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The ρ-parameter in supersymmetric models

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Abstract

The electroweak ρ-parameter is examined in a general class of supersymmetric models. Formulae are given for one-loop contributions to $\Delta\rho$ from scalar quarks and leptons, gauge-Higgs fermions and an extra doublet of Higgs scalars. Mass differences between members of isodoublet scalar quarks and leptons are constrained to be less than about 200 GeV.

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I. Introduction

There is a hope that the gauge hierarchy problem is solved in supersymmetric gauge theories[1]. Supersymmetric partners of leptons and many other particles may soon be discovered. Such new particles affect the low energy phenomenology of weak interactions through one-loop effects. In particular the ratio of neutral to charged weak current amplitudes is sensitive to the existence of heavy particles[2,3]. The purpose of this paper is to examine the contributions of new particles to the parameter ρ in a general class of supersymmetric models. We shall study the effects of the following three types of particles: (i) scalar quarks and scalar leptons, (ii) gauge-Higgs fermions, and (iii) extra Higgs doublet scalars.

Masses of superpartners vary significantly depending on models. Some of them may be as large as one TeV. We shall give general formulae for the deviation of ρ from unity, $\Delta\rho = \rho-1$, which can cope with Majorana fermions. It is known[3] that the breaking of a global SU(2) is responsible for $\Delta\rho \neq 0$. We shall evaluate $\Delta\rho$ in various limits where several sources of the breaking can be separated. We shall also examine the limit where supersymmetry breaking mass scale is much larger than M_W. In order to relate our results to observables, such as neutrino cross sections, one needs to add gauge boson contributions and process-dependent corrections. Some of these contributions have been estimated[4].

We shall consider a general class of softly broken supersymmetric $SU(2)_L \times U(1)$ model[5]. This model can be regarded as an effective low-energy theory of currently successful grand unified models[1,6,7,8].

In Sec. II we introduce a general class of $SU(2)_L \times U(1)$ model with soft supersymmetry breaking terms and discuss the breaking of the global symmetry SU(2). In Sec. III we give the general expression for $\Delta\rho$ and evaluate it in various limits.

When writing this paper we received a preprint by R. Barbieri and L. Maiani[9] in which the same subject has been addressed in a somewhat different class of models. The contributions of scalar quarks and gauge-Higgs fermions to $\Delta\rho$ have also been discussed by L. Alvarez-Gaumé, J. Polchinski and M. B. Wise[7] in an interesting specific case (case (c) in Sec. III of our paper). We have learned recently that R. Arnowitt and E. Elliasson are also studying the ρ-parameter in supersymmetric models[10].

II. The model and the ρ-parameter

We shall work in a general class of softly broken supersymmetric $SU(2)_L \times U(1)$ model[5,7]. For simplicity we suppress generation indices and denote u- and d-type quark supermultiplets (left-handed) as

$$Q = (S_q,\ q) = \left(\begin{pmatrix} S_u \\ S_d \end{pmatrix},\ \begin{pmatrix} u \\ d \end{pmatrix} \right) ,$$
$$\bar{U} = (\bar{S}_u,\ \overline{u_R}) , \quad \bar{D} = (\bar{S}_d,\ \overline{d_R}) , \tag{1}$$

where S and $\bar{S}$ denote scalar quarks. Lepton supermultiplets can be dealt with in exactly the same way as quarks apart from the color factor. In supersymmetric models, a pair of Higgs doublet supermultiplets H_D and H_U are needed to supply masses to both u- and d-type quarks. It is useful to define a 2×2 matrix H

$$H \doteq (H_D, H_U) , \tag{2}$$

whose scalar component and fermionic component are denoted as ϕ_H and ψ_H respectively

$$\begin{aligned} \phi_H &= (\phi_{H_D}, \phi_{H_U}) = \begin{pmatrix} \phi_D^0 & \phi_U^+ \\ \phi_D^- & \phi_U^0 \end{pmatrix} \\ \psi_H &= (\psi_{H_D}, \psi_{H_U}) = \begin{pmatrix} \psi_D^0 & \psi_U^+ \\ \psi_D^- & \psi_U^0 \end{pmatrix}_L . \end{aligned} \tag{3}$$

Here L denotes left-handed chirality. We do not introduce a singlet chiral supermultiplet, since its coupling to other chiral multiplets may invalidate the naturalness of the gauge-hierarchy[11]. In addition to these chiral supermultiplets the model contains gauge supermultiplets $V^a = (A_\mu^a, \lambda^a)$ and $V = (B_\mu, \lambda)$, corresponding to $SU(2)_L$ and U(1) respectively.

