EQUILIBRIUM AND STABILITY STUDIES FOR HIGH BETA PLASMAS IN TORSATRON/HELIOTRON DEVICES*

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ABSTRACT

The equilibrium and stability properties of high β plasmas in torsatron/heliotron devices have been investigated. Three numerical approaches have been used to study plasma equilibria for a range of coil configurations. The method of averaging permits fast equilibrium and stability calculations. Two fully 3-D codes, namely the Chodura-Schluter code, and the NEAR code recently developed at ORNL, are used to explore selected regions of parameter space. The resulting equilibria calculated with different methods are in good agreement. This validates the average method approach and enhances its usefulness. Results are presented for configurations with different aspect ratios and number of field periods. The role of the vertical field has also been studied in detail. The main conclusion is that for moderate aspect ratios ($A_p \lesssim 8$), the self-stabilizing effect of the magnetic axis shift is large enough to open a direct path to the second stability regime.

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I. INTRODUCTION

We have performed detailed equilibrium and stability studies of $\ell = 2$ Torsatron/Heliotron configurations over a wide range of parameters. The main motivation was to find an optimal configuration on which to base the design of the Advanced Toroidal Facility (ATF). Two methods, implemented in three systems of codes, were used for such calculations. One of them, the method of averaging [1,2], has allowed the mapping of magnetohydrodynamical properties of a large number of configurations in a fairly short time. The second method, based on the minimization of the potential energy, is carried out in a full three dimensional geometry either with the Chodura-Schluter code [3] or the NEAR code [4], recently developed at ORNL. They are used to explore selected cases and at present are limited to equilibrium studies. The three codes use the same input, the vacuum fields produced by a given coil configuration. This has permitted detailed comparisons between the results of the three systems of codes. Different ways of implementing the averaging method have also been considered which basically differ in the way the toroidal effects were included. For the range of parameters studied, the averaging method reproduces in a satisfactory way the three-dimensional calculations. The results are not very sensitive to the particular method of implementation. This has lent strong support to such calculations.

The two three-dimensional codes use the minimization of the potential energy as the way to reach equilibrium. The Chodura-Schluter code carries out the minimization on a fixed Cartesian grid, and is approximately flux conserving. This code does not permit high resolution calculations but is suitable for the study of configurations with magnetic islands or a bifurcated magnetic axis. The NEAR code uses a method similar to Chodura-Schluter, but it uses vacuum flux coordinates, with Fourier representation in the angular variables. This gives an increased radial resolution and higher speed in the calculation.

II. EQUILIBRIUM CALCULATIONS

To characterize the different configurations we will refer to the following parameters: coil aspect ratio $A_c = R_c/a_c$, the number of field periods M, the pitch parameter $p = M/\ell A_c$ and the vertical field B_V. By changing these parameters we can vary the main physics characteristics of the vacuum configurations. Many of the equilibrium and stability studies have been done for $p \simeq 1.4$. For instance, Fig. 1 shows one of the configuration scans for $p = 1.4$, changing M. A sample of equilibria for these configurations, and for $\beta_0 = 6\%$, is shown in Fig. 2. These have been obtained with the Chodura-Schluter code. A sequence of equilibria with varying β calculated with the NEAR code, for a fixed configuration ($M = 12$), is shown in Fig. 3. For this same configuration, which is close to the one chosen for the ATF design, we have done detailed comparisons of the equilibria calculated using the different codes. Fig. 4 summarizes these comparisons, showing the magnetic axis shift at different values of beta.

The question of a limiting equilibrium beta is difficult to resolve precisely. The limit has often been taken to be the point at which the shift attains a value of one-half the minor radius. However, this limit is an arbitrary convention. Another possibility is to consider as a limit the failure to find an equilibrium, as evidenced by lack of convergence in the stellarator expansion or by magnetic surface breakup in the 3-D calculations. This limit may be indicative of the encroachment of a separatrix on the flux surfaces, but it can also be due to numerical problems. We are currently investigating the possibility of separating the limitations on β due to numerical resolution from intrinsic β-limitations. The highest β equilibrium obtained with the present codes using the stellarator expansion for the configuration scan with $p = 1.4$ is shown in Fig. 5. The NEAR code is able to attain equilibria for the same configurations with higher β values than the ones given in Fig. 5.

