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ABSTRACT

Newton's world model may have a physical meaning if the graviton has small non-zero mass and if the observable part of the Universe is the interior of a giant finite body. Both possibilities are in principle allowed.

АННОТАЦИЯ

Ньютоновская модель Мира может иметь физическое содержание, в том случае, если масса гравитона малое ненулевое значение и если наблюдаемая часть Вселенной является внутренней областью великого конечного тела. В принципе допускаются оба случая.

KIVONAT

A newtoni világmodellnek lehet fizikai tartalma, ha a gravitonnak a tömege kicsi nem-zérus és ha az Univerzum megfigyelt része egy óriás, véges test belseje. Mindkét eset elvben megengedett.

The oldest and simplest model of the complete physical world is Newton's: the whole Universe is a static, homogeneous and isotropic four-dimensional manifold with the same global and topological properties as Minkowskian space, and the mean matter density is non-zero. (As a matter of fact some ideas relating to such a model had already been formulated by Giordano Bruno.) However, it is well-known that this model conflicts with Newton's and with Einstein's theory of gravitation.

In this paper we show that Newton's world model may have a physical meaning if the graviton has a small non-zero mass m (thus the gravitation here essentially differs from Einstein's gravitation on the scales $\gtrsim 1/m$; $\hbar=c=1$) and if the observable part of the Universe is the interior of a finite "island". The first possibility cannot be excluded, while the second one even seems to be probable. Thus both possibilities are in principle allowed.

At present, even though it is widely assumed that the complete physical world is described by a Friedmannian model, this model cannot be verified - even in principle - by observations, because the regions beyond the particle horizon are not observable. Therefore extrapolation of the properties of the observable part of the Universe beyond the present particle horizon is questionable. Nevertheless, it is not necessary that this extrapolation be valid. It is not prohibited that there is an edge to matter that will be seen in some future era [1]. Some cosmological considerations even suggest this possibility [2,3]. If there were an edge beyond the particle horizon, then the observable region could be the interior of a giant body ("island") of size r. This hypothetical body in ref. [3] is known as the M-galaxy. Obviously, r would need to be much bigger than the present Hubble radius.

Here we a priori assume the existence of this island. This then gives rise to the question: What about the exterior of the M-galaxy? Of course, at present this question still seems to be an academic one though its study is not completely without meaning. Any considerations about the complete Universe may be of use with regard to our view of Nature.

The simplest possibility seems to be the following: there is a vacuum in the exterior of the M-galaxy and the space-time is asymptotically flat. Unfortunately, this model is not satisfactory because it conflicts with Mach's principle. (A particle escaping from the M-galaxy would move into flat space, arbitrarily far from all matter, yet its inertial properties would not change, contrary to the idea that inertia is generated by the matter in the Universe; see [4], p. 12.) In view of this, we have to assume the existence of other islands. The characteristic distance between any two islands should be $s \gg r$. The most natural possibility is then obvious: the mean density of these islands and thus of their matter is homogeneous and isotropic on the scale $p \gg s$. Can this matter be static? Obviously, it would be nice to have a static world, i.e. a world of maximum symmetry.

General relativity allows only one reasonable possibility, viz. Einstein's static model, but this no longer seems to be satisfactory since (i) it needs a given density, which leads to instabilities (see [4], p. 14); (ii) it needs a non-zero cosmological term, which conflicts with the identity of gravitation and of standard spin 2 field [5].

The situation essentially changes if the graviton has a small non-zero mass m. (This possibility is not new; see e.g. [6]. In Ref. [7] a finite quantum gravity is proposed that needs non-zero mass for the graviton. In any case $1/m$ must be much bigger than the present Hubble radius. We assume $1/m \gtrsim p$.) In this case we have a remarkable possibility: Newton's model is in principle allowed.

To show this, consider a manifold that has the same global and topological properties as Minkowskian space. Introducing the unobservable flat background with the metric tensor $\eta^{ij} = \eta_{ij} \equiv \mathrm{diag}(1,-1,-1,-1)$ we obtain the contravariant metric tensor g^{ij}:

$$g^{ij} = \eta^{ij} + fU^{ij}, \qquad f = \sqrt{32\pi G}\,, \tag{1}$$

where U^{ij} is the potential of spin 2 field and G is the gravitational constant (for details see [5]). If the graviton has small non-zero mass m, then Einstein equations are substituted by [7]

$$(\Box + m^2)U^{ij} - U^{k(i,j)}{}_{,k} + U^{,ij} + \eta^{ij}U^{km}{}_{,km} - \eta^{ij}(\Box + \frac{m^2}{2})U = \frac{f}{2}(T^{ij} + t^{ij}); \tag{2}$$
$$U \equiv U^{ij}\eta_{ij}; \quad \Box \equiv \partial^i\partial_i \,,$$