Supersymmetric part of our lagrangian involves a superpotential W given by

$$W = -f_D Q H_D \bar{D} + f_U Q H_U \bar{U} + m_H H_D H_U . \tag{4}$$

Here $SU(2)_L$ indices have been suppressed. The Fayet-Illiopoulos D term for U(I) gauge multiplet should not exist in grand unified theories at least perturbatively. The lagrangian also contains soft supersymmetry breaking terms,

$$
\begin{aligned}
L_{S.B.} = & -\mu^2 S_q^\dagger S_q - \bar{\mu}_D^2 |\bar{S}_d|^2 - \bar{\mu}_U^2 |\bar{S}_u|^2 \\
& - (-f_D M_D S_q \phi_{H_D} \bar{S}_d + f_U M_U S_q \phi_{H_U} \bar{S}_u + h.c.) \\
& - \tfrac{1}{2}\{ m \overline{(\lambda_L^a)^c} \lambda_L^a + m' \overline{(\lambda_L)^c} \lambda_L + h.c.\} \\
& - (m_1^2 - m_H^2) \phi_{H_D}^\dagger \phi_{H_D} - (m_2^2 - m_H^2) \phi_{H_U}^\dagger \phi_{H_U} + m_3^2 (\phi_{H_D} \phi_{H_U} + h.c.) \ ,
\end{aligned} \tag{5}
$$

where the trilinear terms are necessary to assure the renormalizability of the theory[5]. The parameter can be set to be real, without loss of generality.

Though local $SU(2)_L$ symmetry is broken spontaneously, under some circumstance there remains a global symmetry $SU(2)_V$, characterized by the following transformation properties of supermultiplets,

$$
\begin{aligned}
& Q \to GQ \ , \quad \bar{Q} \equiv \begin{pmatrix} \bar{U} \\ \bar{D} \end{pmatrix} \to G^* \bar{Q} \ , \quad H \to GHG^\dagger \ , \\
& V^a T^a \to G(V^a T^a) G^\dagger \ , \quad V \to V \ ,
\end{aligned} \tag{6}
$$

where G denotes an element of $SU(2)_V$ and T^a represent SU(2) generators. Let us note that $SU(2)_L$ gauge bosons form a triplet representation of $SU(2)_V$ group. The parameter $\rho \equiv M_W^2/M_Z^2\cos^2\theta_W$ can deviate from unity only if the $SU(2)_V$ is broken[3]. In our lagrangian there are several sources of the breakdown of the global symmetry $SU(2)_V$:

(a) asymmetric Yukawa coupling constants, $f_D \neq f_U$

(b) weak-hypercharge interactions

(c) asymmetric vacuum expectation values, $\langle\phi_D^0\rangle \neq \langle\phi_U^0\rangle$

(d) asymmetric mass terms, $\bar{\mu}_D^2 \neq \bar{\mu}_U^2$ and $M_D^2 \neq M_U^2$

Since two Higgs doublets are needed, in contrast to the case of non-supersymmetric $SU(2)_L \times U(1)$ model, their asymmetric vacuum expectation values (associated with $m_1^2 \neq m_2^2$) may become a new source of $SU(2)_V$ breakdown in our model.

If the mass parameters in eq. (5) satisfy[5]

$$m_1^2 + m_2^2 > 2|m_3^2| , \quad m_3^4 > m_1^2 m_2^2 , \tag{7}$$

the two Higgs doublets acquire vacuum expectation values $v_U \equiv \langle\phi_U^0\rangle$ and $v_D \equiv \langle\phi_D^0\rangle$

$$v^2 \equiv v_U^2 + v_D^2 = -\frac{2[m_1^2-m_2^2+(m_1^2+m_2^2)\cos2\theta_v]}{(g^2+g'^2)\cos2\theta_v} , \tag{8}$$

$$\tan\theta_v \equiv v_U/v_D , \quad \sin2\theta_v = 2m_3^2/(m_1^2+m_2^2) , \tag{9}$$

which breaks $SU(2)_L \times U(1)$ gauge symmetry.