III. STABILITY STUDIES

The physics parameters of principal interest for stability studies are the radial profiles of ι and V''. In particular, the location of the zero of V'' relative to the location of the dangerous low order rational surfaces ($q = 1$, $q = 2$, etc.) is of crucial importance. Figure 6(a) shows the relationship of the low order rational surfaces to ι_c, the transform at the critical surface, for the vacuum field of the configurations in the constant pitch scan described above. The shaded regions show the sections of "configuration space" where low order rational surfaces lie inside the plasma but outside the critical surface (where V'' is destabilizing). In these regions, the only stabilizing influence is shear. Figure 6(b) shows how the critical surfaces move outward as beta increases and the magnetic well deepens. The changing relationship between the critical surface, the low order rational surfaces, and the well depth is the key element in determining stability. Note that for all these configurations the diamagnetic contribution to V'' is negligible compared with the contribution due to the magnetic axis shift.

Linear and nonlinear stability studies of large scale, fixed boundary modes in these configurations were performed using Strauss's equations [5] implemented in the RST code [6]. The stellarator expansion equilibria in their two-dimensional form were used as input to RST. Since these equilibria can be rapidly generated for a wide variety of configurations, this method was used for systematic studies. The HERA [7] code was used to determine growth rates for helically symmetric equilibria (no toroidal effects) for comparison with the averaged method studies in the helically symmetric limit.

Fig. 7 shows the $n = 1$ and $n = 2$ growth rates for the configuration scan. Here the configurations in which low order resonances fall in the region of destabilizing V' are indicated by cross-hatching. As the scan crosses $M = 18$, the growth rate drops sharply as the mode becomes more localized and is finally stabilized as the $q = 1$ surface is excluded from the plasma. At a plasma aspect ratio of about 7, the $M = 12$ (ATF) configuration is stable to both $n = 1$ and $n = 2$ modes. The modes become unstable in the higher A_c, higher M configurations having the same pitch as ATF, with growth rates increasing with aspect ratio. The stability of the $M = 12$ case and the saturation of growth rate as β_0 increases for $M > 14$ (Fig. 8) suggest that the $M = 12$ configuration may lie at the edge of a second stability regime.

Numerical calculations for high n modes and semianalytical calculations for localized modes confirmed these results. They show that the combination of magnetic well, produced by the Shafranov shift, near the plasma axis with the shear near the plasma edge can stabilize the ideal mode for $M \lesssim 13$. The $M = 12$ configuration lies in the stable region and its maximum achievable β is determined by equilibrium. (Fig. 9).

The effect of a vertical field on the equilibrium and stability of the plasma is very important. As the vertical field is increased to produce an outer shift of the vacuum flux surfaces the quality of the equilibrium deteriorates as its stability properties improve. This effect is reversed when an inward shift is produced. For the $M = 12$ configuration, the plasma can become unstable by increasing the inward shift. Therefore, this configuration is suitable for testing the plasma stability properties.

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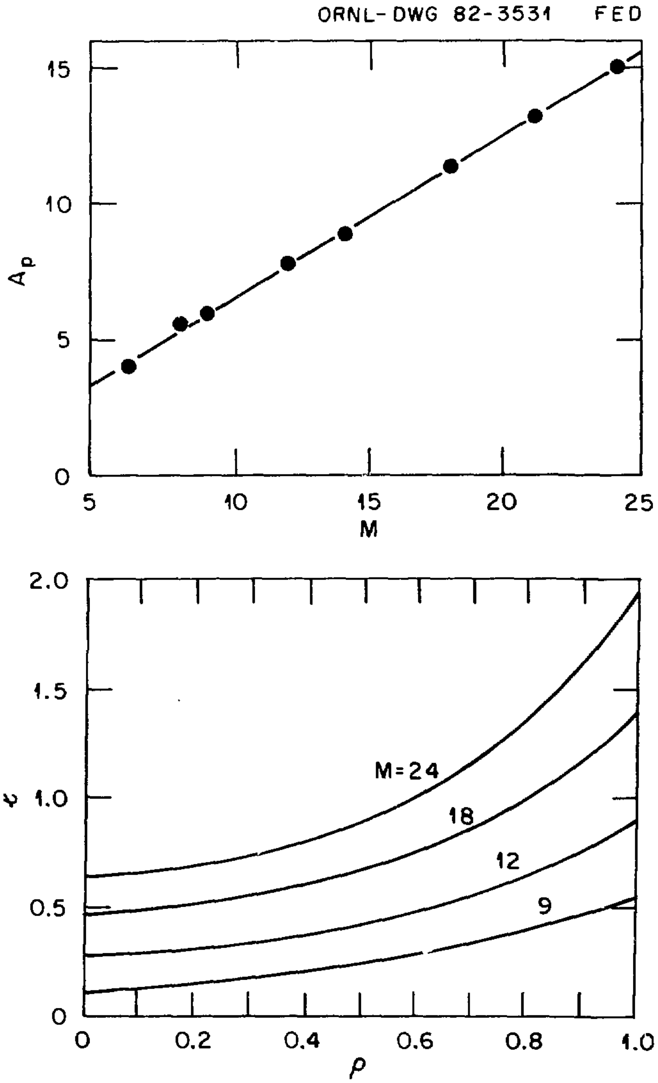

