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THE $c\bar{c}$ AND $b\bar{b}$ SPECTROSCOPY IN THE TWO-STEP POTENTIAL MODEL *

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ABSTRACT

We investigate the spectroscopy of the charmonium ($c\bar{c}$) and bottonium ($b\bar{b}$) bound states in a static flavour independent nonrelativistic quark-antiquark ($q\bar{q}$) two-step potential model proposed earlier. Our predictions are in good agreement with experimental data and with other theoretical predictions.

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We investigate the spectroscopy of the charmonium ($c\bar{c}$) and bottonium ($b\bar{b}$) bound states [1,2] in a static, flavour independent, non-relativistic quark-antiquark ($q\bar{q}$) two step potential*) proposed earlier [3].

In quantum chromodynamics (QCD), the interquark potential is essentially of the Coulomb type at short distances. Lattice gauge theory and the string model suggest that the $q\bar{q}$ force at large distances is completely independent of the interquark distance. This gives rise to a linear confining potential. Our two-step potential [3] reads:

$$V(r) = -\beta/r \quad , \; r \le B$$
$$= -V_0 + Kr \quad , \; r \ge B \quad . \tag{1}$$

Demanding the continuity of $V(r)$ and of its first derivative at $r = B$, we obtain

$$B = \sqrt{\beta/K} \text{ and } V_0 = 2\beta/B = 2KB \quad . \tag{2}$$

Thus the number of free parameters in Eq.(1) is reduced to two (we eliminate B and V_0 in favour of β and K). Our model thus has <u>three free</u> parameters, namely β, K and the quark mass m_q which is, of course, different for different $q\bar{q}$ systems.

We solve the relevant Schrödinger equation

$$\left[-\frac{1}{r}\frac{d^2}{dr^2}r + m_q\,[V(r) - E] + \frac{\ell(\ell+1)}{r^2}\right] R(r) = 0 \tag{3}$$

with $V(r)$ given by Eq.(1) using a standard numerical method [4-9]. We have tested our computational procedure by reproducing very accurately the results of Ono [5-8] for $c\bar{c}$ and $b\bar{b}$ bound states.

To fix the free parameters of our model we fit simultaneously the experimental data [10] for the masses of the $n\,^3S_1$ states of $c\bar{c}$ and $b\bar{b}$.

*) For an earlier treatment of the two-step potential, see e.g. Ref. 3.

As a result of this we obtain

$$\beta = 0.63 \quad , \qquad K = 0.247\ \mathrm{GeV}^2 ,$$
$$m_c = 1.575\ \mathrm{GeV}, \qquad m_b = 4.975\ \mathrm{GeV}. \qquad (4)$$

It is important to note here that β and V_0 (as given by Eq.(2)) are not independent parameters in our model.
The present two-step potential (Eq.(1) is shown in Fig.1, for $r \leq 1.5$ fm, with some other quarkonium potentials [7, 8, 11-15] where a suitable constant is added so that all potentials have the same value at $r = 0.4$ fm.
The r.m.s. radii of the observed S states of $c\bar{c}$ and $b\bar{b}$ states calculated in the present model are also shown.

Fig.2 shows short-distance behaviour of the present potential (Eq.(1)) together with that of some other quarkonium potentials [7, 8, 11-16] for $r \leq 0.15$ fm., where again a suitable constant is added so that all potentials have the same value at $r = 0.15$ fm.

One observes from Fig.1 that our potential (Eq.(1)) (shown in the figure by a solid line) as well as the other quarkonium potentials [7, 8, 11-15] seem to agree in the region $0.1\ \mathrm{fm} \leq r \leq 1.0$ fm, which is probed by the present quarkonium families (up to Υ) (cf. Fig.1 and Tables I and II). However, the short-distance behaviour of each potential as shown in Fig.2 is different.
The predictions of the present model for the various properties such as the mass spectra, ratios of leptonic decay widths, r.m.s. radii, and the mean square velocities, for the spin -triplet states of $c\bar{c}$ and $b\bar{b}$ systems are displayed in Tables I and II respectively together with the experimental data [1, 10, 17].

From Table I one observes that for charmonium the agreement between the predictions of the present model for the masses of the lower-lying (viz., 1S, 1P, 2S and 1D) states and the experimental data [10] is better than that for the higher-lying (viz., 3S, 2D and 4S) states.