Here we summarize the mass eigenvalues and corresponding eigenstates of new particles.

(i) scalar quarks

There arises a mixing between S and $\bar{S}$ through the scalar trilinear terms in $L_{S.B.}$, with angles θ_u and θ_d defined by

$$\tan 2\theta_u = \frac{2m_u(M_U+\cot\theta_v m_H)}{\mu^2-\bar{\mu}_U^2+(\frac{1}{2}-\frac{5}{6}\tan^2\theta_W)\cos 2\theta_v M_W^2}\ , \quad \tan 2\theta_d = \frac{2m_d(M_D+\tan\theta_v m_H)}{\mu^2-\bar{\mu}_D^2+(-\frac{1}{2}+\frac{1}{6}\tan^2\theta_W)\cos 2\theta_v M_W^2}\ , \tag{10}$$

where m_u and m_d are quark masses and $M_W^2 = g^2v^2/2$. The mass eigenvalues of *scalar quarks* are given by

$$\begin{aligned} m^2_{S_u, \bar{S}_u} &= \frac{1}{2}\Big(2m_u^2+\mu^2+\bar{\mu}_U^2+ \frac{1}{2}(1+\tan^2\theta_W)\cos 2\theta_v M_W^2 \\ &\mp [(\mu^2-\bar{\mu}_U^2+(\frac{1}{2}-\frac{5}{6}\tan^2\theta_W)\cos 2\theta_v M_W^2)^2+4m_u^2(M_U+\cot\theta_v m_H)^2]^{1/2}\Big)\ , \\ m^2_{S_d, \bar{S}_d} &= \frac{1}{2}\Big(2m_d^2+\mu^2+\bar{\mu}_D^2- \frac{1}{2}(1+\tan^2\theta_W)\cos 2\theta_v M_W^2 \\ &\mp [(\mu^2-\bar{\mu}_D^2+(-\frac{1}{2}+\frac{1}{6}\tan^2\theta_W)\cos 2\theta_v M_W^2)^2+4m_d^2(M_D+\tan\theta_v m_H)^2]^{1/2}\Big)\ . \end{aligned} \tag{11}$$

(ii) gauge-Higgs fermions

The mass matrix for gauge-Higgs fermions is somewhat complicated, being admixture of Dirac and Majorana mass terms. To avoid inessential complications we shall consider hereafter only two typical cases for the values of gauge-fermion Majorana masses m and m', the Higgs mass m_H in the superpotential and the ratio $\tan\theta_v$ of Higgs vacuum expectation values in eq. (9):

(1) $m = m' = m_H = 0$,

(2) $\tan\theta_v = 1$ $(m = m' = m_H)$.

In both of these cases the mass-squared matrix for neutral gauge- and Higgs- fermions is automatically diagonal. The masses of charged fermions $w_1^\pm$ and $w_2^\pm$ and neutral fermions z and λ_γ are given by

$$m_{w_1}^2 = 2\cos^2\theta_v M_W^2 + m^2 \ , \quad m_{w_2}^2 = 2\sin^2\theta_v M_W^2 + m^2 \ ,$$

$$m_z^2 = M_Z^2 + m^2 \ , \qquad m_\gamma^2 = m^2 \ . \tag{12}$$

(iii) extra Higgs doublet

It is convenient to define a new basis of Higgs doublets ϕ and χ so that $\langle\phi\rangle = v$ and $\langle\chi\rangle = 0$. We have five physical Higgs fields $\chi^\pm$, χ_i , χ_r and ϕ_r (indices i and r denote imaginary and real parts of neutral Higgs fields respectively). The mass eigenstates are $\chi^\pm$, χ_i and two linear combinations of χ_r and ϕ_r with a mixing angle θ_H

$$\tan 2\theta_H = \frac{\sin 4\theta_v M_Z^2}{m_{\chi_i}^2 - \cos 4\theta_v M_Z^2} \ . \tag{13}$$

