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ON THE SPIN SATURATION AND THERMAL PROPERTIES OF NUCLEAR MATTER*

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ABSTRACT

The binding energy and the incompressibility of nuclear matter with degree of spin saturation D is calculated using the Skyrme interaction and two forms of a velocity dependent effective potential. The effect of the degree of spin saturation D on the thermal properties of nuclear matter is also discussed. It is found that generally the pressure decreases with increasing D.

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I. INTRODUCTION

The problem of spin saturation, namely that each occupied state contains a nucleon with spin-up and a nucleon with spin-down in the case of nuclear matter was discussed before by Chang[1] using Skyrme interactions. This problem is important in discussing unsaturated nuclei such as ^{14}N. Chang has shown that not all Skyrme interactions give spin saturation. In order to get such saturation it is necessary to add a density dependent term[2,3] to the Skyrme interaction. In this work we discuss the property of spin saturation using the velocity dependent effective potential of s-wave interaction which was developed by Dzhibuti and Mamasakhlisov[4] and then modified by Dzhibuti and Sallam[5]. We also used the Skyrme interaction, too. Hassan et al.[6] calculated the properties of nuclear matter using two forms of the velocity dependent effective potential of s-wave interaction namely VG and VY. It was found that the binding energy per particle for VG interaction which saturates nuclear matter at $k_F = 1.35\ F^{-1}$ is -17.78 MeV while for VY interaction it is -16.13 MeV at $k_F = 1.8\ F^{-1}$. Hassan and Ramadan[7] calculated the properties of nuclear matter with an excess of neutrons, of spin-up neutrons and of spin-up protons using the VG and VY interactions and they obtained reasonable values for the isospin, spin and spin-isospin symmetry energies.

Hassan and Ramadan[8] also deduced the isospin, spin and spin-isospin incompressibilities using VG, VY and Skyrme interactions.

The thermal properties of nuclear matter have been considered by several authors[9,10,11] for symmetric and asymmetric cases. We study here the effect of spin saturation on the thermal properties of nuclear matter using VY, VG and Skyrme interactions[12].

In section II we present the theory of spin saturation at zero and finite temperatures as well as the description of the VG and VY interactions. Section III contains the results and discussion.

II. THEORY AND CALCULATIONS

2.1 Theory of spin saturation at zero temperature.

We consider a simple model[1] of nuclear matter in which the ground state single-particle (plane-wave) levels with $|\vec{k}| \leq k_L$ are occupied by spin-up, spin-down pairs of nucleons while levels with $k_L \leq |\vec{k}| \leq k_F$ are occupied by one nucleon and one proton with spin up. The parameter $D = k_L/k_F$ then provides a measure of the degree of spin saturation. The density of nuclear matter in this model can be written in the form:

$$\rho = \frac{(1 + D^3)}{3\pi^2} k_F^3 . \qquad (1)$$

The effective potential takes the following form[6).]

$$V_{eff}(r) = \tfrac{1}{2} \left\{ V_{real}(r) \exp\left[-\vec{a} \cdot \frac{\partial}{\partial \vec{r}}\right] + \exp\left[\vec{a} \cdot \frac{\partial}{\partial \vec{r}}\right] V_{real}(r) \right\}_{\vec{a} \to \vec{r}} + \lambda(A) \frac{\hbar^2}{M} \left\{ \delta(\vec{r}) \nabla^2 + \nabla^2 \delta(\vec{r}) \right\} , \qquad (2)$$

where $V_{real}(r)$ is the initial realistic potential parametrized in accordance with the free two-nucleon problem. The realistic part was considered first to be of the Yukawa form (VY) with parameters determined from the free nucleon-nucleon scattering at low energies with identical parameters for the singlet and triplet central forces,

$$V_{real}(r) = - V_0 \frac{e^{-\mu r}}{\mu r} , \qquad (3)$$

where $V_0 = 48.1$ MeV , $\mu = 0.86$ fm.

