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Automatic Camera Tracking for Remote Manipulators

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AUTOMATIC CAMERA TRACKING FOR REMOTE MANIPULATORS*

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ABSTRACT. The problem of automatic camera tracking of mobile objects is addressed with specific reference to remote manipulators and using either fixed or mobile cameras. The technique uses a kinematic approach employing 4 × 4 coordinate transformation matrices to solve for the needed camera PAN and TILT angles. No vision feedback systems are used, as the required input data are obtained entirely from position sensors from the manipulator and the camera-positioning system. All hardware requirements are generally satisfied by currently available remote manipulator systems with a supervisory computer.

The system discussed here implements linear plus on/off ("bang-bang") closed-loop control with a ±2-deg deadband. The deadband area is desirable to avoid operator "seasickness" caused by continuous camera movement. Programming considerations for camera control, including operator interface options, are discussed. The example problem presented is based on an actual implementation using a PDP 11/34 computer, a TeleOperator Systems SM-229 manipulator, and an Oak Ridge National Laboratory (ORNL) camera-positioning system.

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1. INTRODUCTION

Servomanipulators are used for remote maintenance and repair work in hazardous and/or inaccessible environments such as high radiation, under water, or outer space. These systems conist of similar master and slave manipulators. Each manipulator has two 6-degree-of-freedom (DOF) arms with end effectors. As the operator moves the master arms the slave arms follow the motions of the master in real time. Remote closed-circuit television (CCTV) cameras send pictures of the working environment back to television screens for the operator to view.

This paper presents a technique for driving these cameras to follow automatically the motion of the slave end effector. As the main use for the cameras is viewing the actions of the end effector, automatic tracking significantly reduces the burden placed on the operator. Continuous work is possible, as intermittent camera adjustments are no longer needed. The technique is a combination of software transformations to determine end-effector location, and hardware servo-activation to position the cameras properly.

The TeleOperator Systems[1] (TOS) Model SM-229 servomanipulator with boom-mounted onboard cameras is shown pictorially in Fig. 1 and schematically in Fig. 2. This system is in operation at the ORNL Remote Systems Development Facility (RSDF), where the technique described here was developed. The servomanipulator has two servo-actuated mechanical arms mounted on the telescoping hoist of a three-axis positioner, thus allowing the arms to reach anywhere within the remote environment. Two cameras are also mounted on the hoist tube. The camera positioner can extend and rotate about the hoist axis to position the camera base as well as pan and tilt to direct the camera orientation. These motions are shown in Fig. 2.

II. KINEMATICS

The kinematics are handled systematically by expressing each motion and system parameter (see Table 1), whether it is a constant distance transfer or a variable angle, in terms of a 4 × 4 transformation matrix.[2] In this manner, different paths starting and ending at arbitrary common points can be described in matrix form. Since both paths have the same starting and ending point, the matrix elements representing the x, y, and z positions of each transformation are equal. This yields three equations, which can be used to solve for the camera PAN and TILT angles, and also the distance from the camera to the end effector.

The position of the end effector relative to the hoist axis (shoulder pivot elevation) can be described by a path through the manipulator (see Fig. 3). Given a starting position and orientation on the hoist axis, this path begins with a (variable) rotation about the hoist (x) axis (shoulder rotation), followed by a constant distance transfer (d_{1z}) to the shoulder pivot point. Next is a variable rotation (shoulder pitch) about the Z_0 axis, Z_0 followed by a (constant) 90-deg rotation about the X_0 axis. The purpose of this is merely to shift the reference axes in such a way that the next variable rotation (in this case, the shoulder roll angle) is about the new (Z_1) axis.

The path proceeds with a variable rotation (shoulder roll) about the Z_1 axis, a constant distance trans (d_2) to the elbow pivot, and another 90-de rotation to align the new Z_2 axis with the elb pitch axis.

Table 1. Parameter definitions

Symbol*	Definition	Distance (in.)
Parameters		
d_{1y}	Hoist axis to shoulder pivot (y-dir)	6.0
d_{1z}	Hoist axis to shoulder pivot (z-dir)	13.5
d_2	Manipulator upper arm length	22.00
d_3	Offset of forearm axis from elbow pivot	2.5
d_4	Forearm length	26.5
d_6	Tong gripper length	6.5
X_1	X dir offset of Camera 1 axis from shoulder pivot	9.0
X_2	X dir offset of Camera 2 axis from shoulder pivot	0.0
Y_1	Y dir offset of Camera 1 axis from shoulder pivot	9.75
Y_2	Y dir offset of Camera 2 axis from shoulder pivot	9.75
d_y	Offset of camera axis from camera pivot	7.25

Symbol	Definition	Range
z_1	Camera 1 extension from hoist axis	(33-1/2 in. to 47-1/8 in.)
z_2	Camera 2 extension from hoist axis	(28-7/8 in. to 42-1/4 in.)
ROT1	Camera 1 rotation about hoist axis	(±180 deg)
ROT2	Camera 2 rotation about hoist axis	(±180 deg)
SHRT	Rotation of shoulder about hoist axis	(±180 deg)
θ_1	Shoulder pitch (150 deg, -100 deg)	
θ_2	Shoulder roll (±45 deg)	
θ_3	Elbow pitch (+50 deg, -135 deg)	
θ_4	Elbow roll (YAW) (±180 deg)	
θ_5	Wrist pitch (+120 deg, -45 deg)	
θ_6	Wrist roll (360 deg)	

*See Fig. 2.

Each of the four remaining degrees of freedom (DOFs) of the manipulator arm are handled similarly; that is, a variable rotation about the Z axis, a constant distance transfer (to the axis of the next DOF), and a constant rotation to align the new Z axis with the axis of rotation of the next DOF. It is important to note that after the final DOF (wrist roll), a constant rotation is made which is equal and opposite to the sum of all the previous constant rotations. Thus, the net shift of the reference axis is zero, and the final reference axes are realigned with the initial reference axes.

