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ABSTRACT

If the graviton has a very small non-zero mass, then the existence of six additional massive gravitons with very big masses leads to a finite quantum gravity. There is an acausal behaviour on the scales that are determined by the masses of additional gravitons.

АННОТАЦИЯ

Если гравитон имеет очень малую ненулевую массу, то при существовании шести дальнейших массивных гравитонов с очень большими массами квантовая гравитация конечна. В этом случае имеется некоторое нарушение причинности на масштабе, определяемом этими дальнейшими гравитонами.

KIVONAT

Ha a graviton tömege kicsi nem-zéró, akkor hat további nagyon nagy tömegű graviton léte véges kvantumgravitációhoz vezet. Ez esetben bizonyos akauzalitásunk van azon a skálán, melyet ezek a további gravitonok határoznak meg.

1. INTRODUCTION

The Lorentz-covariant quantum gravity [1] is a non-renormalizable theory of massless spin 2 particle (graviton). In order to obtain a finite quantum gravity one usually supposes the existence of other particles together with the ordinary graviton, and the system of these particles should give a finite S-matrix. For example, the supergravity [2] supposes the existence of gravitinos that have helicities 3/2 or 5/2.

We propose here a new route that leads to a finite quantum gravity. We suppose here the existence of six massive gravitons together with the ordinary graviton. It is remarkable that a small non-zero mass is necessary for the ordinary gravition too.

We follow the procedure of Lorentz-covariant quantum gravity, in which the notion of the Minkowskian background has an essential importance. The metric tensor of this flat background has the form $\eta^{ij}=\eta_{ij}\equiv\mathrm{diag}(1,-1,-1,-1)$. We use the system $\hbar = c = 1$.

2. EINSTEIN'S GRAVITY WITH MASSIVE GRAVITON

In order to write down the Einstein equations in the form

$$\Box(U^{ij} - \frac{1}{2}\eta^{ij}U) = t^{ij}; \quad t^{ij}{}_{,j} = 0; \quad \Box \equiv \partial^i\partial_i, \tag{1}$$

we use the coordinate conditions

$$U^{ij}{}_{,j} = \frac{1}{2}U^{,i}; \quad \eta_{ij}U^{ij} \equiv U, \tag{2}$$

where the quantities U^{ij}, which indices are moved by η^{ij} and η_{ij}, are given by relations

$$g^{ij} = \eta^{ij} + f\,U^{ij}; \quad f = \sqrt{32\pi G}. \tag{3}$$

g^{ij} is the contravariant metric tensor, G is the gravitational constant. t^{ij} and g_{ij} are given by infinite series (for details see [1]). We have the field equations of a self-interacting massless spin 2 field.

Now breaking the gauge freedom we introduce a non-zero mass $\lambda > 0$ by the obvious substitution $\Box \rightarrow \Box + \lambda^2$ in the left-hand-side of (1). ($1/\lambda$ must be bigger than the sizes investigated by the present-day cosmology.) For $\lambda \neq 0$ equations (1) and (2) are equivalent with

$$(\Box + \lambda^2)U^{ij} - U^{k(i,j)}{}_{k} + U^{,ij} + \eta^{ij}\,U^{km}{}_{,km} - \eta^{ij}(\Box + \frac{\lambda^2}{2})U = t^{ij}. \quad (4)$$

In order to explain the properties of massive graviton we consider the free field, i.e. we take $t^{ij} = 0$. Then U^{ij} fulfils the Klein-Gordon equation and in momentum space we have

$$U^{ij} = (2\pi)^{-3/2}\int\frac{d\vec{k}}{\sqrt{2k_0}}(\bar{U}^{ij}(\vec{k})e^{-ik^m x_m} + \overset{+}{U}^{ij}(\vec{k})e^{ik^m x_m}). \quad (5)$$

It is convenient to write

$$\overset{\pm}{U}^{ij}(\vec{k}) = \frac{\overset{\pm}{a}(\vec{k})}{\sqrt{2}}(e^i e^j - f^i f^j) + \frac{\overset{\pm}{b}(\vec{k})}{\sqrt{2}}(e^i f^j + f^i e^j) +$$

$$+ \frac{\overset{\pm}{c}(\vec{k})}{\sqrt{2}}\frac{1}{\lambda}(e^i(k_0 n^j + |\vec{k}|m^j) + (k_0 n^i + |\vec{k}|m^i)e^j) +$$

$$+ \frac{\overset{\pm}{d}(\vec{k})}{\sqrt{2}}\frac{1}{\lambda}(f^i(k_0 n^j + |\vec{k}|m^j) + (k_0 n^i + |\vec{k}|m^i)f^j) +$$