Afterwards, the realistic part was considered by Dzhibuti and Sallam to be of the Gaussian form (VG)

$$V_{real}(r) = \left[a_\tau (\vec{\tau}_1 \cdot \vec{\tau}_2) + a_{\sigma\tau} (\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2) \right] e^{-r^2/r_c^2}$$

with $a_\tau = 2.096$ MeV, $a_{\sigma\tau} = 7.767$ MeV and $r_c = 2.18$ fm.

The ground state energy of nuclear matter with degree of spin saturation D, in the case of VY interaction is:

$$E(k_F,D)/A = \frac{3}{10} \frac{\hbar^2}{M} \frac{(1+D^5)}{(1+D^3)} k_F^2 + \frac{1}{20} \frac{\lambda}{\pi^2} \frac{\hbar^2}{M} \frac{(1+2D^3+2D^5+D^8)}{(1+D^3)} k_F^5 - \frac{12 V_0}{\mu\pi} k_F \left[\frac{1}{(1+D^3)} \int_0^\infty j_1^2 \left(\frac{k_F r}{2}\right) \frac{e^{-\mu r}}{r} dr + \frac{4D^2}{(1+D^3)} \int_0^\infty j_1\left(\frac{Dk_F r}{2}\right) \times j_1\left(\frac{k_F r}{2}\right) \frac{e^{-\mu r}}{r} dr + \frac{D^4}{(1+D^3)} \int_0^\infty j_1^2 \left(\frac{Dk_F r}{2}\right) \frac{e^{-\mu r}}{r} dr \right] \qquad (4)$$

and for VG interaction is:

$$E(k_F,D)/A = \frac{3}{10} \frac{\hbar^2}{M} \frac{(1+D^5)}{(1+D^3)} k_F^2 + \frac{1}{20} \frac{\hbar^2}{M} \frac{\lambda}{\pi^2} \frac{(1+2D^2+2D^5+D^8)}{(1+D^3)} k_F^5 - \frac{36 k_F}{\pi} \frac{4 a_{\sigma\tau} D^2}{(1+D^3)} \int_0^\infty j_1\left(\frac{Dk_F r}{2}\right) j_1\left(\frac{k_F r}{2}\right) e^{-r^2/r_c^2} dr + \frac{(a_\tau + a_{\sigma\tau})}{(1+D^3)} \left\{ D^4 \int_0^\infty j_1^2\left(\frac{Dk_F r}{2}\right) e^{-r^2/r_c^2} dr + \int_0^\infty j_1^2\left(\frac{k_F r}{2}\right) e^{-r^2/r_c^2} dr \right\} \qquad (5)$$

for the Skyrme interaction see Ref.. (1).

The incompressibility of nuclear matter can be calculated using the relation

$$K = k_F^2 \frac{\partial^2 (E/A)}{\partial k_F^2} . \qquad (6)$$

The effect of degree of spin saturation D on both the energy and the incompressibility will be discussed in section III.

2.2 <u>Theory of spin saturation at finite temperatures</u>

Following the procedure given by Kupper et al.[10)] we can write the integral energy per particle in the form:

$$e(T,\rho) = e(T=0,\rho) + \frac{1}{\rho} \int_0^T dT' \, C_v(T') , \qquad (7)$$

where C_v is the specific heat,

$$C_v = \rho T \frac{\partial S}{\partial T} \qquad (8)$$

and the entropy per particle S can be written in the form

$$S = \pi^2 \frac{m^*}{\hbar^2 k_F^2} \frac{1+D}{1+D^3} T . \qquad (9)$$

m* is the effective mass. $e(T = 0, \rho)$ may be given by equations (4) and (5). The free energy can be defined as

$$
\mathcal{f} = e(T, \rho) - ST \tag{10}
$$

The pressure can be calculated from the relation:

$$
P = \rho^2 \left(\frac{\partial \mathcal{f}}{\partial \rho} \right)_T \tag{11}
$$