The position of the end effector relative to the hoist axis can be described by a path through the camera positioner and along the camera axis to the end effector (see Fig. 4). Given the same starting position and orientation as the previous path, this path begins with a variable rotation about the hoist (x) axis (the camera rotation angle). Note that as the shoulder and camera rotation are about the same axis, the relative angle can be substituted for shoulder rotation angle, and this step can be eliminated.

A distance transfer to the camera pivot point follows. Referring to Fig. 4, note that the camera arm can extend a variable distance Z_2. The variable PAN rotation about the x axis follows, and then a variable rotation about the z axis. With the proper PAN and TILT angles, the camera is now pointing directly at the end effector. For this system, the camera pivot point is a short distance off the camera axis. Thus, this path is concluded with a constant distance transfer to the camera axis, and an unknown distance transfer along the camera axis to the end effector.

The analytical problem can be stated as follows: Given the values of the parameters and variables of the manipulator and camera positioner, what PAN and TILT angles are needed to aim the camera at the end effector?

The common starting point for the two paths to the end effector was arbitrarily chosen as the hoist axis at shoulder pivot elevation in order to conveniently separate the kinematics of the manipulator and the camera positioner. The equations for the needed PAN and TILT angles are obtained by selecting a common starting point at the camera pivot point, with the TILT angle associated with one path and a PAN angle associated with the second path. This uncouples the unknowns when the x, y, and z positions of each path are equated. The kinematics equations are derived and solved in detail in Appendix A.

III. IMPLEMENTATION

Requirements for implementation of the tracking algorithm include appropriate equipment, a position control algorithm, user friendly man/machine interfacing, and the design of a real-time camera control software package as described below.

The position information for each independent variable is needed and can be obtained easily from potentiometers. Rotary potentiometers are standard equipment on the TOS SM 229, and are also provided on the camera-positioning system. A host computer with analog input and output capability is required.

Proper drive signals to the camera pan and tilt drives must be applied in order to achieve the desired camera angles. A simple control algorithm consisting of linear control plus bang-bang control with a deadband was implemented for the following reasons:

1. Because a steady camera is desirable, camera drives generally have friction characteristics that dominate their inertia. Therefore, when the camera drive signal stops, very little additional motion occurs.

2. It is desirable to place a deadband of about ±2 deg around the desired camera position so that small movements of the end effector will not cause camera motion; continuous camera motion can cause the operator to feel "seasick."

3. A pure time delay exists between the time the position data is read and the time the drive signal is sent. The magnitude of the bang-bang drive signal is set so that time delay cannot cause the camera to overshoot the deadband. Note that in this way the camera stops at a random point inside of the deadband, and thus the average error is about half the deadband width.

A block diagram for the control system is shown in Fig. 5.[3]

Four modes were implemented for automatic camera tracking: One-time point-to-point, continuous point-to-point, fully continuous, and voice command. Two main factors were considered in evaluating the different modes. First, excessive camera movement, particularly with rapid directional changes, can cause the operator to feel seasick. Second, the purpose of automatic tracking is to reduce operator work load, and therefore operator switching requirements should be minimized. The four modes of implementing automatic tracking are summarized in Table 2.

In one-time point-to-point tracking, the initiating prompt was the touching of a switch on the camera control console. The system variables were read once, and the camera(s) driven to the calculated PAN and TILT angles. Although this resulted in zero excessive camera movement, operator input was required for every camera adjustment.

Continuous point-to-point tracking was similar to the one-time mode, except that the routine repeated itself upon reaching the last calculated orientation until the switch was turned off. The smooth camera motion and deadband minimized excessive camera movement, and operator input was considerably reduced. It was observed, however, that significant lag occurred between actual and predicted location due to the slow speed of camera positioner motions and the elapsed time between calculations and reaching the actual desired position.

Fully continous tracking was similar to continous point-to-point tracking, except that the desired PAN and TILT angles were updated with each pass through the loop. This allowed the cameras to follow the end effector faster in real time.

Table 2. Man-machine interface (MMI) modes

Mode	Advantages	Disadvantages
One-time point-to-point	No excessive camera movement.	Frequent manual input required.
Continuous point-to-point	Little excessive camera movement; little operator input required.	Slightly reduced real-time speed.
Fully continuous	Optimum real-time speed.	Excessive camera movement.
Voice command	No manual input required; no excessive camera movement.	Reduced real-time speed.

To implement voice command tracking, a voice recognition package was integrated into the camera control software, with one-time point-to-point tracking initiated by the voice command "Camera One track" or "Camera Two track". This offered the distinct advantage of allowing the operator to switch in automatic tracking without releasing the manipulators.

The software for real-time camera control was written in FORTRAN FOUR PLUS and implemented on a PDP 11/34 computing system. The package handles all manipulator transformations and controls for the on-board cameras. A flowchart for the task is shown in Fig. 6.

Communication between the host and the remote environment is provided over a multiplexing link. The initiating prompt and drive signals are sent from the host to an 8085 microprocessor in the remote cell. The 8085 performs the servo-activation of the camera DOFs and returns camera position information and limit status to the host. Voltages from the manipulator position potentiometers are input directly to the PDP 11/34 A/D ports. Input from the operator station includes direct analog voltages from manual joystick potentiometers, and digital inputs from switches. Software switching was designed to allow complete manual override of automatic tracking.

The required calculation time was measured by putting each subroutine in a tight "DO" loop for 500 iterations and using a system subroutine (SECNDS) to find the total time required. It was found that subroutine TRACK, which receives the raw position data and calculates the needed PAN and TILT angles for both cameras, required 0.0115 s per loop. Subroutine AUTDRV, which receives the needed and actual PAN and TILT angles and generates the appropriate drive signals, required only 0.0013 s per loop. Thus, the total calculation time was considerably less than the time delays specified in the code used to slow total loop speed to about 3 Hz. (Three Hz was considered to give satisfactory performance, and slowing the task to this speed allowed greater freedom for other real-time tasks.) Note that this measured calculation time does not include the data acquisition time, which is quite device dependant.