Their mass eigenvalues turn out to be

$$m_{\chi_i}^2 = m_1^2 + m_2^2 \ , \qquad m_{\chi^+}^2 = m_{\chi_i}^2 + M_W^2 \ ,$$

$$m_{\phi_r,\chi_r}^2 = \frac{1}{2}[m_{\chi_i}^2 + M_Z^2 \mp \sqrt{(m_{\chi_i}^2 + M_Z^2)^2 - 4(\cos^2 2\theta_v) m_{\chi_i}^2 M_Z^2}] \ . \tag{14}$$

III. The contributions to $\Delta\rho$ from new particles

Let us now discuss the one-loop contribution of the new particles to $\Delta\rho$. We first give formulae for these contributions following the method of Ref. 3. The deviation of ρ from unity can be given in terms of the difference of polarization tensors

π^{ab} of charged and neutral gauge bosons, $\Delta\rho = \rho - 1 = (\pi^{+-} - \pi^{33})/M_W^2$. The contributions to π^{ab} (a,b = 1, 2, 3) from scalar quarks, gauge-Higgs fermions and extra Higgs doublet can be expressed in terms of $SU(2)_L$ generators T^a and propagators Δ for the particles in question as follows*):

(i) scalar quarks

$$\pi_S^{ab} = -\frac{3}{2} i g^2 \int \frac{d^n k}{(2\pi)^n} k^2 \, \mathrm{Tr}\,([T_S^a, \Delta_S]\cdot[T_S^b, \Delta_S]) . \tag{15}$$

The color factor 3 is explicitly shown.

(ii) gauge-Higgs fermions

$$\pi_f^{ab} = -\frac{i}{2} g^2 \int \frac{d^n k}{(2\pi)^n} \mathrm{Tr}\big[2\mathrm{Re}\{T_f^a \Delta_f M_f + (T_f^a \Delta_f M_f)^T\} \times \{T_f^b \Delta_f M_f + (T_f^b \Delta_f M_f)^T\}^\dagger + k^2 [T_f^a, \Delta_f][T_f^b, \Delta_f]\big] , \tag{16}$$

(iii) extra Higgs doublet

$$\pi_H^{ab} = -\frac{i}{4} g^2 \int \frac{d^n k}{(2\pi)^n} k^2 \, \mathrm{Tr}\,([T_H^a, \Delta_H]\,[T_H^b, \Delta_H]) . \tag{17}$$

Here propagators, $\Delta_i = (k^2 - M_i^2)^{-1}$ for i = S, H and $\Delta_f = (k^2 - M_f M_f^\dagger)^{-1}$, correspond to the mass matrices given by

$$L_{mass} = -\phi_S^\dagger M_S^2 \phi_S - \frac{1}{2}[\bar{\psi}_f M_f \psi_f^c + \text{h.c.}] - \frac{1}{2}\phi_H^\dagger M_H^2 \phi_H , \tag{18}$$

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*) We use the convention of Bjorken and Drell.

in the bases

$$\Phi_S = \begin{pmatrix} S_u \\ S_d \\ \bar{S}_u^* \\ \bar{S}_d^* \end{pmatrix}, \quad \psi_f = \begin{pmatrix} \lambda^1 \\ \lambda^2 \\ \lambda^3 \\ \lambda \\ \psi_D^0 \\ \psi_D^- \\ \psi_U^+ \\ \psi_U^0 \end{pmatrix}_L, \quad \Phi_H = \begin{pmatrix} \phi^+ \\ \phi^- \\ \phi_r \\ \phi_i \\ \chi^+ \\ \chi^- \\ \chi_r \\ \chi_i \end{pmatrix}. \tag{19}$$

The $SU(2)_L$ generators T_i^a (i = S, f, H) are also in these bases. The formula given in Ref. 3 has had to be modified so that it is applicable to the gauge-Higgs fermions possessing Majorana mass terms as well as Dirac mass terms. In fact the first term of the right-hand side of eq. (16) does contribute to $\Delta\rho$, in contrast to the case of quarks and leptons.