The pressure for VG interaction can be written as[6)]

$$
\begin{aligned}
P = \Bigg\{ & \frac{2 k_f^4}{9 \pi^2} \Bigg(\frac{3}{5} \cdot \frac{\hbar^2}{M} \frac{(1 + D^5)}{(1 + D^3)} k_f \\
& + \frac{1}{4} \frac{\lambda}{\pi^2} \frac{\hbar^2}{M} \frac{(1 + 2 D^3 + 2 D^5 + D^8)}{(1 + D^3)} k_f^4 \\
& - \frac{36}{\pi} \Bigg[\frac{4 a_{\sigma\tau} D^2}{(1 + D^3)} \Bigg[\int_0^\infty J_1(D \frac{k_f r}{2}) J_1(\frac{k_f r}{2}) e^{-r^2/r_c^2} dr \\
& + \frac{1}{6} \int_0^\infty r k_f J_1 (D \frac{k_f r}{2}) J_o (\frac{k_f r}{2}) e^{-r^2/r_c^2} dr \\
& - \frac{1}{3} \int_0^\infty r k_f J_1 (D \frac{k_f r}{2}) J_2 (\frac{k_f r}{2}) e^{-r^2/r_c^2} dr \\
& - \frac{1}{6} \int_0^\infty Dr k_f J_1(\frac{k_f r}{2}) J_o (D \frac{k_f r}{2}) e^{-r^2/r_c^2} dr \\
& - \frac{1}{3} \int_0^\infty Dr k_f J_1(\frac{k_f r}{2}) J_2 (D \frac{k_f r}{2}) e^{-r^2/r_c^2} dr \Bigg] \\
& + \frac{(a_\tau + a_{\sigma\tau}) D^4}{(1 + D^3)} \Bigg[\int_0^\infty J_1^2(D \frac{k_f r}{2}) e^{-r^2/r_c^2} dr \\
& + \frac{1}{3} \int_0^\infty Dr k_f J_1(D \frac{k_f r}{2}) J_o(D \frac{k_f r}{2}) e^{-r^2/r_c^2} dr \\
& - \frac{2}{3} \int_0^\infty Dr k_f J_1(D \frac{k_f r}{2}) J_2(D \frac{k_f r}{2}) e^{-r^2/r_c^2} dr \Bigg] \\
& + \frac{(a_\tau + a_{\sigma\tau})}{(1 + D^3)} \Bigg[\int_0^\infty J_1^2(\frac{k_f r}{2}) e^{-r^2/r_c^2} dr
\end{aligned}
$$

$$
\begin{aligned}
& + \frac{1}{3} \int_0^\infty r k_f J_1(\frac{k_f r}{2}) J_o(\frac{k_f r}{2}) e^{-r^2/r_c^2} dr \\
& - \frac{2}{3} \int_0^\infty r k_f J_1(\frac{k_f r}{2}) J_2(\frac{k_f r}{2}) e^{-r^2/r_c^2} dr \Bigg]\Bigg]\Bigg) \\
& + \frac{2}{9} \frac{M}{\hbar^2} k_f \frac{(1 + D)}{(1 + D^3)} (KT)^2 \Bigg\} .
\end{aligned} \tag{12}
$$