IV. RESULTS

For the TOS SM-229 system with a host PDP 11/34, the calculated PAN and TILT angles for each camera were manually checked by centering the end effector in the corresponding TV screens and requesting the host computer to print the actual and desired PAN and TILT angles. For the full range of manipulator and camera configurations, the average error was 0.9 deg and the maximum error was 2.0 deg. Adding a constant correction factor of up to ±1 deg to each PAN and TILT angle resulted in an average error of 0.4 deg and a maximum error of 1.1 deg, which was considered satisfactory. In fact, this error is much smaller than the deadband values of ±2 deg.

The camera variable calibration was one error source. The pan, tilt, rotate, and extend variables were calibrated using hand-drawn chalk lines between plumbobs, protractors, rulers, and visual sightings along the camera axis. Uncertainties were estimated at about ±2.5 deg, 1.0 deg, 1.5 deg, and 0.5 in. for the pan, tilt, rotate, and extend respectively. Cantilever bending of the camera booms was another source of error. This resulted in a slight position change and angular offset, so that the pan and tilt angles were not truly about the horizontal and vertical axes. Note that this could be measured and included in the kinematic model if desirable. The manipulator arm angles were measured with bubble balance protractors accurate to ±0.5 deg, resulting in minimal uncertainty in these variables.

V. SUMMARY

A successful automatic camera tracking system has been demonstrated at the Remote Systems Development Facility. Utilizing 4 × 4 matrix representation of manipulator and camera locations and orientations, the needed camera PAN and TILT angles are determined in real time. The response of manipulator operators to this new development has been very positive as it reduces both mental and manual operator activity by automating camera positioning tasks. Similar tracking techniques will be applied to future manipulator systems developed by the Remote Control Engineering Group at the Oak Ridge National Laboratory.