Fig. 1. Configurations studied in the constant pitch scan, (p = 1.4) and sample rotational transform profiles.

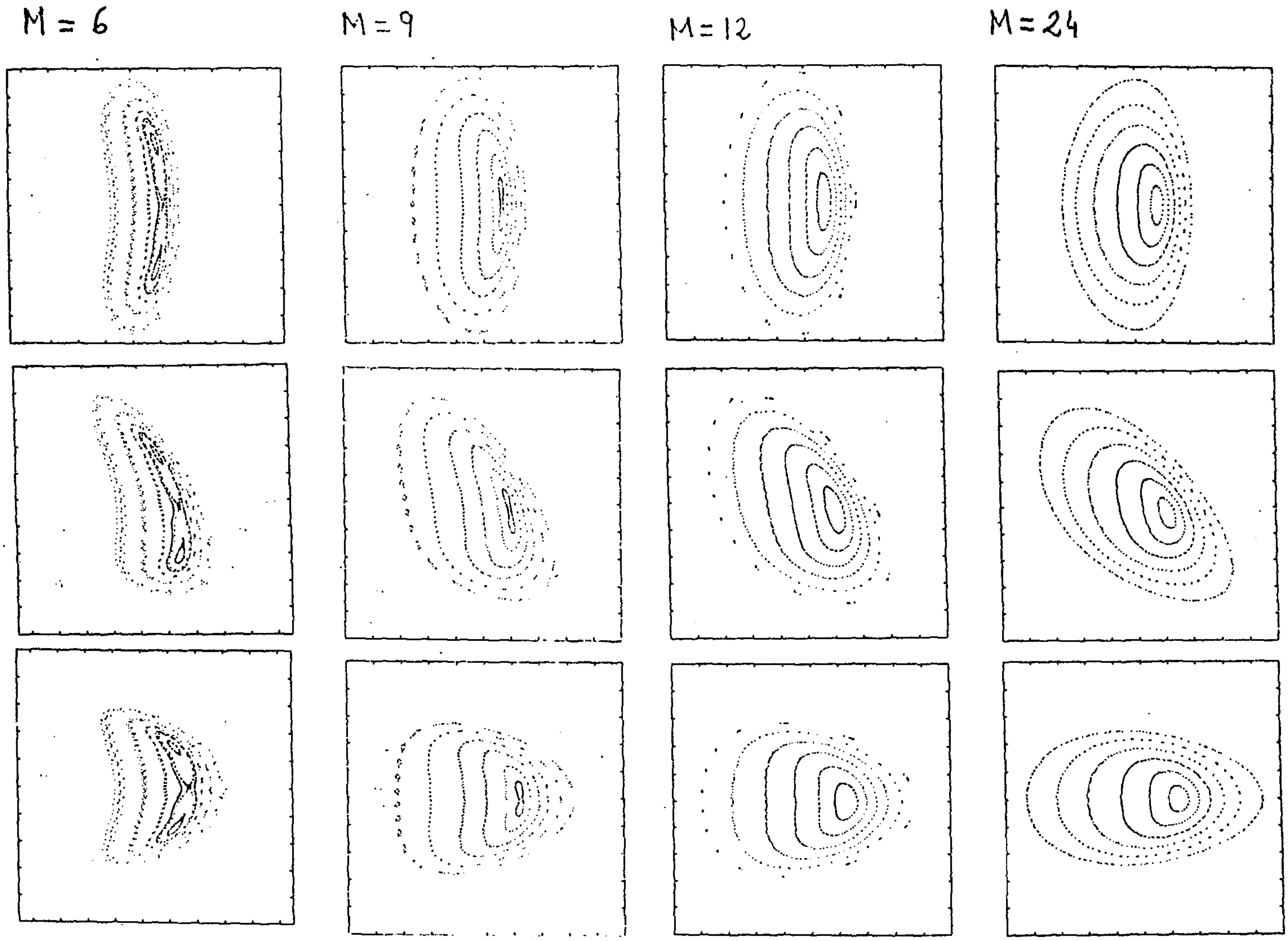

Fig. 2. Sample of equilibria at a constant beta value for configurations with p = 1.4.