For VY interaction, we get

$$
\begin{aligned}
P = \Bigg\{ & \frac{2 k_f^4}{9 \pi^2} \Bigg(\frac{3}{5} \frac{\hbar^2}{M} \frac{(1 + D^5)}{(1 + D^3)} k_f \\
& + \frac{1}{4} \frac{\lambda}{\pi^2} \frac{\hbar^2}{M} \frac{(1 + 2 D^3 + 2 D^6 + D^8)}{(1 + D^3)} k_f^4 \\
& - \frac{12 V_o}{\mu \pi} \Bigg[\frac{1}{(1 + D^3)} \Bigg[\int_0^\infty J_1^2 (\frac{k_f r}{2}) \frac{e^{-\mu r}}{r} dr \\
& + \frac{1}{3} \int_0^\infty k_f J_1(\frac{k_f r}{2}) J_o(\frac{k_f r}{2}) e^{-\mu r} dr \\
& - \frac{2}{3} \int_0^\infty k_f J_1(\frac{k_f r}{2}) J_2(\frac{k_f r}{2}) e^{-\mu r} dr \Bigg] \\
& + \frac{D^4}{(1 + D^3)} \Bigg[\int_0^\infty J_1^2(D \frac{k_f r}{2}) \frac{1}{r} e^{-\mu r} dr \\
& + \frac{1}{3} \int_0^\infty D k_f J_1(D \frac{k_f r}{2}) J_o(D \frac{k_f r}{2}) e^{-\mu r} dr \\
& - \frac{2}{3} \int_0^\infty D k_f J_1(D \frac{k_f r}{2}) J_2(D \frac{k_f r}{2}) e^{-\mu r} dr \Bigg] \\
& + \frac{4 D^2}{(1 + D^3)} \Bigg[\int_0^\infty J_1(D \frac{k_f r}{2}) J_1 (\frac{k_f r}{2}) \frac{e^{-\mu r}}{r} dr \\
& + \frac{1}{6} \int_0^\infty k_f J_1 (D \frac{k_f r}{2}) J_o(\frac{k_f r}{2}) e^{-\mu r} dr \\
& + \frac{1}{6} \int_0^\infty D k_f J_1(\frac{k_f r}{2}) J_o(\frac{D k_f r}{2}) e^{-\mu r} dr \\
& - \frac{1}{3} \int_0^\infty k_f J_1 (\frac{D k_f r}{2}) J_2 (\frac{k_f r}{2}) e^{-\mu r} dr \\
& - \frac{1}{3} \int_0^\infty D k_f J_1(\frac{k_f r}{2}) J_2(\frac{D k_f r}{2}) e^{-\mu r} dr \Bigg]\Bigg]\Bigg) \\
& + \frac{2}{9} \frac{M}{\hbar^2} k_f \frac{(1 + D)}{(1 + D^3)} (KT)^2 \Bigg\}
\end{aligned} \tag{13}
$$

and for Skyrme interaction, we get,

$$P = \left\{ \frac{2 k_f^4}{9\pi^2} \left[\frac{3}{5} \frac{\hbar^2}{M} \frac{(1 + D^5)}{(1 + D^3)} k_f + \frac{3}{4} \frac{t_0}{\pi^2} \left\{ \tfrac{1}{2} (1 + D^3) + \frac{1}{3} (x_0 - \tfrac{1}{2}) \frac{(1 - D^3)^2}{(1 + D^3)} \right\} k_f^2 + \frac{1}{20\pi^2} \left\{ (3 t_1 + 5 t_2)(1 + D^5) + (t_2 - t_1) \frac{(1 - D^5)(1 - D^3)}{(1 + D^3)} \right\} k_f^5 + \frac{t_3 D^3}{6\pi^4} k_f^5 \right] + \frac{2}{9} \frac{M}{\hbar^2} k_f \frac{(1 + D)}{(1 + D^3)} (KT)^2 \right\} . \tag{14}$$

III. RESULTS AND DISCUSSION

The binding energy per particle as a function of k_F and for different degrees of spin saturation D, is calculated using equations (4) and (5) for VY and VG interactions with λ = 1.1 and 3.9 F^3, respectively and also for Skyrme interaction. The results are displayed in Figs. (1) and (2) for VY and VG. From these figures we notice that the binding energy increases by increasing D for both interactions. It is interesting to notice that the equilibrium Fermi momentum shows a maximum for both interactions. The stability of the spin saturated ground state is tested by applying the condition[1]:

$$\left[\frac{\partial^2 (E/A)}{\partial k_F \, \partial D} \right]^2 - \frac{\partial^2 (E/A)}{\partial k_F^2} \frac{\partial^2 (E/A)}{\partial D^2} \leq 0 . \tag{15}$$

This condition is satisfied for VY interaction at D = 1 and $k_F = 1.8\ F^{-1}$ and at D = 1, $k_F = 1.36\ F^{-1}$ for VG interaction. Concerning the Skyrme interaction Chang[1] has shown that the interactions SKI, III and VI, which were found to yield an unstable spin-saturated HF ground state of ^{16}O and to permit overbinding due to spin realignment in N[14], violate the condition (15) for the stability of the spin-saturated ground state of nuclear matter. These interactions are those with the strong three-body term that seems to be required to reproduce the observed density of single-particle levels near the Fermi surface of spherical nuclei[13]. In particular the spin-stability condition is violated by the interaction SK III that is in best overall accord with experiment.