The explicit evaluation of these formulae gives $\Delta\rho^i$ from each set of particles (i = S, f, H) as follows:

(i) scalar quarks

$$\Delta\rho^S = \frac{3}{4}\frac{g^2}{(4\pi)^2}\frac{1}{M_W^2}\{\cos^4\theta_u m_{S_u}^2 + \cos^4\theta_d m_{S_d}^2 + \sin^4\theta_u m_{\bar{S}_u}^2 + \sin^4\theta_d m_{\bar{S}_d}^2$$

$$-2\left(\cos^2\theta_u\cos^2\theta_d\,\frac{m^2_{S_u}m^2_{S_d}}{m^2_{S_u}-m^2_{S_d}}+\sin^2\theta_u\cos^2\theta_d\,\frac{m^2_{\bar{S}_u}m^2_{S_d}}{m^2_{\bar{S}_u}-m^2_{S_d}}+\cos^2\theta_u\sin^2\theta_d\,\frac{m^2_{S_u}m^2_{\bar{S}_d}}{m^2_{S_u}-m^2_{\bar{S}_d}}\right.$$

$$\left.+\sin^2\theta_u\sin^2\theta_d\,\frac{m^2_{\bar{S}_u}m^2_{\bar{S}_d}}{m^2_{\bar{S}_u}-m^2_{\bar{S}_d}}\right)\times\ \ln\frac{m^2_{S_u}}{m^2_{S_d}} \tag{20}$$

$$+2\sin^2\theta_u\left(\cos^2\theta_u\,\frac{m^2_{S_u}m^2_{\bar{S}_u}}{m^2_{S_u}-m^2_{\bar{S}_u}}+\cos^2\theta_d\,\frac{m^2_{\bar{S}_u}m^2_{S_d}}{m^2_{\bar{S}_u}-m^2_{S_d}}+\sin^2\theta_d\,\frac{m^2_{\bar{S}_u}m^2_{\bar{S}_d}}{m^2_{\bar{S}_u}-m^2_{\bar{S}_d}}\right)\ln\frac{m^2_{S_u}}{m^2_{\bar{S}_u}}$$

$$+2\sin^2\theta_d\left(\cos^2\theta_d\,\frac{m^2_{S_d}m^2_{\bar{S}_d}}{m^2_{S_d}-m^2_{\bar{S}_d}}-\cos^2\theta_u\,\frac{m^2_{S_u}m^2_{\bar{S}_d}}{m^2_{S_u}-m^2_{\bar{S}_d}}-\sin^2\theta_u\,\frac{m^2_{\bar{S}_u}m^2_{\bar{S}_d}}{m^2_{\bar{S}_u}-m^2_{\bar{S}_d}}\right)\ln\frac{m^2_{S_d}}{m^2_{\bar{S}_d}}\ \Big],$$

(ii) gauge-Higgs fermions

case (1): $m = m' = m_H = 0$

$$\Delta\rho^f = \frac{1}{4}\,\frac{g^2}{(4\pi)^2}\,\frac{1}{M_W^2}\ \Big[-3(m^2_{w_1}+m^2_{w_2}) + \cos^2 2\theta_v\, m_z^2$$

$$+\ \{4m^2_{w_1}+(-1+4\sin^2\theta_W-\cos 2\theta_v)m_z^2\}\ \frac{m^2_{w_1}}{m^2_{w_1}-m_z^2}\ \ln\frac{m^2_{w_1}}{m_z^2} \tag{21}$$

$$+\ \{4m^2_{w_2}+(-1+4\sin^2\theta_W+\cos 2\theta_v)m_z^2\}\ \frac{m^2_{w_2}}{m^2_{w_2}-m_z^2}\ \ln\frac{m^2_{w_2}}{m_z^2}\ \Big],$$

case (2): $\tan\theta_v = 1 \quad (m = m' = m_H \neq 0)$

$$\Delta\rho^f = \frac{1}{2}\,\frac{g^2}{(4\pi)^2}\,\frac{1}{M_W^2}\Big[-3m_w^2-2m_\gamma^2+\cos^2\theta_W(3m_z^2+2m_\gamma^2)\frac{m_z^2}{m_w^2-m_z^2}\,\ell n\,\frac{m_w^2}{m_z^2}$$

$$+5\sin^2\theta_W\,\frac{m_\gamma^4}{m_w^2-m_\gamma^2}\,\ell n\,\frac{m_w^2}{m_\gamma^2}\Big] \quad , \tag{22}$$

where $m_w \equiv m_{w_1} = m_{w_2}$.