$$+\sqrt{\frac{2}{3}}\,\overset{\pm}{g}(\vec{k})(-\eta^{ij} + \frac{k^i k^j}{\lambda^2} - \frac{3}{2}(e^i e^j + f^i f^j)) + \frac{\overset{\pm}{h}(\vec{k})}{\sqrt{12}}(\eta^{ij} + 2\,\frac{k^i k^j}{\lambda^2}), \quad (6)$$

where the wave vector k and the four-vectors e, f, n, m have the following properties:

$$e^i e^j + f^i f^j + n^i n^j - m^i m^j = -\eta^{ij}; \qquad e^i e_i = f^i f_i = n^i n_i = -m^i m_i = -1;$$

$$e^i f_i = e^i n_i = e^i m_i = f^i n_i = f^i m_i = n^i m_i = 0; \quad k^i k_i = (k^0)^2 - |\vec{k}|^2 = \lambda^2;$$

$$k^i \equiv n^i|\vec{k}| + m^i k_0; \quad e^i \equiv [0,\vec{e}]; \quad f^i \equiv [0,\vec{f}]; \quad n^i \equiv [0, \frac{\vec{k}}{|\vec{k}|}];$$

$$m^i \equiv [1, 0, 0, 0]. \quad (7)$$

After the quantisation one obtains

$$[\bar{a}(\vec{k}),\overset{+}{a}(\vec{q})] = [\bar{b}(\vec{k}),\overset{+}{b}(\vec{q})] = [\bar{c}(\vec{k}),\overset{+}{c}(\vec{q})] = [\bar{d}(\vec{k}),\overset{+}{d}(\vec{q})] = [\bar{g}(\vec{k}),\overset{+}{g}(\vec{q})] =$$

$$= [\bar{h}(\vec{k}),\overset{+}{h}(\vec{q})] = \delta(\vec{k} - \vec{q}), \quad (8)$$

and hence

$$[\bar{U}^{ij}(\vec{k}),\overset{+}{U}^{ps}(\vec{q})] = \frac{1}{2}\,\delta(\vec{k}-\vec{q})((\eta^{ip}-\frac{k^ik^p}{\lambda^2})(\eta^{js}-\frac{k^jk^s}{\lambda^2}) + $$
$$+(\eta^{is}-\frac{k^ik^s}{\lambda^2})(\eta^{jp}-\frac{k^jk^p}{\lambda^2}) - \frac{2}{3}(\eta^{ij}-\frac{k^ik^j}{\lambda^2})(\eta^{ps}-\frac{k^pk^s}{\lambda^2}) +$$
$$+\frac{1}{6}(\eta^{ij}+\frac{2k^ik^j}{\lambda^2})(\eta^{ps}+\frac{2k^pk^s}{\lambda^2})). \qquad (9)$$

In the standard way (see e.g. [3]) one obtains the four-momentum

$$p^i = \int d\vec{k}k^i(\overset{+}{a}(\vec{k})\bar{a}(\vec{k}) + \overset{+}{b}(\vec{k})\bar{b}(\vec{k}) + \overset{+}{c}(\vec{k})\bar{c}(\vec{k}) + \overset{+}{d}(\vec{k})\bar{d}(\vec{k}) +$$
$$+\overset{+}{g}(\vec{k})\bar{g}(\vec{k}) + \overset{+}{h}(\vec{k})\bar{h}(\vec{k})) , \qquad (10)$$

and the spin angular momentum $(\alpha,\beta = 1, 2 , 3)$

$$S^{\alpha\beta} = \int S^{\alpha\beta}(\vec{k})d\vec{k} = 2i\int d\vec{k}(\overset{+}{U}^{j\alpha}(\vec{k})\bar{U}^{\beta}_{j}(\vec{k}) - \overset{+}{U}^{j\beta}(\vec{k})U^{\alpha}_{j}(\vec{k})). \qquad (11)$$

Taking specially $e^i = [0,0,1,0]$, $f^i = [0,1,0,0]$, $n^i = [0,0,0,1]$ one has

$$S^{12}(\vec{k}) = i(2(\overset{+}{a}(\vec{k})\bar{b}(\vec{k}) - \overset{+}{b}(\vec{k})\bar{a}(\vec{k})) + \overset{+}{c}(\vec{k})\bar{d}(\vec{k}) - \overset{+}{d}(\vec{k})\bar{c}(\vec{k})). \qquad (12)$$

Thus it is obvious that the spin of six polarizations are ±2, ±1, 0, 0. If one did not consider the terms determined by $\overset{+}{h}(\vec{k})$, $\bar{h}(\vec{k})$, one would have $\overset{\pm}{U}^{ij}(\vec{k})k_j = 0$ and $\overset{\pm}{U}(\vec{k}) = 0$, and thus one would have a standard massive spin 2 field. Therefore, as a matter of fact one has a system of spin 2 and spin 0 massive fields.