BIBLIOGRAPHY

1. TeleOperator Systems Corp., Bohemia, N.Y.

2. R. P. Paul, *Robot Manipulatore*, Cambridge, Mass.: MIT Press, 1981.

3. K. Agata, *Modern Control Engineering*, Englewood Cliffs, N.J.: Prentice-Hall, 1970, p. 540.

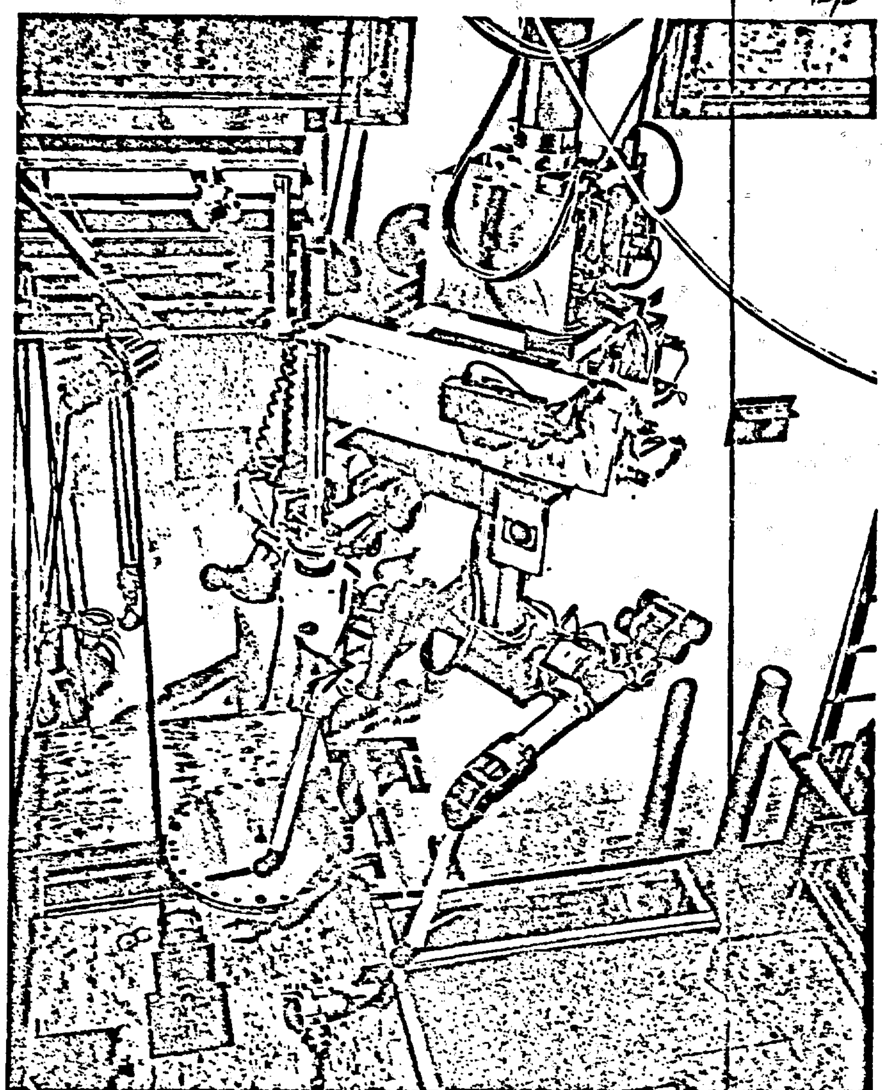

Fig. 1

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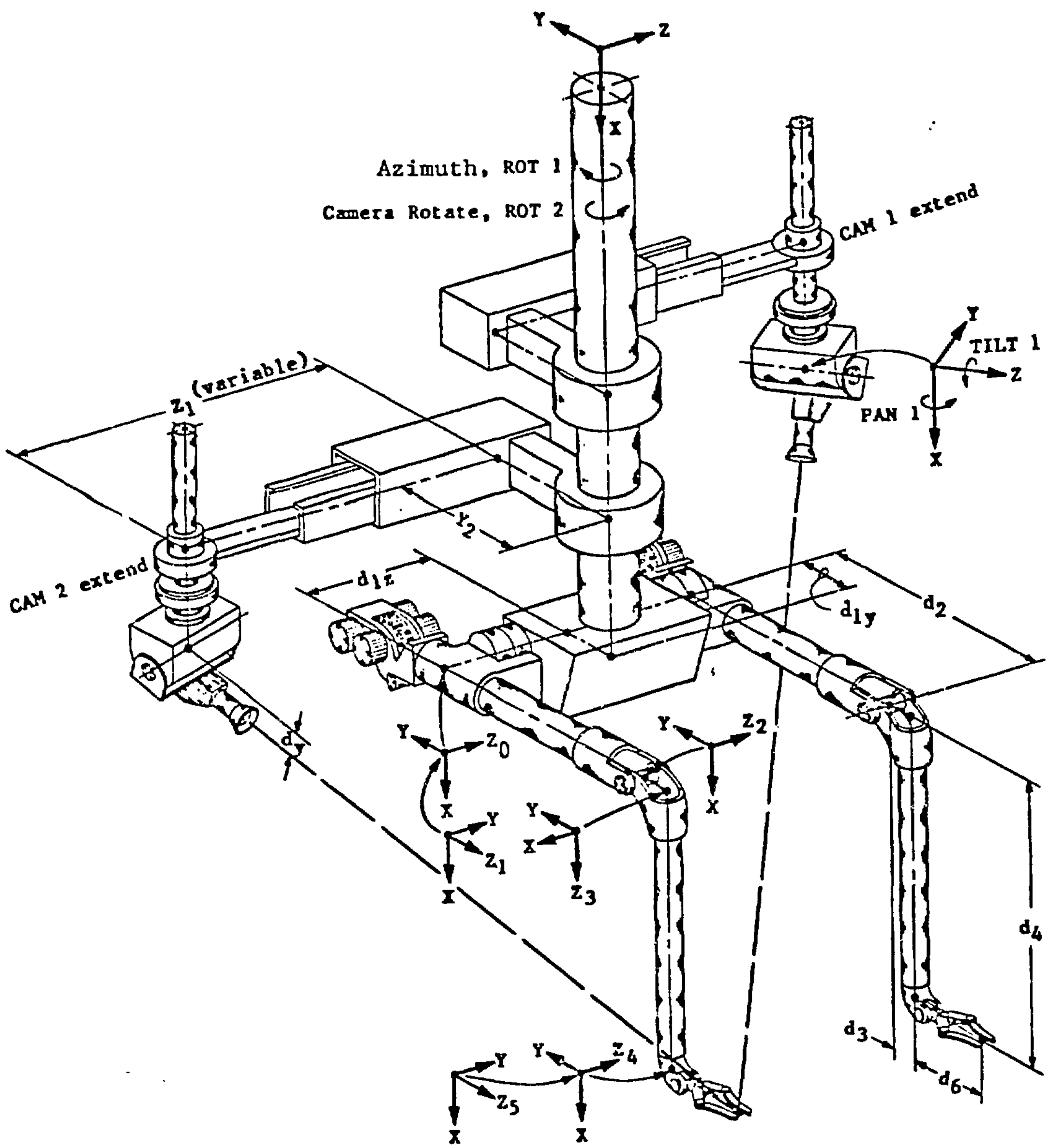

FIG. 2 - KINEMATICS OF SM-229 WITH BOOM MOUNTED CAMERAS.

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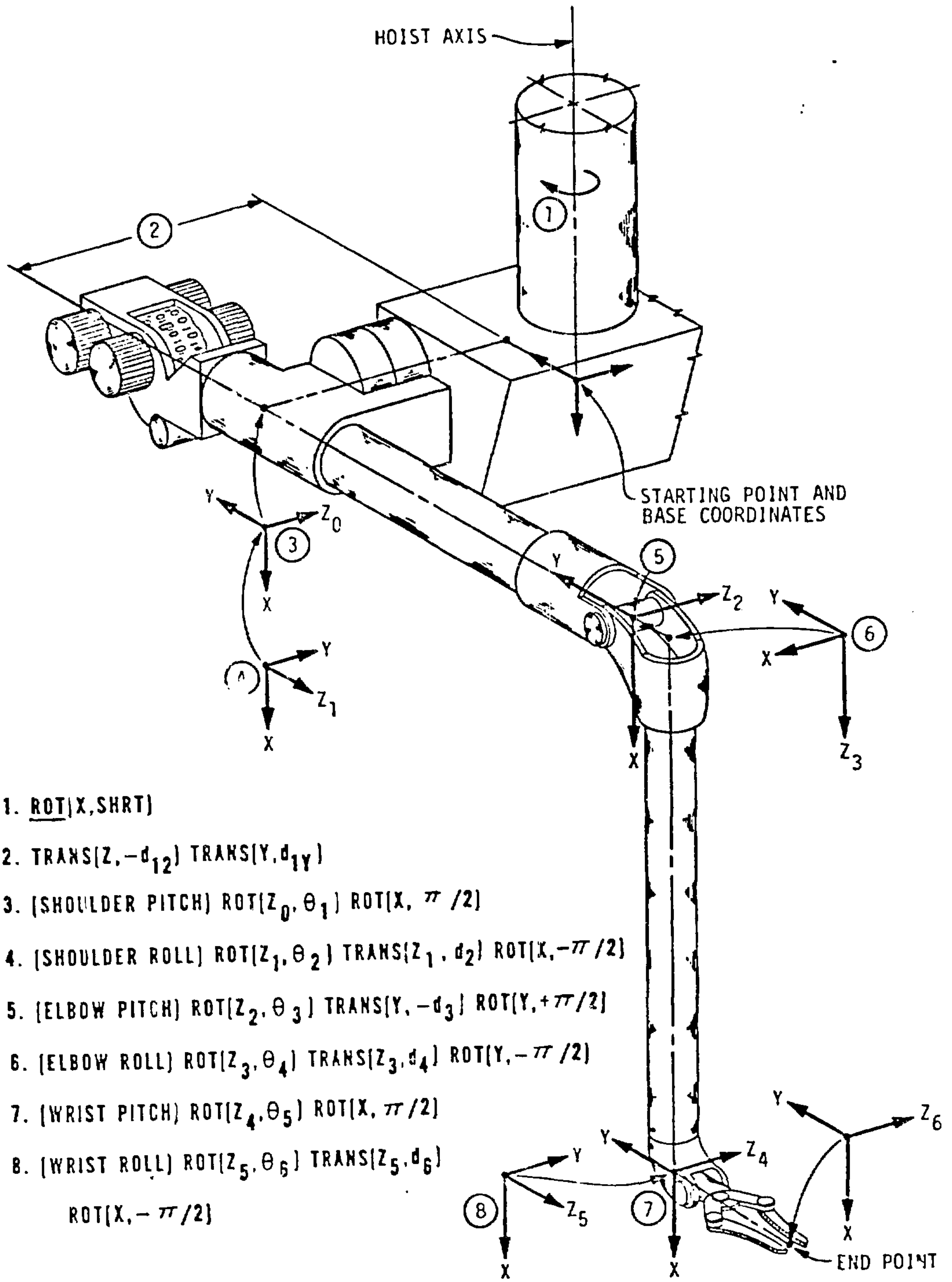

