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CHARGE EXCHANGE OF MUONS IN GASES: EXPERIMENTAL IMPLICATIONS FROM RATE THEORY

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Abstract

The effects of the charge exchange process on muon spin dynamics have been investigated using a density operator formalism with special interest placed upon the diamagnetic muon and paramagnetic muonium signals observed after thermalization. In the charge exchange region the dynamics of the spin density operator is assumed to be determined by the muonium hyperfine interaction and by electron capture and loss processes for muons. Analytical expressions are obtained for the amplitudes and phases of the diamagnetic muon and paramagnetic muonium signals as a function of the duration of the charge exchange region, t_c, which is inversely proportional to the number density of the moderating gas. The theoretical signals exhibit three features which have, as yet, to be experimentally observed, namely: i) that the amplitudes associated with the muonium Larmor frequency and with the hyperfine frequency are not, in general, equal, ii) that all the amplitudes are, in general, damped oscillatory functions of t_c (temperature/pressure) and iii) that phase jumps occur when an amplitude decreases to zero and then increases with falling pressure. Fits to the experimental argon data are discussed in light of the above points.

I. INTRODUCTION

The mechanism for the slowing down of a high energy charged particle in gases has been studied for decades. There are essentially four regions in this stopping process, namely: i) the Bethe-Bloch regime, ii) the charge exchange regime, iii) the thermalization regime and iv) the thermal equilibrium region. In the Bethe-Bloch region, above roughly 40 keV, collisions between the muon and gas atoms result in ionization of the gas atoms. Thus the muon remains as a bare μ^+ with the motion of its spin vector given by the muon Larmor precession about the external magnetic field. For energies of roughly 40 keV to 50 eV charge exchange collisions dominate as the Massey criterion holds, that is, the muon velocity and the velocity of the valence electrons of the moderating gas are comparable. In this region the muon is undergoing a series of charge exchange cycles between the free muon state and the paramagnetic muonium state. Thus the motion of the muon spin vector is alternately described by the free muon and the paramagnetic muonium spin Hamiltonians. In the thermalization regime (roughly between 50 eV and 0.025 eV) elastic, inelastic and reactive collisions are dominant processes. There may be single charge exchange collisions in this third region but there are no more cycles. Finally, the last region involves only thermal processes. The emphasis in the present study is placed on an attempt to understand the effect of the second region on the spin polarization of the muon observed in the last region.

II. THEORY AND FIT TO ARGON DATA

Experimentally [1], the time dependence of the muon spin is measured in the fourth region. After proper consideration is made for the overall normalization, the constant background and the muon decay life time, the muon polarization is expressed by

$$P_X(t) = P_D \cos(\omega_D t+\theta_D) + P_M\cos(\omega_M t-\theta_M) + P_M^0\cos(\omega_0 t-\theta_M^0). \qquad (1)$$

This general signal involves a hyperfine frequency term, the so-called "singlet" muonium signal, which cannot be measured by the standard μSR technique because of the limited time resolution of the experiments. It is commonly assumed that this "singlet" amplitude is equal to that of the observed muonium precession term, the so called "triplet" signal. The amplitudes and phases of the various terms in this expression are related to the density operators which describe the coupled diamagnetic and paramagnetic muon spin states that emerge from the charge exchange region at time t_2. For example, the three amplitudes in eq. (1),

$$P_D = |c[\rho^D_{12}(t_2) + \rho^D_{43}(t_2)] + s[\rho^D_{23}(t_2) - \rho^D_{14}(t_2)]|,$$

$$P_M = |2s\rho^M_{23}(t_2) + 2c\rho^M_{12}(t_2)|,$$

$$P^o_M = |-2s\rho^M_{14}(t_2) + 2c\rho^M_{43}(t_2)|, \quad (2)$$

can be expressed in terms of matrix elements of the density operators, $\rho^D(t_2)$, and $\rho^M(t_2)$, whose motion is described by the following pair of coupled first-order linear differential equations:

$$d\rho^D/dt = -R_C\rho^D(t) + R_L\rho^M(t),$$

$$d\rho^M/dt = -[R_L + iL_o]\rho^M(t) + R_C\rho^D(t). \quad (3)$$

Here, L_o describes the motion generated by the hyperfine interaction while $R_C = K_CP$ and $R_L = K_LP$ are the rates of electron capture and loss, respectively. These rates are products of the rate constants K_C and K_L and the pressure, P. The basis functions used in eq. (2) are the eigenfunctions of the muonium spin Hamiltonian, that is, $|1\rangle = |\alpha\alpha\rangle$, $|2\rangle = s|\alpha\beta\rangle + c|\beta\alpha\rangle$, $|3\rangle = |\beta\beta\rangle$ and $|4\rangle = c|\alpha\beta\rangle - s|\beta\alpha\rangle$, where $\alpha\alpha$ refers to the (1/2) eigenstate of the muon times the (1/2) eigenstate of the electron, in the direction of the transverse field, and where the functions $s = [1-c^2]^{1/2}$ and $c = [1+X/(1+X^2)^{1/2}]^{1/2}/2^{1/2}$ are standard normalizations. They involve the reduced field strength $X = (\omega_e+\omega_D)/\omega_o = 6.3 \times 10^{-4}B$. Qualitatively, the electron capture and loss rate constants determine the relative amounts of diamagnetic and paramagnetic muon that are produced while the hyperfine interaction leads to a loss of polarization through free flight spin dephasing. It should be emphasized that the hyperfine interaction enters the equations of motion in an asymmetric fashion. That is, the equations for the matrix elements, namely, $\rho^{M}_{23}(t)$ and $\rho^{M}_{12}(t)$, which give rise to the amplitude associated with the Larmor frequency term, do not involve the hyperfine frequency directly. On the other hand, the matrix elements, $\rho^{M}_{14}(t)$ and $\rho^{M}_{43}(t)$, of eqs. (3) which give rise to the amplitude associated with the "singlet" signal directly involve the hyperfine frequency. It is the coupling of these equations through the rate operators that leads to spin dephasing of all three signals. In particular, the rate constant for loss of an electron is diagonal in the muonium basis while the rate constant for electron capture is diagonal in the $\alpha\alpha$ basis. Thus the coupling between the diamagnetic and paramagnetic states is a result of the fact that the eigenfunctions of the diamagnetic and paramagnetic spin Hamiltonians are not orthogonol. The charge exchange collisions, themselves, are assumed to have no effect on the spin states of the muon or the electron. In general, the amplitudes are found to be the magnitude of a sum of damped sinusoidal functions of t_c.

For rates large compared to the hyperfine frequency the amplitudes are given by

$$P_M = [K_C/4K_S]\ \exp(-\lambda\omega_0^2 t_c/2)\left|\cos(\omega_0\ \nu t_c/2)\right| = P_M^0\ , \tag{4}$$

and $$P_D = [K_L/K_S]\ \exp(-\lambda\omega_0^2 t_c/2)\{\cos^2(\omega_0 \nu t_c/2) + \sinh^2[\lambda\omega_0^2 t_c(c^2-s^2)/2]\ \}^{1/2}. \tag{5}$$

Here the polarization of the incoming muon beam has been normalized to unity. It was found that the following linear combinations of these rate constants are particularly useful for the purpose of fitting, $K_S = K_L+K_C/2$ and $K_D = (K_L-K_C/2)/K_S$. Finally, the relaxation rate λ is

$$\lambda = K_S P(1-K_D{}^2)/4(1+K_L{}^2P^2) \tag{6}$$

while the associated frequency is

$$\nu = -K_S K_L P^2(1-K_D{}^2)/4(1+K_L{}^2P^2). \qquad (7)$$

For these large rate parameters the amplitudes associated with the Larmor and hyperfine frequencies are equal. This result does not hold when the size of the rates approaches the magnitude of the hyperfine frequency. Phase jumps occur because the amplitudes are, by definition, positive. This reflects the possibility that the initial experimentally observed signal, $P_X(0)$ of eq. (1), may change from positive to negative (or vice versa) as a function of pressure.