Einstein's gravity with massive graviton is a system of self-interacting massive spin 2 and spin 0 fields that have the same masses. Note here that in [4] Einstein's gravity with massive graviton is similarly introduced too. Nevertheless, there is no unambiguous introduction of the mass term there.

3. ADDITIONAL GRAVITONS

Given the Lagrangian of the massive Einstein's gravity

$$L = -\frac{2}{f^2}\, g^{ik}(\Gamma^m{}_{kj}\Gamma^j{}_{mi} - \Gamma^j{}_{ik}\Gamma^m{}_{jm}) - \frac{\lambda^2}{2}(U^{ij}-\frac{1}{2}\eta^{ij}U)(U_{ij}-\frac{1}{2}\eta_{ij}U) =$$
$$= \frac{1}{2}\,U^{ij,k}U_{ij,k} - U^{ij,k}U_{ik,j} + U^{ij}{}_{,j}U_{,i} -$$
$$-\frac{1}{2}\,U^{,i}U_{,i} - \frac{\lambda^2}{2}(U^{ij}-\frac{1}{2}\,\eta^{ij}U)(U_{ij}-\frac{1}{2}\,\eta_{ij}U) + L^{int}(U^{ij};U^{ij,k}) =$$
$$= L^0(U^{ij};\lambda) + L^{int}(U^{ij};U^{ij,k}), \qquad (13)$$

where L^{int} is an infinite series containing the potentials U^{ij} and their first derivatives. (For the simplicity we consider the self-interacting gravitation only. This is a non-essential restriction here.) In accordance with (9) one has

$$[g^{ij}(x),g^{ps}(y)] = f^2[U^{ij}(x),U^{ps}(y)] = \frac{f^2}{2}\left((\eta^{is} + \frac{\partial^i\partial^s}{\lambda^2})(\eta^{jp} + \frac{\partial^j\partial^p}{\lambda^2}) + \right.$$

$$\left. + (\eta^{ip} + \frac{\partial^i\partial^p}{\lambda^2})(\eta^{js} + \frac{\partial^j\partial^s}{\lambda^2}) - \frac{1}{2}\eta^{ij}\eta^{ps} - \eta^{ij}\frac{\partial^p\partial^s}{\lambda^2} - \eta^{ps}\frac{\partial^i\partial^j}{\lambda^2}\right) D((x-y);\lambda);$$

$$D(x;\lambda) = \frac{1}{2\pi}\epsilon(x^o)\delta(x^i x_i) - \frac{1}{4\pi\sqrt{x^i x_i}}\epsilon(x^o)\theta(x^i x_i)J_1(\lambda\sqrt{x^i x_i}).$$

$D(x;\lambda)$ is the well-know Pauli-Jordan function.

Now we suppose that there are six other massive gravitational fields beside the ordinary Einstein's gravitation. These fields are called as additional gravitations and the relevant particles as additional gravitons. They have masses M_A; $A = 1, 2, \ldots, 6$; and $M_1 < M_2 < \ldots < M_6$ holds. We suppose that M_1 is much bigger than the usual masses of the present-day experimental particle physics. We assume that the contravariant metric tensor is given by

$$g^{ij} = \eta^{ij} + f\,U^{ij} + f\sum_{A=1}^{6}\sqrt{c_A}\,V_A^{ij}\,. \tag{15}$$

where c_A are positive non-zero constants. V_A^{ij} is the potential of the A-th additional gravitation. The complete Lagrangian of seven gravitations is obtainable from (13) by substitution $U^{ij} \to U^{ij} + \sum_{A=1}^{6}\sqrt{c_A}\,V_A^{ij}$, and therefore

$$L = L^o(U^{ij};\lambda) + \sum_{A=1}^{6} c_A\,L^o(V_A^{ij};M_A) + L^{int}((U^{ij} + \sum_{A=1}^{6}\sqrt{c_A}\,V_A^{ij});$$
$$;(U^{ij} + \sum_{A=1}^{6}\sqrt{c_A}\,V_A^{ij})^{,k}), \tag{16}$$

where

$$L^o(V_A^{ij};M_A) = \frac{1}{2}V_A^{ij,k}V_{Aij,k} - V_A^{ij,k}V_{Aik,j} + V_A^{ij}{}_{,j}V_A{}^{,i} - \frac{1}{2}V_A{}^{,i}V_{A,i} -$$
$$-\frac{M_A^2}{2}(V_A^{ij} - \frac{1}{2}\eta^{ij}V_A)(V_{Aij} - \frac{1}{2}\eta_{ij}V_A)\,. \tag{17}$$