The incompressibility has been calculated using equations (4), (5) and equation (4) in Ref. (1) with equation (6). The results are given in table 1. We note that the incompressibility increases with increasing D for VG, VY, SK IV and V interactions where they obey spin saturation condition (15). However SK III shows an opposite behaviour.

The equations of state p = p (V,T,D), ($V = \frac{1}{\rho}$) for nuclear matter are calculated using equation (11) for VG, VY and Skyrme interactions. The effective mass m* which appears in equation (9) is considered to be equal to the free particle mass. This choice of effective mass is justified by the fact that the effective mass at the Fermi surface in nuclear matter may be well approximated by the free nucleon mass. This is in agreement with the results of the investigation of heavy nuclei[14]. The isothermal curves are drawn in Figs. (3) and (4) for VG and VY interactions (which represent the family of interactions that give spin saturation). Figure (5) shows that of SK III (one of the family of interactions which does not give spin saturation). The two families resemble formally the classical Van der Waals isotherms of classical gas-fluid systems[9,10,15]. The isotherms of low temperature have two points of intersection with the v-axis. By increasing the temperature, the curves are shifted upwards. Also, we notice that in both cases, the pressure decreases with increasing the degree of spin saturation D. However the minimum in the first case (VG and VY family) is not so pronounced as in the second case (SK III family). Also the minimum in the first case decreases with decreasing D which is opposite to the behaviour of the second case.

It is worth mentioning that from those figures we notice that the effect of decreasing the degree of spin saturation resembles the effect of increasing the temperature. This happens only with potentials which obey the stability condition (equation (15)).

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TABLE 1

Potential \ D	K(MeV)			
	0.7	0.8	0.9	1.0
VG	150	215	235	245
VY	97	160	215	225
SK I	-	-	360	345
SK II	-	325	350	320
SK III	-	400	370	335
SK IV	240	300	310	330
SK V	155	210	270	325
SK VI		380	360	305

Values of the incompressibility K at different values of the degree of spin saturation D for Skyrme, VY and VG interactions.

Fig. (1): Binding energy per particle versus k_F at different values of D for VY interaction.

Fig. (2): Binding energy per particle versus k_F at different values of D for VG interaction.

Fig. (3): Nuclear matter isotherms in the (v,p_D) diagram for VY interaction for different degrees of spin saturation D.

Fig. (4): Nuclear matter isotherms in the (v,p_D) diagram for VG interaction for different degrees of spin saturation D.

Fig. (5): Nuclear matter isotherms in the (v,p_D) diagram for SK III interaction for different degrees of spin saturation D.