Both the diamagnetic and paramagnetic experimental signals [1-3] for argon have been fitted, independently, to the theoretical expressions, see figs. 1 and 2, respectively. The average value of K_S is $8.0\pm4.0\ \omega_0 = 2.3\pm1.0\times10^{11}$/s-atm while the average value of K_D is -0.40 ± 0.10. Inverting these functions leads to the following results for the rate constants of electron loss and capture, $2.4\pm2.0\ \omega_0 = 0.67\pm0.5\times10^{11}$/s-atm and $11\pm2.0\ \omega_0 = 3.1\pm0.5\times10^{11}$/s-atm, respectively. The time span of the charge exchange regime has been written as $t = W/P$ where W is a fitting parameter. Its average value is $1.4\pm0.5/\omega_0 = 0.050\pm0.02$ ns/atm. For pressures greater than about 3 atm the sum of the amplitudes is equal to the initial polarization. Thus there is no lost fraction at high pressures, cf. liquids. Both signals have a cusp around 0.2 atm.

In conclusion, a theoretical treatment of the effects of the charge exchange regime on the dynamics of the muon spin vector has predicted three properties of the signals that, as yet, have not been confirmed or denied by experiments. These effects are: i) that, in general, the amplitudes associated with the muonium Larmor and hyperfine signals are not equal; ii) that the amplitudes are damped oscillatory functions of the number density; and iii) that phase jumps occur around the cusps of the, by definition, positive amplitudes. In particular, for low number densities, the oscillatory nature of both the diamagnetic and muonium Larmor amplitudes should be resolvable. On the other hand, for large number densities, the diamagnetic amplitude should monotonically increase while the muonium Larmor amplitude should exhibit a broad maximum and then slowly decrease with increasing number density.

References

[1] D.G. Fleming, R.J. Mikula and D.M. Garner, Phys. Rev. A26 (1982) 2527.

[2] R.J. Mikula, Ph.D. thesis, University of British Columbia (1981).

[3] K.P. Arnold, Diplomarbeit, Universität Heidelberg (1980).

Figure Captions

1. Muon amplitude for argon. A 10% uncertainty in the experimental numbers has been assumed. The reduced field X is 0.05 (75 G) while the fitting parameters are as follows: $K_S = 5.4\ \omega_o$; $K_D = -0.51$ and $W = 1.36/\omega_o$.

2. Muonium amplitude for argon. The reduced field is 0.0 (8 G) while the fitting parameters are as follows: $K_S = 11.\ \omega_o$; $K_D = -0.28$ and $W = 1.48/\omega_o$.

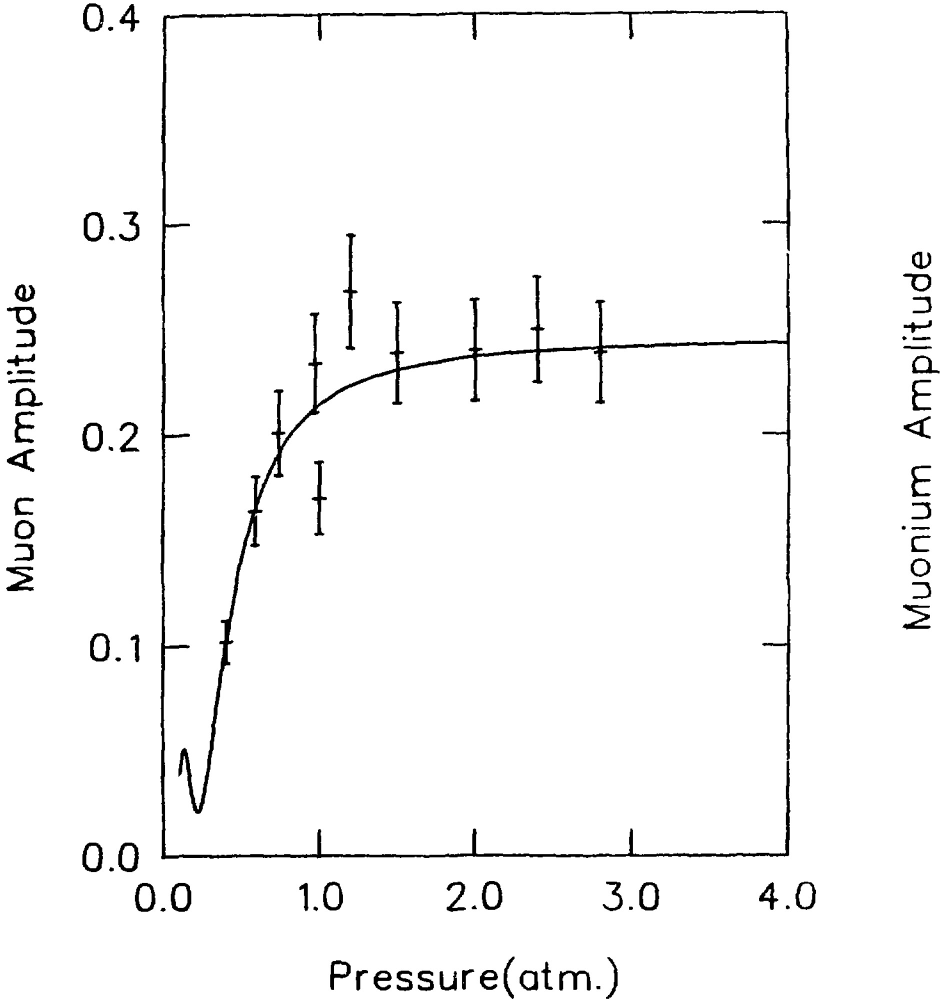
Muon Amplitude
0.0
0.1
0.2
0.3
0.4
0.0
1.0
2.0
3.0
4.0
Pressure(atm.)
Muonium Amplitude