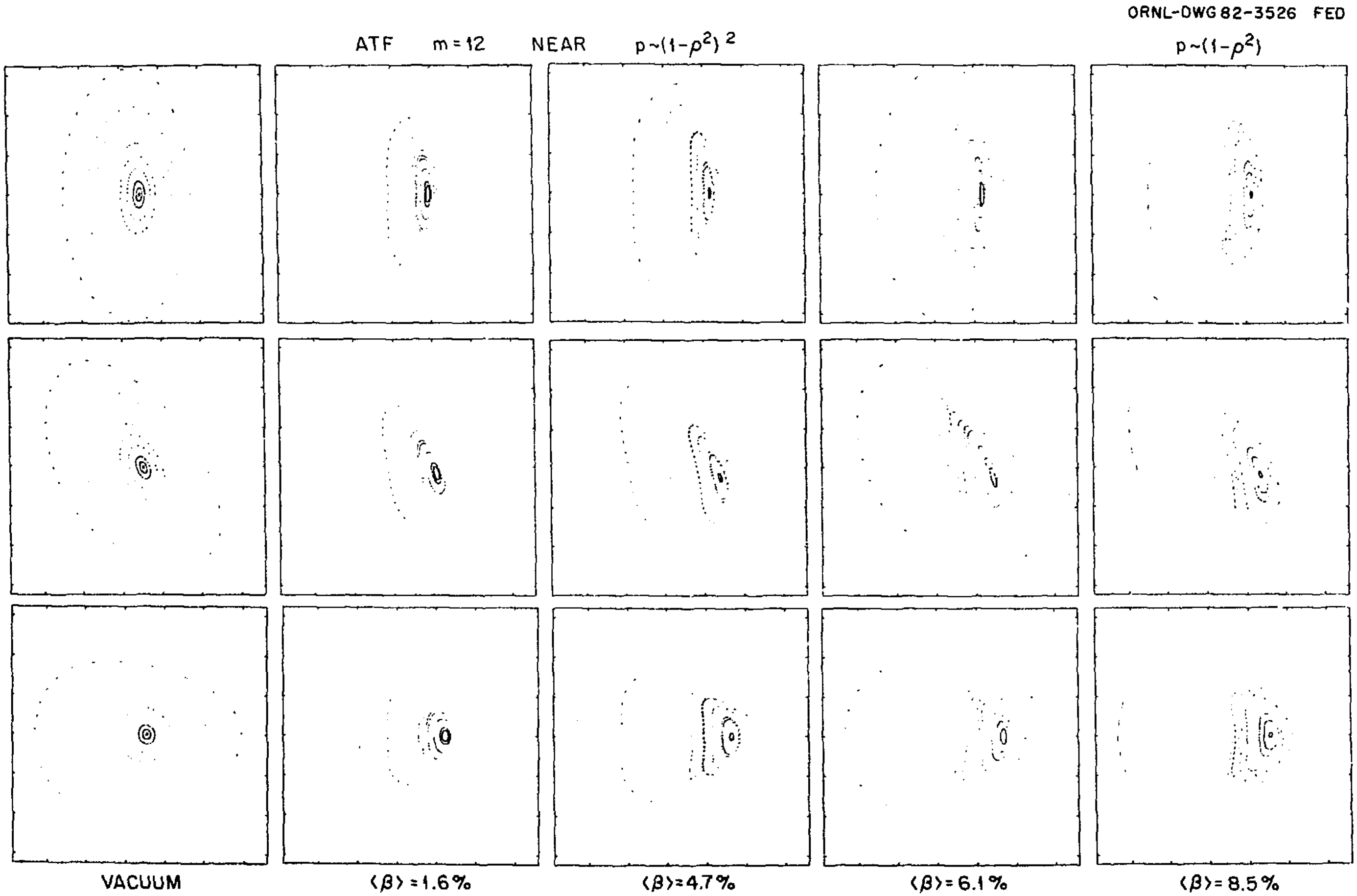

Fig. 3. Equilibria for the M = 12, p = 1.4, configuration (ATF).