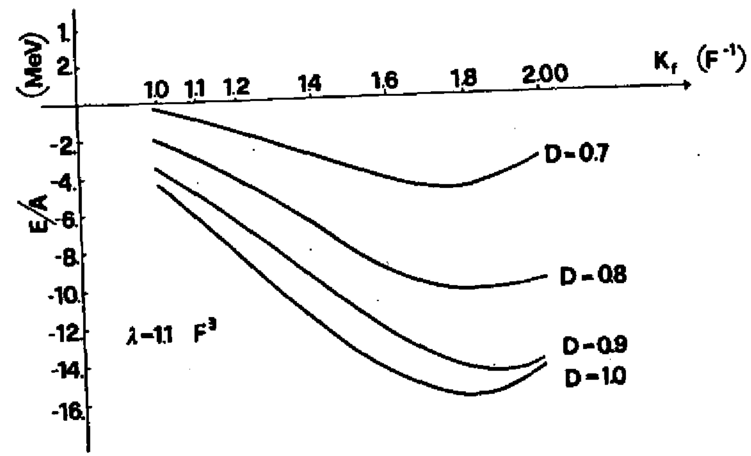
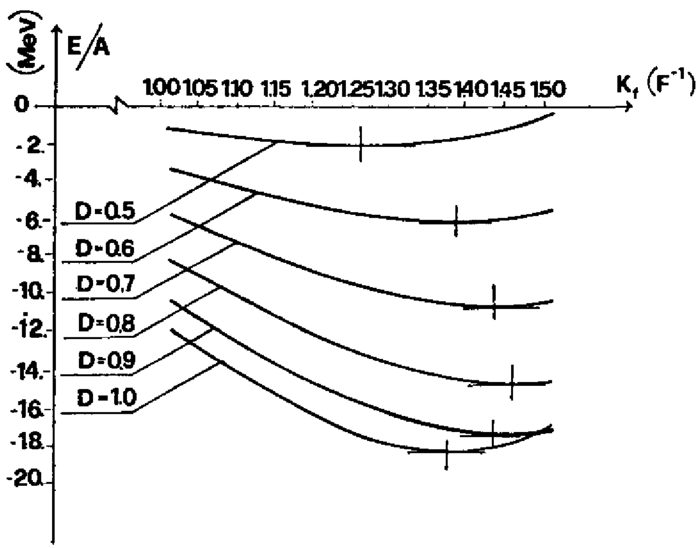
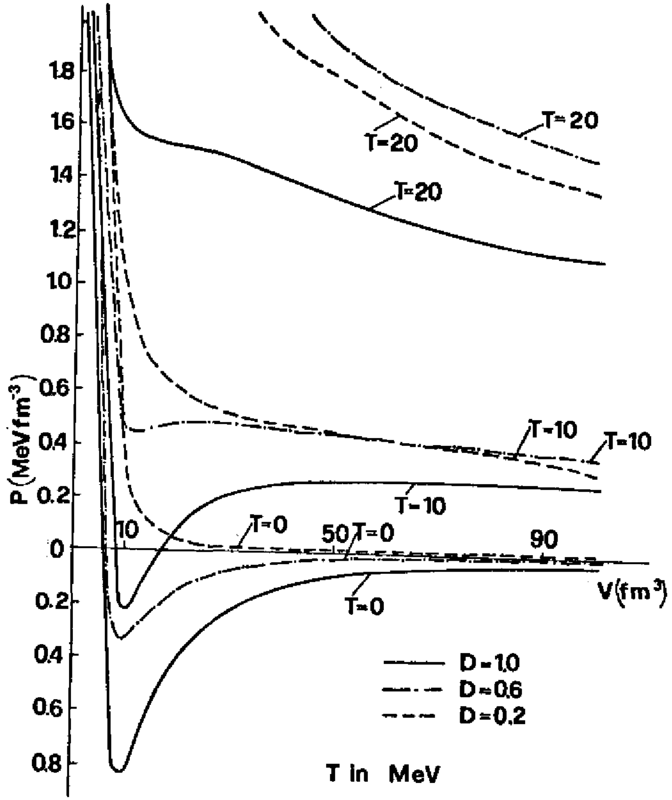