(iii) extra Higgs doublet

$$\Delta\rho^H = \frac{1}{4}\,\frac{g^2}{(4\pi)^2}\,\frac{1}{M_W^2}\Big[m_{\chi^+}^2 - \cos^2\theta_H\Big\{\frac{1}{m_{\chi_i}^2-m_{\chi_r}^2}\Big(\frac{m_{\chi^+}^2-m_{\chi_r}^2}{m_{\chi^+}^2-m_{\chi_i}^2}\,m_{\chi_i}^4\,\ell n\,\frac{m_{\chi^+}^2}{m_{\chi_i}^2}$$

$$-\frac{m_{\chi^+}^2-m_{\chi_i}^2}{m_{\chi^+}^2-m_{\chi_r}^2}\,m_{\chi_r}^4\,\ell n\,\frac{m_{\chi^+}^2}{m_{\chi_r}^2}\Big)\Big\} \quad -\sin^2\theta_H\Big\{\frac{1}{m_{\chi_i}^2-m_{\phi_r}^2}\Big(\frac{m_{\chi^+}^2-m_{\phi_r}^2}{m_{\chi^+}^2-m_{\chi_i}^2}\,m_{\chi_i}^4\,\ell n\,\frac{m_{\chi^+}^2}{m_{\chi_i}^2}$$

$$-\frac{m_{\chi^+}^2-m_{\chi_i}^2}{m_{\chi^+}^2-m_{\phi_r}^2}\,m_{\phi_r}^4\,\ell n\,\frac{m_{\chi^+}^2}{m_{\phi_r}^2}\Big)\Big\}\Big] \quad . \tag{23}$$

In these expressions the masses and mixing angles of new particles are mutually interrelated, as given in the previous section. We have given the expression for $\Delta\rho^H$ in the *Landau gauge* ($\xi \to \infty$ in the R_ξ *gauge*). *In* this gauge $\Delta\rho$ has no contributions from unphysical scalars.

Now let us examine how large $\Delta\rho$ may become in supersymmetric models compared to the experimental bound. We shall see below that the potentially important effect comes only from the large mass splitting between up- and down-quark supermultiplets (or charged lepton and neutrino supermultiplets). Thus the situation is similar to the case of non-supersymmetric standard model, although the presence of many new particles bring about a number of modifications of the previous results[2].

As explained in the previous section there are four sources (a), (b), (c) and (d) of the breaking of the $SU(2)_V$. It will be shown in the end of this section that the source (d) does not give a significant contribution to $\Delta\rho \neq 0$. The remaining three sources have the following characteristic mass-squared differences:

(a) $m_u^2 - m_d^2$,
(b) $M_Z^2 - M_W^2$,
(c) $m_{w_1}^2 - m_{w_2}^2$,

respectively. If these mass-squared differences are not much larger than M_W^2, their contributions to $\Delta\rho = (\pi^{+-} - \pi^{33})/M_W^2$ cannot be much larger than $\alpha/4\pi$. Then $\Delta\rho$ is comfortably within the present experimental upper bound[12] $\Delta\rho_{exp} = 0.002 \pm 0.015$. Only the mass differences between up- and down-type quarks and/or scalar quarks can be significantly larger than M_W and can give a large contribution to $\Delta\rho$.

To illustrate the magnitude of $\Delta\rho^i$ in typical situations we shall evaluate the contributions from each set of new particles (i = S, f, H) in three extreme cases in which

(a) $f_U \neq 0$, $f_D = 0$ ($\tan\theta_V = 1$, $\theta_W = 0$) ,

(b) $\theta_W \neq 0$ ($f_U = f_D$, $\tan\theta_V = 1$) ,

(c) $\tan\theta_V = \infty$ ($\theta_W = 0$).

To see the typical situation in simple terms, we neglect all mass parameters in $L_{S.B.}$ and m_H when we evaluate $\Delta\rho$ in the following. Consequently the result can be expressed in terms of masses of ordinary particles and θ_W.

Case (a)

$$\Delta\rho^S = \frac{g^2}{(4\pi)^2} \frac{3}{4} \frac{m_u^2}{M_W^2} , \qquad \Delta\rho^f = \Delta\rho^H = 0 . \tag{24}$$

The scalar quarks give exactly the same contributions as those of ordinary quarks[2], e.g., $\Delta\rho^S = 3.5\times10^{-3}$ for $m_u = 100$ GeV. If we use the experimental bound $|\Delta\rho| < 0.015$, we obtain $m_u < 200$ GeV.