In fact every potential (except the Martin potential [13, 14]) predicts higher masses for ψ(3S) and ψ(4S). This discrepancy may have two reasons. First, the overestimate can be related to the strong coupling between $c\bar{c}$ states and $c\bar{q} + \bar{c}q$. Eichten et al.[11, 12] have shown that the channel coupling effects become very important for the quarkonium states which are close to or above the threshold.

The reason that the Martin [13,14] potential predicts lower values for the masses of ψ(4S) and ψ(2D) is that the potential has the single power $r^{0.1}$ and it does not rise as fast as more conventional potentials which have a linear long range part (cf. Fig.1). Therefore the Martin potential predicts smaller values for higher excited states.
Secondly, it has been proposed recently by Ono [9] that the experimentally observed states ψ(4030), ψ(4159) and ψ(4415) which are identified as ψ(3S), ψ(2D) and ψ(4S) respectively are not pure $c\bar{c}$ states but mixed states of $c\bar{c}$ and $c\bar{c}g$ [9].

Table II shows that the predictions of the present model for the mass spectra of $b\bar{b}$ bound states are, in general, in good agreement with the experimental data [1,10,17] and with other theoretical predictions [7,8,11-16]. The present model predictions for the excitation energies of the $n^3S_1(b\bar{b})$ states with respect to the $1^3S_1(b\bar{b})$ state, are 524 MeV, 883 MeV and 1190 MeV, for n = 2,3 and 4 respectively, which are to be compared with the corresponding experimental values [1,10] of 559 $\pm$ 3 MeV, 891 $\pm$ 4 MeV and 1113 $\pm$ 4 MeV. our predictions for the centroids (COG) of 1^3P_J and 2^3P_J states of $b\bar{b}$ at 9918 MeV and 10264 MeV respectively match very well with the corresponding experimental values [17] of 9900 $\pm$ 3 MeV and 10256 $\pm$ 5 MeV.

The excitation energies of the centroids of 1^3P_J and 2^3P_J states with respect to the $1^3S_1(b\bar{b})$ state predicted in our model as 458 MeV and 804 MeV, also agree well with their corresponding experimental values [17] 444 $\pm$ 3 MeV and 801 MeV.

We calculate the leptonic (e^+e^-) decay widths of the n^3S_1 states using the Van Royen-Weisskopf [18] decay width formula:

$$\Gamma(n^3S_1 \to e^+e^-) = \frac{16\pi\,\alpha^2\,e_q^2}{M_n^2\,(q\bar{q})}\,|\psi_n(0)|^2 \tag{5}$$

Here $\alpha \simeq 1/137$ is the fine structure constant, $e_q = 2/3$ for $c\bar{c}$ and $-1/3$ for $b\bar{b}$, M_n is the mass of the vector meson and $|\psi_n(0)|^2$ is the square of the wave function at the origin. However, it is more meaningful to study the ratios of the decay widths $\Gamma_{e^+e^-}(nS)/\Gamma_{e^+e^-}(1S)$, because the QCD corrections cancel for the ratios. These ratios for $c\bar{c}$ and $b\bar{b}$ systems as calculated in the present model are also shown in Tables I and II together with the available experimental data [1,10].

Looking at Tables I and II we see that for the $b\bar{b}$ system, the predictions of the present model for the ratios of the decay widths are in good agreement with the experimental data [1,10]. However, for the $c\bar{c}$ system, the predictions of the present model are somewhat higher than the available experimental data [1,10].

Further, our predictions for these ratios for the lower-lying states of the $c\bar{c}$ system are in better agreement with the data [1,10] than those for the higher-lying states. It is, however, important to notice that the same is true for most of the potential models [7,8,11,12,15,16].

In conclusion, we find that our present two-step potential (Eq.(1)) explains the $c\bar{c}$ and $b\bar{b}$ spectroscopy in reasonably good agreement with the experimental data. A detailed study of the various heavy quarkonium ($c\bar{c}$, $b\bar{b}$, $b\bar{c}$, $t\bar{t}$, $t\bar{c}$ and $t\bar{b}$) systems, employing the present potential, is relegated to a separate communication.