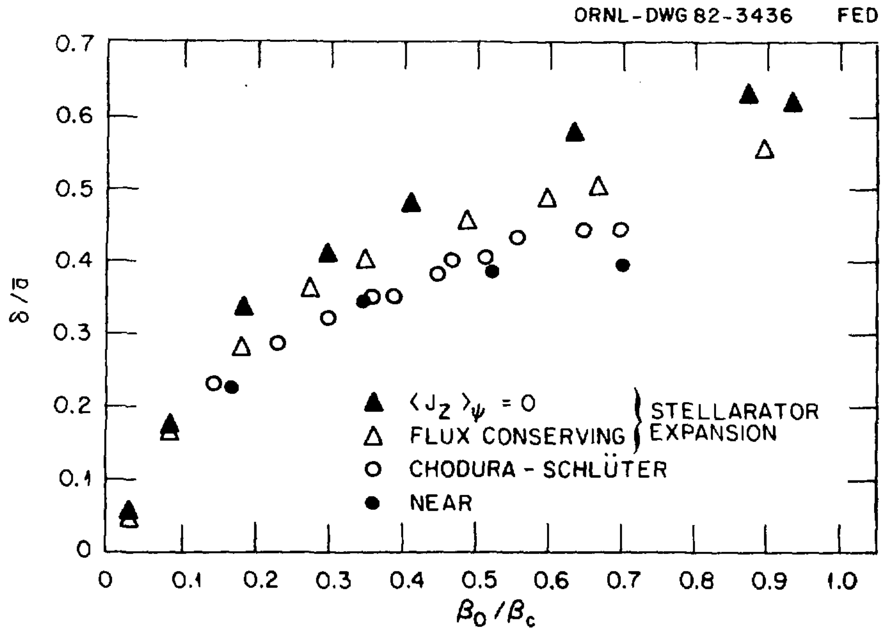

Fig. 4. Normalized equilibrium shift for the $M = 12$ configuration as a function of β_0. Here $\beta_c = 2\iota^2(a)/A_p$.

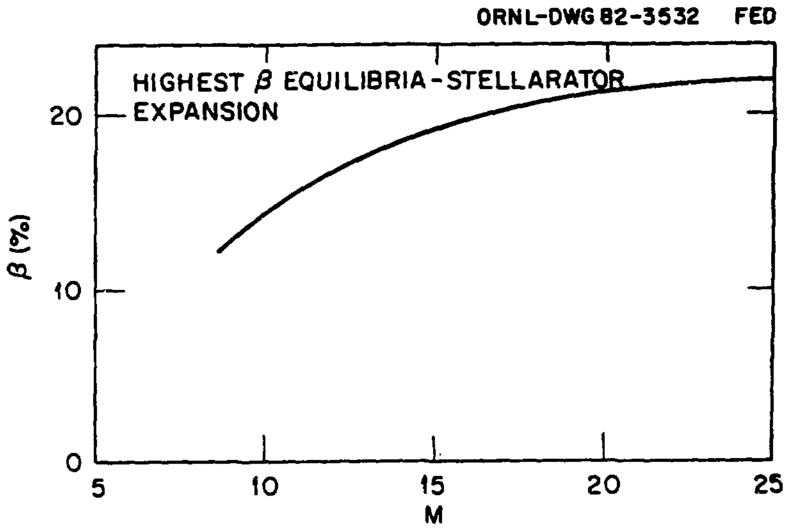

Fig. 5. Maximum β_0 for which the stellarator expansion equilibrium converged, with present numerical techniques.