1. ROT(X,SHRT)
2. TRANS$(Z,-d_{12})$ TRANS(Y,d_{1Y})
3. (SHOULDER PITCH) ROT(Z_0,Θ_1) ROT$(X,\pi/2)$
4. (SHOULDER ROLL) ROT(Z_1,Θ_2) TRANS(Z_1,d_2) ROT$(X,-\pi/2)$
5. (ELBOW PITCH) ROT(Z_2,Θ_3) TRANS$(Y,-d_3)$ ROT$(Y,+\pi/2)$
6. (ELBOW ROLL) ROT(Z_3,Θ_4) TRANS(Z_3,d_4) ROT$(Y,-\pi/2)$
7. (WRIST PITCH) ROT(Z_4,Θ_5) ROT$(X,\pi/2)$
8. (WRIST ROLL) ROT(Z_5,Θ_6) TRANS(Z_5,d_6)

   ROT$(X,-\pi/2)$

FIG. 3 - PATH 1 THRU MANIPULATOR ARM.

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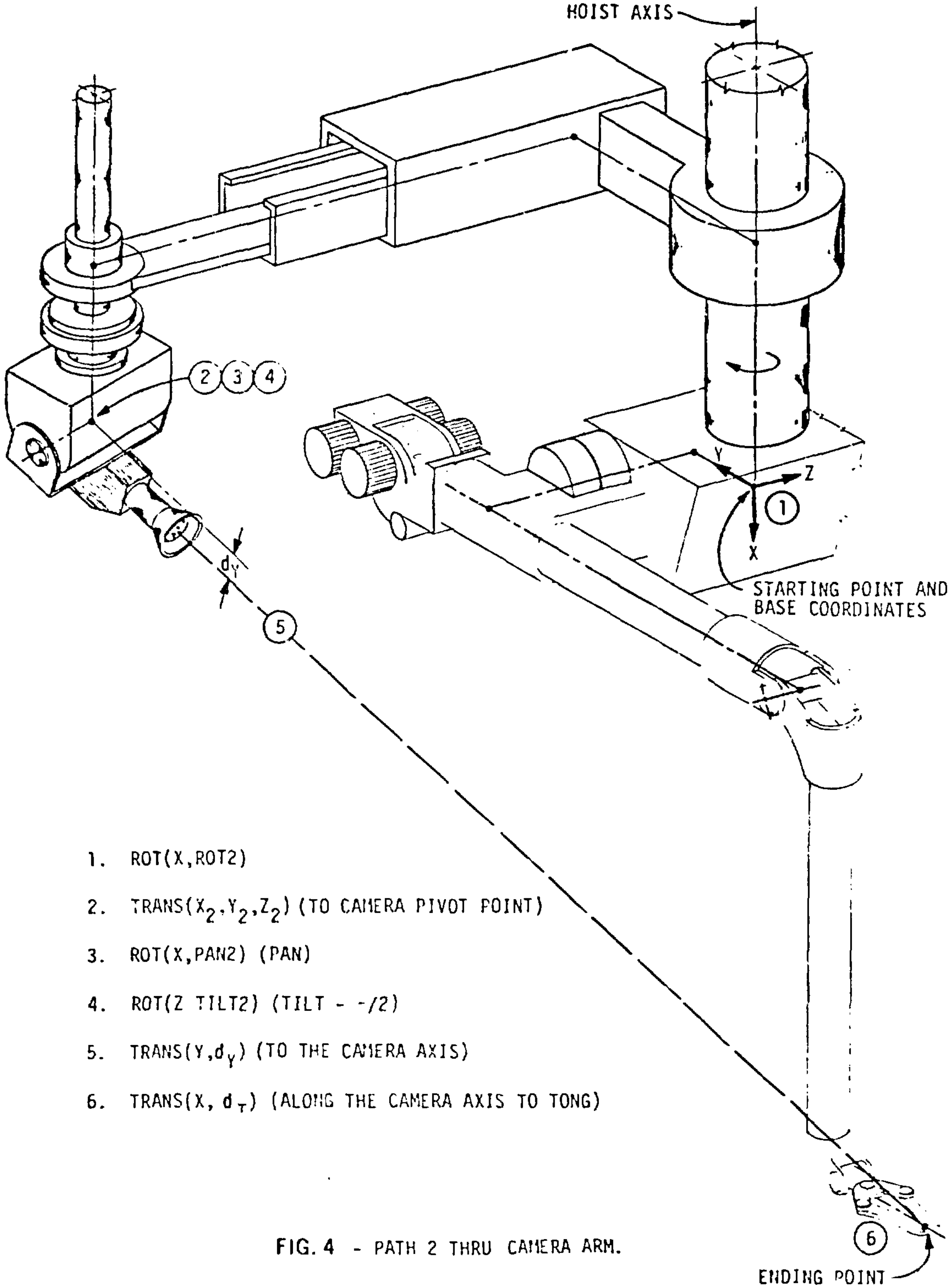