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TABLE I: Predictions of the present model for the $c\bar{c}$ spectrum: masses, ratios of leptonic decay widths, r.m.s.radii, and the mean square velocities. Experimental data [1,10] is also shown.

State	Mass: present MeV	Mass: Experimental data (Refs. 1,10) MeV	$\Gamma_{e^+e^-}(nS)/\Gamma_{e^+e^-}(1S)$: present	$\Gamma_{e^+e^-}(nS)/\Gamma_{e^+e^-}(1S)$: Experimental data (Refs. 1, 10)	$\langle r^2 \rangle^{1/2}$ (present) fm	$\langle v^2/c^2 \rangle$ (present)
1S	3096	3096.9±0.1	1	1	0.42	0.21
1P	3495	3521			0.64	0.22
2S	3698	3686.0±0.1	0.57	0.45 ± 0.08	0.78	0.27
1D	3798	3770 ± 3			0.80	0.25
2P	4009				0.95	0.28
3S	4189	4030 ± 5	0.41	0.16 ± 0.04	1.07	0.31
2D	4266	4159 ± 20			1.08	0.30
4S	4622	4415 ± 6	0.31	0.11 ± 0.04	1.32	0.34

TABLE II: Predictions of the present model for the $b\bar{b}$ spectrum: masses, ratios of leptonic decay widths, r.m.s. radii, and the mean square velocities. Experimental data [1,10,17] is also shown.

State	Mass: present (MeV)	Mass: Experimental data (Refs. 1, 10,17) (MeV)	$\Gamma_{e^+e^-}(nS)/\Gamma_{e^+e^-}(1S)$: present	$\Gamma_{e^+e^-}(nS)/\Gamma_{e^+e^-}(1S)$: Experimental data (Refs. 1,10,17)	$\langle r^2\rangle^{1/2}$ fm	$\langle \frac{v^2}{c^2} \rangle$
1S	9460	9456 $\pm$ 10	1	1	0.21	0.108
1P	9918	9900 $\pm$ 3			0.42	0.058
2S	9984	10016 $\pm$ 10	0.35	0.46 $\pm$ 0.03	0.49	0.079
1D	10139				0.54	0.065
2P	10264	10256 $\pm$ 5			0.63	0.079
3S	10343	10347 $\pm$ 10	0.26	0.33 $\pm$ 0.03	0.70	0.093
2D	10455				0.73	0.084
3P	10569				0.81	0.096
4S	10650	10569 $\pm$ 10	0.21	0.23 $\pm$ 0.02	0.87	0.106
3D	10740				0.90	0.100
5S	10926		0.18		1.03	0.119
6S	11181		0.16		1.18	0.130

TABLE CAPTIONS

Table I: Predictions of the present model for the $c\bar{c}$ spectrum: masses, ratios of leptonic decay widths, r.m.s.radii, and the mean square velocities. Experimental data [1,10] is also shown.

Table II: Predictions of the present model for the $b\bar{b}$ spectrum: masses, ratios of leptonic decay widths, r.m.s.radii, and the mean square velocities. Experimental data [1,10,17] is also shown.

FIGURE CAPTIONS

Fig. 1: The present two-step potential (Eq.(1)), for $r \leq 1.5$ fm, together with some other phenomenologically successful potentials [7,8,11-15] for the quarkonium where a suitable constant is added so that all potentials have the same value at $r = 0.4$ fm. The r.m.s. radii of the observed S states of $c\bar{c}$ and $b\bar{b}$ states calculated in the present model are shown by markers.

Fig. 2 : Short distance behaviour of the present potential (Eq.(1)) together with that of some other potentials [7,8,11-16] for $r \leq 0.15$ fm., where again a suitable constant is added so that all potentials have the same value at $r = 0.15$ fm.