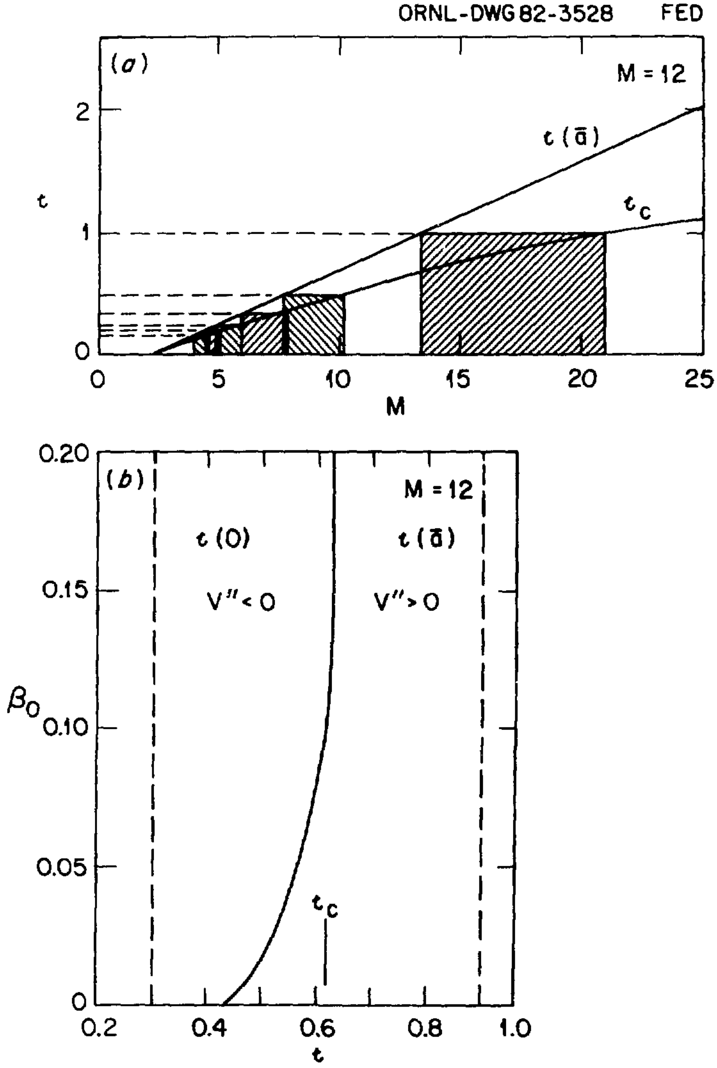

Fig. 6. (a) Location of low order rational surface relative to critical surfaces of configurations in constant pitch scan. (b) Dependence of critical surface location on beta (flux-conserving sequence).

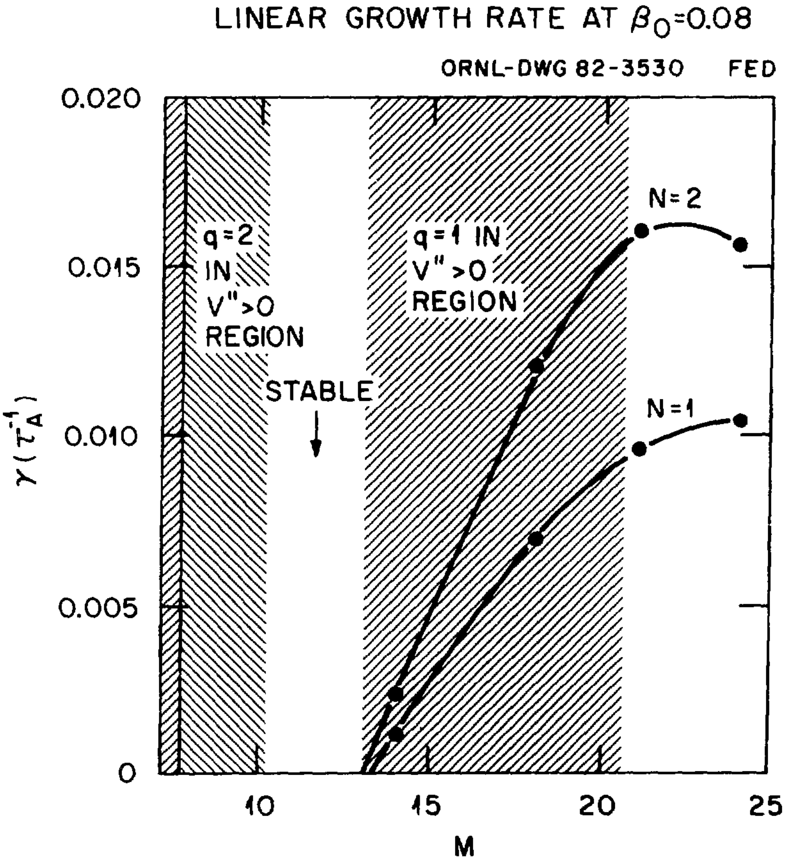

Fig. 7. Linear growth rates of the n = 1 and n = 2 modes for constant pitch configuration scan at fixed value of beta (β_0 = 0.12).