1. ROT(X,ROT2)
2. TRANS(X_2,Y_2,Z_2) (TO CAMERA PIVOT POINT)
3. ROT(X,PAN2) (PAN)
4. ROT(Z TILT2) (TILT - -/2)
5. TRANS(Y,d_Y) (TO THE CAMERA AXIS)
6. TRANS(X, d_T) (ALONG THE CAMERA AXIS TO TONG)

FIG. 4 - PATH 2 THRU CAMERA ARM.

Fig. 5
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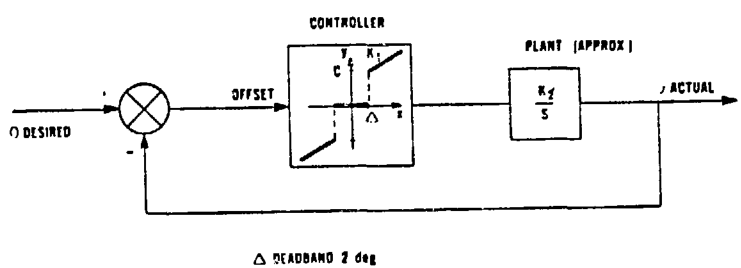

Fig 5. Closed-loop Remote manipulator control [3]

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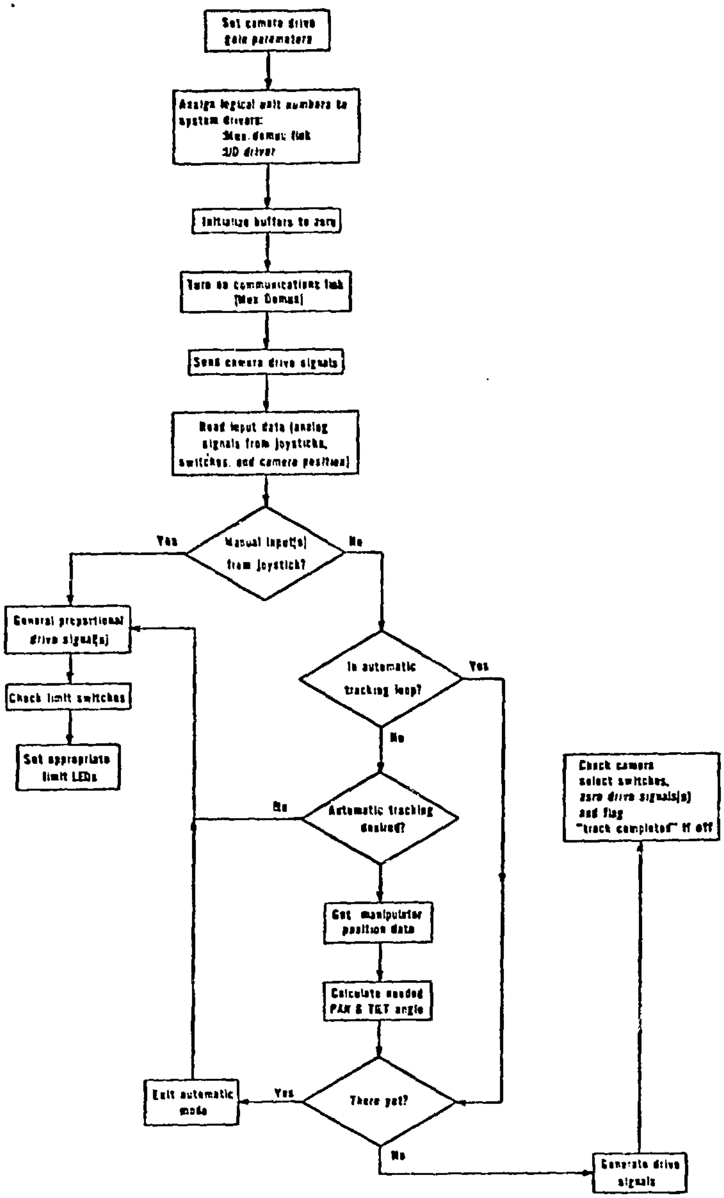

Fig. 6. Camera control program for RSDF system.

APPENDIX A

This appendix contains the equations for the PAN and TILT angles for the RSDF system with the TOS Model SM-229 manipulator.

Notation and Nomenclature:

$Rot(x,\theta_n)$ = rotation about the X axis an angle θ_n

$Trans(d_x,d_y,d_z)$ = translation a distance d_x in the x direction, d_y in the y direction, and d_z in the z direction

d_{1y} = hoist axis to shoulder pivot distance (y direction)

d_{1z} = hoist axis to shoulder pivot distance (z direction)

d_2 = shoulder pivot - elbow pivot distance, inches

d_3 = forearm axis offset from elbow, inches d_4 = elbow pivot - wrist pivot distance along forearm, inches

d_4' = wrist offset from forearm axis, inches

d_6 = wrist pivot - tong end distance, inches

X1, Y1, Z1 = x, y, and z distances from the hoist axis at shoulder pivot elevation to the camera one pivot point (all positive)

X2, Y2, Z2 = same as above but for Camera 2

PAN1, PAN2 = pan angles for Cameras 1 and 2

TILT1, TILT2 = tilt angles - π/2 for cameras 1 and 2

d_y = offset of camera axis from camera pivot point (same for both cameras)

d_T = distance from camera to tong gripper

ROT1, ROT2 = rotation angles for cameras 1 and 2

SHRT = rotation of the shoulder

S_x, C_x = SIN(x), COS(x)

T_6 = total manipulator arm transformation matrix

T_x,T_yT_z = x, y, and z position of the tong relative to the shoulder pivot as determined in $T_6 = (T_{14},T_{24},T_{34})$

DERIVATION OF MANIPULATOR TRANSFORM, T_6

Each motion of the arm is represented by an "A" transform. This transform, a 4 × 4 matrix, represents one independent motion. It may also represent characteristics of the physical system, such as the length of one of the linkages, and/or mathematical manipulations which simplify the model, such as a shifting of the reference axes.