Case (b)

$$\Delta\rho^S = 0 , \qquad \Delta\rho^f = -6\Delta\rho^H , \tag{25}$$

where

$$\begin{aligned} \Delta\rho^f &= \frac{g^2}{(4\pi)^2} \left(-\frac{3}{2}\right) \left[1 + \frac{\ln(1-\sin^2\theta_W)}{\sin^2\theta_W}\right] \\ &\simeq \frac{g^2}{(4\pi)^2} \frac{3}{4} \sin^2\theta_W \quad \text{for} \quad \sin^2\theta_W \ll 1 . \end{aligned} \tag{26}$$

Case (c)

$$\Delta\rho^S = \frac{g^2}{(4\pi)^2}\frac{3}{4}\frac{1}{M_W^2}\left[m_u^2 - \frac{(2m_u^2-M_W^2)M_W^2}{2(m_u^2-M_W^2)}\,\ell n\,\frac{2m_u^2-M_W^2}{M_W^2}\right],$$

$$\Delta\rho^f = \frac{g^2}{(4\pi)^2}\frac{1}{4}\,[12\,\ell n\,2 - 5] \quad , \qquad \Delta\rho^H = \frac{g^2}{(4\pi)^2}\frac{1}{4} \quad . \tag{27}$$

From these results we see in fact that the potentially large contribution to $\Delta\rho$ comes from $\Delta\rho^S$ due to the large difference of Yukawa couplings, $f_U \neq f_D$ in case (a), or to the large Yukawa coupling f_U accompanied by the asymmetric vacuum expectation values in case (c).

The supersymmetry breaking mass scale M^2_{SUSY}, characterized by $m^2_{S_q} - m^2_q$, can be as large as one TeV, without spoiling the naturalness[1] of our effective low energy theory. We shall now estimate the effects of super-partners in the limit where $M^2_{SUSY} \gg m^2_{u,d}, M_W^2$. For simplicity, we consider the case $\mu^2, m^2 \gg m^2_{u,d}, M_W^2$ and assume $\tan\theta_v = 1$ and $\theta_{u,d} = 0$ $(M_U = M_D = -m_H)$,

$$\Delta\rho^S \simeq \frac{g^2}{(4\pi)^2}\frac{1}{4}\frac{(m_u^2-m_d^2)^2}{M_W^2 m^2_{S_u}} \qquad m^2_{S_u} \gg m_u^2,\ m_d^2 \ ,$$

$$\Delta\rho^f \simeq \frac{g^2}{(4\pi)^2}\frac{1}{3}\frac{M_Z^2-M_W^2}{m_Y^2} \qquad m_Y^2 \gg M_Z^2,\ M_W^2 \ . \tag{28}$$

Thus we can see the "decoupling" of superpartners from the low-energy sector of our theory, as may be expected.

Finally we study the sensitivity of $\Delta\rho^S$ to the source (d) of the $SU(2)_V$ breaking by turning off the other sources; $m_u = m_d$ and $\tan\theta_V = 1$. Our crude approximation, $\bar{\mu}^2_{U,D} = (M_{U,D}+m_H)^2 >> m_u^2 = m_d^2 = \mu^2$, leads to

$$\Delta\rho^S \simeq \frac{g^2}{(4\pi)^2}\frac{3}{4}\frac{m_u^4}{M_W^2}\left\{\frac{1}{m^2_{\bar{S}_u}} + \frac{1}{m^2_{\bar{S}_d}} - 2\frac{1}{m^2_{\bar{S}_u}-m^2_{\bar{S}_d}}\ln\frac{m^2_{\bar{S}_u}}{m^2_{\bar{S}_d}}\right\} . \qquad (29)$$

Hence the effect of the source (d) is suppressed by an inverse power of $m^2_{\bar{S}_{u,d}} \simeq \bar{\mu}^2_{U,D}$, as was mentioned in the previous section. A similar "decoupling" phenomenon has been noted for superheavy gauge bosons in SU(5) GUT[13].

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