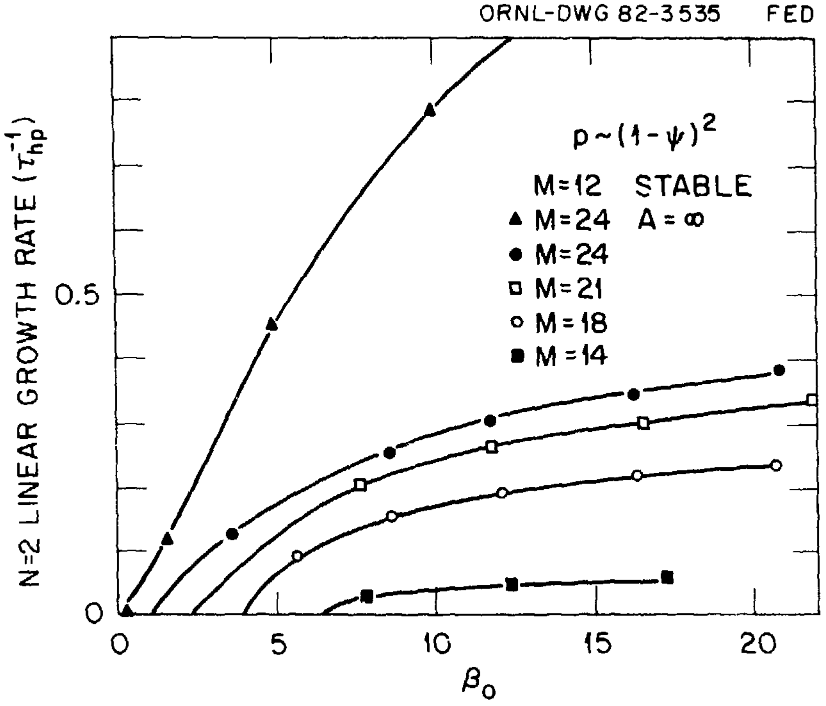

Fig. 8. Linear growth rates for the n = 1 ideal mode in the constant pitch scan (p = 1.4).

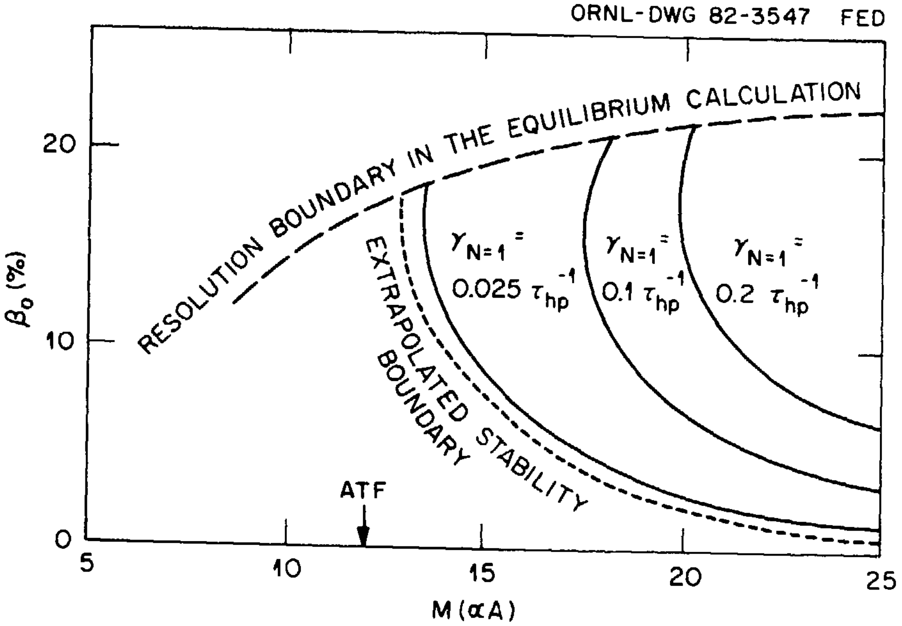

Fig. 9. Combined equilibrium and stability constraints for constant pitch configuration scan, obtained with the averaged method.