The following describes the calculations required to determine the position and orientation of the SH-229 manipulator.

Transform A_n represents the following process:

1) Rotation about the Z_{n-1} axis an angle Θ_n

2) Translation along the Z_{n-1} axis a distance d_n

3) Transform A_y only: Translate along the Y_{n-1} axis a distance d_4^*

4) Rotation of the reference axes an angle α_n about the x axis OR an angle β_n about the y axis. [move left?]

The A transforms are determined as follows:

$$A_3 = Rot(z,\Theta_3)\ Trans(0,-d_3,0)\ Rot(y,\pi/2)$$

The term $Rot(y, \pi/2)$ represents the angle β given in Table A.1. An angle α would correspond to $Rot(x,\alpha)$. These terms are included only for consistency, so that all variable rotations are about the Z axis (helpful when modeling system dynamics).

Table A.1. Transformation parameters

Transform	Variable	α	β	d	Motion
1	θ_1	$\pi/2$	0	0	Shoulder pitch
2	θ_2	$-\pi/2$	0	d_2	Shoulder roll
3	θ_3	0	$\pi/2$	$-d_3$	Elbow pitch
4	θ_4	0	$-\pi/2$	$d_4, -d_4'$	Elbow roll*
5	θ_5	$\pi/2$	0	0	Wrist pitch
6	θ_6	$-\pi/2$	0	d_6	Wrist roll

*d_4' is in the y direction.

$$A_3 = \begin{matrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \quad X \quad \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -d_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \quad X \quad \begin{matrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$A_3 = \begin{matrix} 0 & -S_3 & C_3 & d_3S_3 \\ 0 & C_3 & S_3 & -d_3C_3 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

NOTE: For each matrix, the first three elements of the first column represent the x,y,z components of the new x axis. Similarly, the first three elements of the second and third columns represent the x, y, and z components of the new y and z axes. The fourth column's first three elements represent the x, y, and z distances of the new origin from the base origin in the directions of the base origin.

The rest of the A matrices are as follows:

$$A_1 = \begin{matrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$A_2 = \begin{matrix} C_2 & 0 & -S_2 & 0 \\ S_2 & 0 & C_2 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$A_4 = \begin{matrix} 0 & -S_4 & -C_4 & S_4 d_4' \\ 0 & C_4 & -S_4 & -C_4 d_4' \\ 1 & 0 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$A_5 = \begin{matrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

$$A_6 = \begin{matrix} C_6 & 0 & -S_6 & 0 \\ S_6 & 0 & C_6 & 0 \\ 0 & -1 & 0 & d_6 \\ 0 & 0 & 0 & 1 \end{matrix}$$

The positions and orientations of each joint with respect to the fixed frame or each other are easily found by multiplying the appropriate A transforms.

Elbow matrix = $T_2 = A_1 A_2 =$

$$\begin{matrix} C_1C_2 & -S_1 & -C_1S_2 & d_2S_1 \\ S_1C_2 & C_1 & -S_1S_2 & -d_2C_1 \\ S_2 & 0 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Wrist matrix = $T_4 = A_1A_2A_3A_4$

$$T_4 = \begin{matrix} W_{11} & W_{12} & W_{13} & W_{14} \\ W_{21} & W_{22} & W_{23} & W_{24} \\ W_{31} & W_{32} & W_{33} & W_{34} \\ 0 & 0 & 0 & 1 \end{matrix}$$

where

$$\begin{aligned}
W_{11} &= C_1C_2C_3 - S_1S_3 \\
W_{21} &= S_1C_2C_3 + C_1S_3 \\
W_{31} &= S_2C_3 \\
W_{12} &= -C_1C_2S_3C_4 - S_1C_3C_4 - C_1S_2S_4 \\
W_{22} &= -S_1C_2S_3C_4 + C_1C_3C_4 - S_1S_2S_4 \\
W_{32} &= S_2S_3C_4 + C_2S_4 \\
W_{13} &= C_1C_2S_3S_4 + S_1C_3S_4 - C_1S_2C_4 \\
W_{23} &= S_1C_2S_3S_4 - C_1C_3S_4 - S_1S_2C_4 \\
W_{33} &= S_2S_3S_4 + C_2C_4 \\
W_{14} &= C_1C_2(C_3d_4 + S_3d_3 + S_3C_4d_4') \\
&\quad - S_1\ (S_3d_4 - C_3d_3 - C_3C_4d_4') \\
&\quad + S_1d_2 + C_1S_2S_4d_4' \\
W_{24} &= S_1C_2(C_3d_4 + S_3d_3\ S_3C_4d_4') \\
&\quad + S_3C_4d_4') + C_1\ (S_3d_4 - C_3d_3) \\
&\quad - C_3C_4d_4) - C_1d_2\ C_1S_1S_2S_4d_4' \\
W_{34} &= S_2(C_3d_4 + d_3S_3 + S_3C_4d_4') \\
&\quad - C_2C_4d_4'
\end{aligned}$$

$W_{14,24,34}$ = x,y,z position of the wrist.