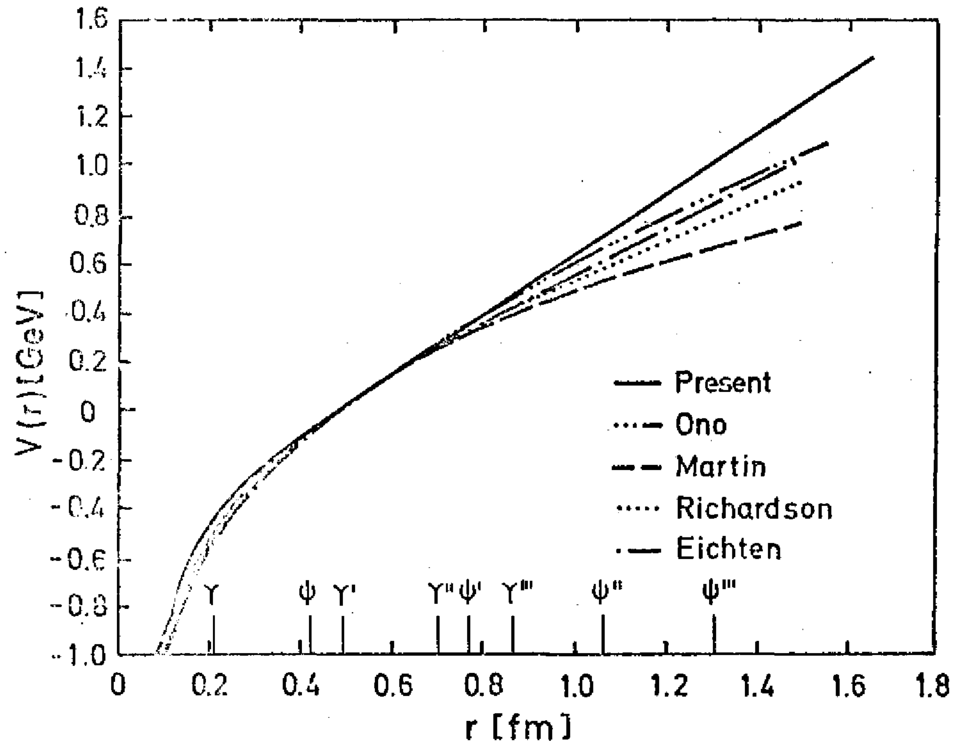
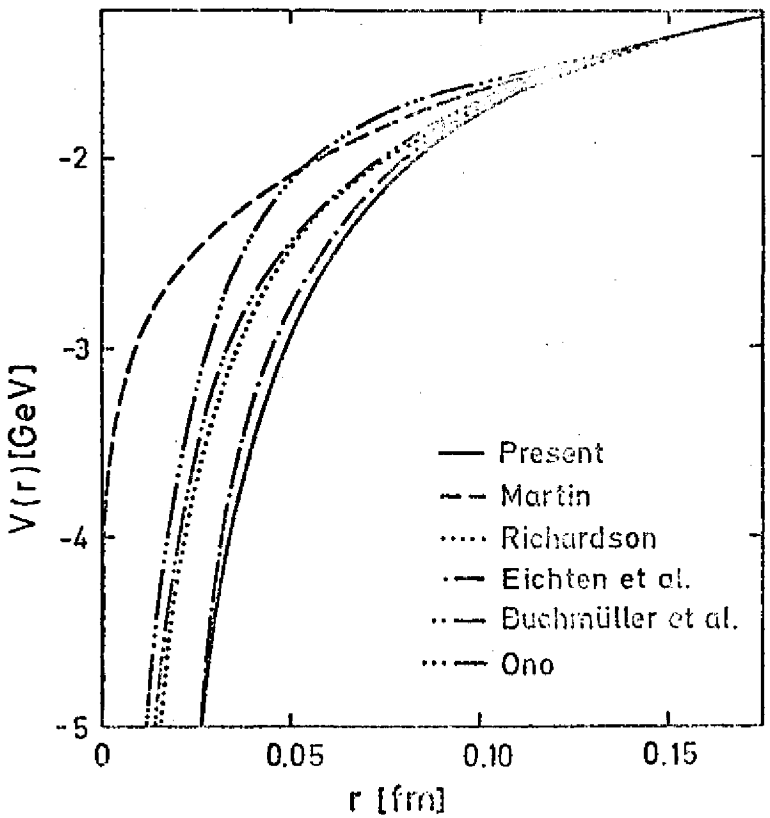


Fig. 1



V(r) [GeV]
-2
-3
-4
-5
0
0.05
0.10
0.15
r [fm]
Present
Martin
Richardson
Eichten et al.
Buchmüller et al.
Ono

Fig. 2