		3.5	{221,221} 63; {203,442} 20; 442 2;
		4.0	{321,321} 88;
0^-	0.328	0.35	301 98;
		1.3	302 77; 303 4; {201,302} 9; {201,303} 4;
		3.8	{221,321} 76; {221,324} 8; 203 2;
		7.8	{201,301} 70;
1^-	0.952	0.95	311 100;
2^-	1.123	1.14	321 96;
		1.8	322 86; {201,323} 10;

The results of calculation for ^{168}Er and ^{228}Th, given in tables 2 and 3, are characteristic in may respects of the well deformed nuclei and those lying on the boundary of the region of deformed nuclei. These data indicate the absence of the low-lying collective two-phonon states.

CONCLUSION

Based on the study of the two-phonon states in doubly even deformed nuclei within the quasiparticle-phonon nuclear model taking into account the Pauli principle in the two-phonon components of the excited state wave functions, we can make the following conclusions;

1. The conclusion of ref.[9] about the shift of centroid energies of the collective two-phonon states in doubly even deformed nuclei by 1-3 MeV towards higher energies is confirmed.

2. The results of calculation with the phonons depending on the sign of the angular momentum projection into the symmetry axis[20] differ slightly from the results obtained in refs.[9,10]. The largest difference implies that one and the same configuration of $\{\lambda_1\mu_1 i_1, \lambda_2\mu_2 i_2\}$ the function $\chi^{K_0}(g_1g_2|g_2g_1)$ and the pole shifts turn out to be different for $K_0 = \mu_1 + \mu_2$ and $K_0 = |\mu_1 - \mu_2|$. For both collective phonons this difference is not large.

3. The shifts of three-phonon poles due to the Pauli principle in the three-phonon components of the wave functions are calculated. It is shown that the shifts take different values from 0.1 to 5 MeV. Inspite of a considerable decrease in the number of three-phonon poles up to the excitation energy of 5 MeV, one may expect a strong fragmentation of two-phonon states, the centroid energies of which are at 3-5 MeV.

4. The conclusion of paper[9] is confirmed, that the collective two-phonon states cannot exist in the deformed nuclei. This conclusion is universal. It concerns all the deformed nuclei and is independent of the choice of the model parameter. This is just the difference from the explanations made in papers[11,19], in which nonobservation of the two-phonon states in a certain energy interval is related to the specific properties of the nuclei considered.

5. To elucidate the situation with the two-phonon states, it is necessary to search for the collective two-phonon states in many deformed nuclei.

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Соловьев В.Г., Ширикова Н.Ю. Положение с коллективными двухфононными состояниями в деформированных ядрах E4-82-300

В рамках квазичастично-фононной модели ядра с операторами фононов, зависящими от знака проекции углового момента, учтен принцип Паули в двухфононных компонентах волновых функций. Рассчитаны центроиды энергии коллективных двухфононных состояний в четно-четных деформированных ядрах и показано, что учет принципа Паули приводит к их сдвигу на 1-3 МэВ в сторону больших энергий. Рассчитаны сдвиги трехфононных полюсов из-за учета принципа Паули в трехфононных компонентах волновых функций. Следует ожидать сильной фрагментации коллективных двухфононных состояний, центроиды энергий которых равны 3-5 МэВ. Подтвержден вывод, что коллективные двухфононные состояния не должны существовать в деформированных ядрах. Проанализировано положение в ^{168}Er и изотопах Th и U.

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Soloviev V.G., Shirikova N.Yu. Situation with Collective Two-Phonon States in Deformed Nuclei E4-82-300

Within the quasiparticle-phonon nuclear model with the operators of phonons depending on the sign of the angular momentum projection, the Pauli principle is taken into account in the two-phonon components of the wave functions. The centroid energies of the collective two-phonon states in doubly even deformed nuclei are calculated. It is shown that the inclusion of the Pauli principle leads to their shift by 1-3 MeV towards high energies. The shifts of three-phonon poles due to the Pauli principle are calculated in the three-phonon components of the wave functions. The collective two-phonon states, the centroid energies of which are 3-5 MeV, are expected to be strongly fragmented. The conclusion is confirmed that the collective two-phonon states should not exist in deformed nuclei. The situation in ^{168}Er and in the Th and U isotopes is analysed.

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