We suppose that the following commutation relations hold:

$$[g^{ij}(x),g^{ps}(y)] = \frac{f^2}{2}((\eta^{ip}\eta^{js} + \eta^{is}\eta^{jp} - \frac{1}{2}\eta^{ij}\eta^{ps})$$

$$(D((x-y);\lambda) + \sum_{A=1}^{6}(-1)^A c_A D((x-y);M_A)) + (\eta^{ip}\partial^j\partial^s + \eta^{js}\partial^i\partial^p +$$

$$+ \eta^{is}\partial^j\partial^p + \eta^{jp}\partial^i\partial^s - \eta^{ij}\partial^p\partial^s - \eta^{ps}\partial^i\partial^j)(\frac{D((x-y);\lambda)}{\lambda^2} +$$

$$+ \sum_{A=1}^{6}\frac{(-1)^A c_A}{M_A^2} D((x-y);M_A)) + 2\partial^i\partial^j\partial^p\partial^s(\frac{D((x-y);\lambda)}{\lambda^4} +$$

$$+ \sum_{A=1}^{6}\frac{(-1)^A c_A}{M_A^4} D((x-y);M_A))) = f^2\tilde{D}^{ijps}(x-y); \qquad (18)$$

$$[v_A^{ij}(x),v_A^{ps}(y)] = \frac{(-1)^A}{2}((\eta^{ip} + \frac{\partial^i\partial^p}{M_A^2})(\eta^{js} + \frac{\partial^j\partial^s}{M_A^2}) +$$

$$+ (\eta^{is} + \frac{\partial^i\partial^s}{M_A^2})(\eta^{jp} + \frac{\partial^j\partial^p}{M_A^2}) - \frac{1}{2}\eta^{ij}\eta^{ps} - \eta^{ij}\frac{\partial^p\partial^s}{M_A^2} - \eta^{ps}\frac{\partial^i\partial^j}{M_A^2})\cdot$$

$$\cdot D((x-y);M_A). \qquad (19)$$

The constants c_A are determined by algebraic equations

$$\lambda^{2(n-3)} + \sum_{A=1}^{6}(-1)^A c_A M_A^{2(n-3)} = 0; \quad n = 1, 2, \ldots, 6. \qquad (20)$$

(Solving this system it is easy to show the positivity of constants c_A.) Hence it follows that $\tilde{D}(x)$; $\tilde{D}(x)_{,i}$; $\tilde{D}(x)_{,ij}$ are singularity free functions, and therefore the S-matrix contains no infrared and ultraviolet divergencies.

4. REMARKS

The potentials v_A^{ij} for odd (A = 1, 3, 5) A give commutation relations with opposite sign. Because $-D(x) = D(-x)$ holds, one may interpret the odd additional gravitations as gravitations for which the coordinates of flat background have opposite meaning. Therefore the future (past; right; left) of odd additional gravitations is identical to the past (future; left; right) of even (A = 2, 4, 6) additional gravitations and of ordinary gravitations. This interpretation leads to an acausal behaviour on scales $\lesssim 1/M_A$. Note that it was conjectured a very similar acausal behaviour of quantum gravity on the microscopic scales [5]. However, the considerations leading to this conjecture were essentially different.

The conjecture of existence of additional gravitations is not quite new, too. In Ref. [6] the possibility of an additional gravitation is discussed. Nevertheless, there the mass of additional graviton is supposed to be big, and the considerations are essentially different.

In our model as a matter of fact the masses of the Pauli-Villars regularization scheme were interpreted as masses of some real particles. It is remarkable that this interpretation needs the condition $\lambda \neq 0$. Suppose that $\lambda = 0$. Then one has [1]

$$[U^{ij}(x), U^{ps}(y)] = \frac{1}{2}(\eta^{ip}\eta^{js} + \eta^{is}\eta^{jp} - \frac{1}{2}\eta^{ij}\eta^{ps}) D((x-y); \lambda = 0). \quad (21)$$

Introducing the Pauli-Villars additional fields with masses M_D; $D=1,2,..,N$; and with potentials W_D^{ij} one obtains for the real coefficients c_D

$$1 + \sum_{D=1}^{N} c_D = 0; \quad \sum_{D=1}^{N} c_D M_D^{2n} = 0; \quad n = 1, 2, 3. \quad (22)$$

If the massive additional fields are fields with physical meaning, then the potentials W_D^{ij} must give (19). Therefore, a singularity free $\tilde{D}(x)$ needs, together with (22), the relations

$$\sum_{D=1}^{N} c_D M_D^{2n} = 0; \quad n = -2, -1, 0, \quad (23)$$

too, which cannot be fulfilled.

5. CONCLUSION

If our model has a physical meaning, then the gravitation essentially differs from the Einstein's one on the scales $\lesssim 1/\lambda$ and $\lesssim 1/M_1$. The non-zero λ may drastically change our image of the whole universe. On the other hand, the acausal behaviour on small scales may have an essential impact on the conditions near the Big Bang.

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