Tong gripper matrix = $T_6 = A_1A_2A_3A_4A_5A_6$

$$T_6 = \begin{matrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \\ 0 & 0 & 0 & 1 \end{matrix}$$

where

$$\begin{aligned}
T_{11} &= W_{11}C_5C_6 + W_{12}S_5C_6 + W_{13}S_6 \\
T_{21} &= W_{21}C_5C_6 + W_{22}S_5C_6 + W_{23}S_6 \\
T_{31} &= W_{31}C_5C_6 + W_{32}S_5C_6 + W_{33}S_6 \\
T_{12} &= W_{12}C_5 - W_{11}S_5 \\
T_{22} &= W_{22}C_5 - W_{21}S_5 \\
T_{32} &= W_{32}C_5 - W_{31}S_5 \\
T_{13} &= -W_{11}C_5S_6 - W_{12}S_5S_6 + W_{13}C_6 \\
T_{23} &= -W_{21}C_5S_6 - W_{22}S_5S_6 + W_{23}C_6 \\
T_{33} &= -W_{31}C_5S_6 - W_{32}S_5S_6 + W_{33}C_6 \\
T_{14} &= W_{11}S_5d_6 - W_{12}C_5d_6 + W_{14} \\
T_{24} &= W_{21}S_5d_6 - W_{22}C_5d_6 + W_{24} \\
T_{34} &= W_{31}S_5d_6 - W_{32}C_5d_6 + W_{34}.
\end{aligned}$$

If one starts at the hoist axis at the elevation of the shoulder pivot, there are two paths:

A) 1. Rotate about the hoist axis the shoulder rotation angles,
   2. Translate to the shoulder pivot,
   3. Rotate/translate along the manipulator arm to the tong gripper.

B) 1. Rotate about the hoist axis the camera rotation angle,
   2. Translate to the camera pivot point,
   3. PAN,
   4. TILT,
   5. Translate to the camera axis, and
   6. Translate along the camera axis to the tong gripper.

The reference axes are as follows:

x: + = downward parallel to hoist axis
y: + = horizontal toward north wall
z: + = horizontal toward east wall

The two paths are described as follows:

$$\mathrm{Rot}(x,\mathrm{SHRT})\mathrm{TRANS}(0,d_{1y},-d_{1z})\,T_6 =$$
$$\mathrm{ROT}(x,\mathrm{ROT1})\mathrm{TRANS}(-X1,Y1,Z1)\mathrm{ROT}\,(x,\mathrm{PAN1})$$
$$\mathrm{ROT}(Z,\mathrm{TILT1})\mathrm{TRANS}(d_T,d_y,0)$$

<u>Note</u>: The above equations are equal in <u>position</u> only, not orientation.

The TILT angle used is the actual TILT-$\pi/2$ (for convenience).

Let SHCH = SHRT-ROT1, then

$$ROT(x,-PAN1)TRANS(X1,-Y1,-Z1)ROT(X,SHCH)$$
$$TRANS(0,d_{1y}, -d_{1z})T_6$$
$$= ROT(Z,TILT1)TRANS(d_T,d_y,0$$

Let: $T_y' = T_y + d_{1y}$.

This matrix equation yields three scaler equations upon equating the x, y, and z positions:

$$(1) \quad T_x + x_1 = d_T COS(TILT1) - d_y SIN(TILT1)$$

$$(2) \quad COS(PAN1)[T_y' COS(SHCH)$$
$$-(T_Z - d_{1Z})SIN(SHCH) - Y1]$$
$$+ SIN(PAN1) \times [T_y' SIN(SHCH) + (T_z - d_{1Z})$$
$$\times COS(SHCH) - Z1] = d_T SIN(TILT1)$$
$$+ d_y COS(TILT1)$$

$$(3) \quad -SIN(PAN1)[T_y' COS(SHCH) - (T_z - d_{1Z})$$
$$SIN(SHCH)-Y_1] + COS(PAN1)$$
$$\times [T_y' SIN(SHCH) + (T_z - d_{1Z})$$
$$\times COS(SHCH) - Z1] = 0$$

The PAN angle can be determined from Eq. (3):

$$PAN1 = TAN^{-1}[(T_y' S_{SHCH} + (T_z - d_{1z})C_{SHCH}$$
$$- Z1)/(T_y' C_{SHCH} - (T_Z - d_{1Z})\ S_{SHCH} - Y1)].$$

The TILT angle is a bit more involved:

Let:

$$K_1 = C_{PAN1}[T_y' C_{SHCH} - (T_z - d_{1z})S_{SHCH} - Y1]$$
$$+ S_{PAN1}[T_y' S_{SHCH} + (T_Z - d_{1Z})C_{SHCH} - Z1]$$

$$x_i' = T_x + X_i$$

$$C_T, S_T = \text{COS(TILT1)}, \text{SIN(TILT1)}$$

Then,

$$d_T C_T - d_y S_T = x_i' \qquad (1)$$

$$d_T S_T + d_y C_T = K_1 \qquad (2)$$

Multiplying Eq. (1) by $\frac{S_T}{C_T}$ and subtracting Eq. (2),

$$-d_y \left(\frac{S_T^2}{C_T} + C_T \right) = \frac{S_T}{C_T} X_i' - K_1.$$

Applying $S_T^2 = 1 - C_T^2$ and squaring both sides,

$$d_y^2 \sec^2(\text{TILT1}) = X'^2 \text{TAN}^2(\text{TILT1}) -$$

$$2K_1 X' \text{TAN}(\text{TILT1}) + K_1^2,$$

$\sec^2 = \tan^2 + 1$, so

$$(X'^2 - d_y^2)\tan^2(\text{TILT1}) - (2K_1X')\tan(\text{TILT1})$$

$$+ K_1^2 - d_y^2 = 0,$$

so

$$\tan(\text{TILT1}) = \frac{K_1 X' - \sqrt{K_1^2 (X')^2 - (K_1^2 - d_y^2)(X'^2 - d_y^2)}}{X'^2 - d_y^2}$$

Note that the $-\sqrt{(\ )}$ root was used. This was determined from the physics of the problem by noting that if $d_y = 0$, the tilt angle would be larger than that of $d_y = +$.