where t^{ij} is the energy-momentum pseudotensor of gravitation and T^{ij} is the usual energy-momentum tensor. For our purpose it is essential that $t^{ij}=0$ holds for U^{ij}=const. We need to search for a solution with U^{ij}=const., T^{ij}=const. Let us assume that

$$T^{ij} = (p+\rho)v^i v^j - pg^{ij}; \quad p = \text{const.} \geq 0; \quad \rho = \text{const.} > 0; \tag{3}$$
$$p \leq \frac{\rho}{3}$$

holds, where $v^i \equiv [\sqrt{g^{00}}, 0, 0, 0]$ is the four-velocity, ρ is the density, p is the pressure. In this case we must have

$$m^2(U^{ij} - \frac{1}{2}\eta^{ij}U) \equiv \frac{f}{2}T^{ij} \,. \tag{4}$$

Because $U^{11}=U^{22}=U^{33}$ is fulfilled, we have

$$m^2(U^{00} - \frac{1}{2}U^{00} + \frac{3}{2}U^{11}) = \frac{f}{2}(1+fU^{00})\rho \;; \quad U^{ij} = 0 \;\; i \neq j \;; \tag{5}$$
$$m^2(U^{11} + \frac{1}{2}U^{00} - \frac{3}{2}U^{11}) = \frac{f}{2}(1+fU^{11})(-p) \;;$$

and hence

$$U^{00} = \frac{fm^2(\rho-3p) - f^3\rho p}{4m^4 - f^2m^2(\rho+p) + f^4\rho p} \;; \tag{6}$$
$$U^{11} = \frac{fm^2(\rho+p) - f^3\rho p}{4m^4 - f^2m^2(\rho+p) + f^4\rho p} \,.$$

Requiring the fulfilment of the physically obvious conditions $fU^{00}>-1$ and $fU^{11}<1$, we have the restrictions

$$p < \frac{m^2}{f^2} ; \quad \rho < \frac{m^2}{f^2} + \frac{m^4}{f^2(m^2-pf^2)} . \tag{7}$$

If these restrictions hold, then in fact we have a "quasi-flat" space-time, i.e. by changing $\bar{x}_0 = \sqrt{g^{00}}x_0$ and $\bar{x}_{1,2,3} = \sqrt{-g^{11}}x_{1,2,3}$ we obtain the Minkowskian space. In other words, in this case the presence of matter changes the time and distances only, but except for this difference the space-time is in fact Minkowskian.

This space-time for very small densities seems to be stable. For this to be apparent, consider a fluctuation on the homogeneous, isotropic and time independent background. Assume that the Jeans approximation is good (see [8], Chapt. 9.1). Obviously here in the Poisson equation the substitution $\Delta \to \Delta - m^2$ must be done. (In other words, Newton's potential is exchanged for Yukawa's.) Except for this difference Jeans' full theory remains unchanged. This says that everywhere the $k^2 \to k^2 + m^2$ (k is the wave vector) substitution must be done. Thus for the critical Jeans length λ_J we have

$$\frac{f^2}{8}\rho - b^2\frac{4\pi^2}{\lambda_J^2} - b^2m^2 = 0 , \tag{8}$$

where b is the velocity of sound. If

$$\rho < \frac{8b^2m^2}{f^2} \tag{9}$$

holds, then there is no critical Jeans length and any fluctuation always behaves like an acoustic wave.

We have arrived at the result: if the graviton has a nonzero mass, then for small densities and pressures (see (7) and (9)) a physically reasonable "quasi-flat" manifold may exist. The question posed by the title may be answered positively. i.e. Newton's model may in principle describe the whole physical Universe. Nevertheless, one has to admit that it is not certain that the whole of Nature is described by this model. Indeed, the validity of the Friedmannian model beyond the present particle horizon is questionable and, therefore, it is not necessary that in the whole of Nature the Friedmannian model be valid. On the other hand, it is also not necessary that Nature be described by Newton's model.

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REFERENCES

[1] Rees M.J.: in Quantum Gravity 2, A Second Oxford Symp., ed. C.J. Isham, R. Penrose, D.W. Sciama, Clarendon Press 1981, p. 273

[2] Fabbri R., Melchiorri F.: Gen. Rel. Grav., 13, 201 (1981)

[3] Mészáros A.: KFKI report 1983-68 (1983)

[4] Peebles P.J.E.: The Large-Scale Structure of the Universe, Princeton Univ. Press 1980

[5] Thirring W.: Ann. Phys. (N.Y.) 16, 96 (1961); Weinberg S.: Phys. Rev. B 138, 988 (1965); Cavalleri G., Spinelli G.: Riv. Nuo. Cim. 3 (1980)

[6] Ogievetsky V.I., Polubarinov I.V.: Ann. Phys. (N.Y.) 35, 165 (1965)

[7] Mészáros A.: to be published

[8] Zeldovich Ya.B., Novikov I.D.: Stroyenyiye i evoluciya vselennoy, Nauka, Moscow 1975

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