Fig. 1

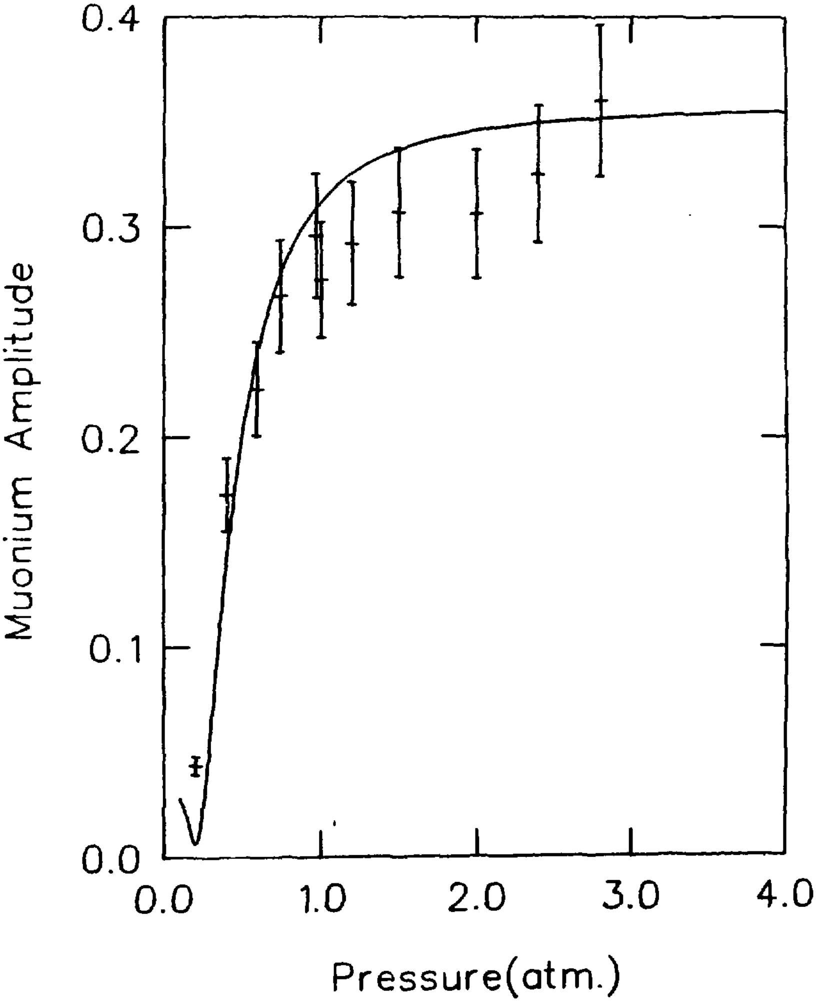

Fig. 2