FIG. 1



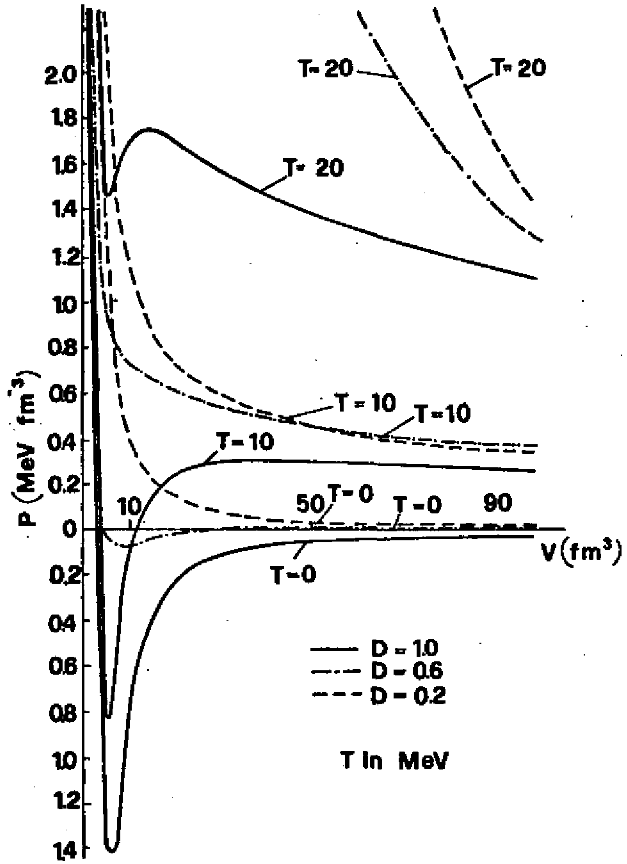
(MeV)
E/A
K_f (F^{-1})
1.00 1.05 1.10 1.15 1.20 1.25 1.30 1.35 1.40 1.45 1.50
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
D=0.5
D=0.6
D=0.7
D=0.8
D=0.9
D=1.0

FIG. 2



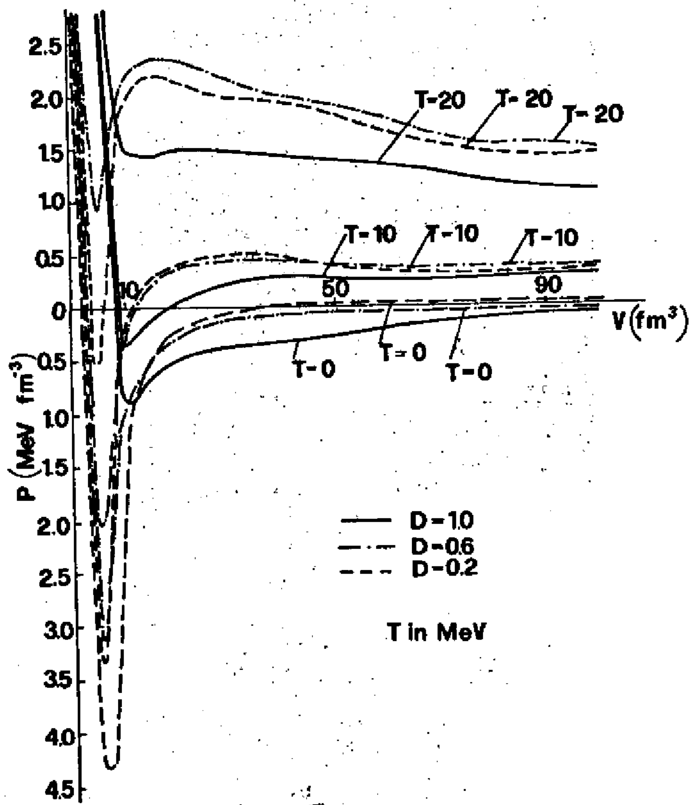
P (MeV fm^{-3})
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
T=20
T=20
T=20
T=10
T=10
T=10
T=0
T=0
T=0
10
50
90
V(fm^3)
D=1.0
D=0.6
D=0.2
T in MeV

FIG. 3



T= 20
T= 20
T= 20
T = 10
T=10
T= 10
T= 0
T =0
T-0
10
50
90
P (MeV fm^{-3})
V (fm^3)
D = 1.0
D = 0.6
D = 0.2
T in MeV

FIG. 4



T=20
T- 20
T- 20
T=10
T-10
T-10
T-0
T=0
T-0
10
50
90
P (MeV fm^{-3})
V (fm^3)
D=1.0
D=0.6
D=0.2
T in MeV

FIG. 5