IC/84/70

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1984 MIRAMARE-TRIESTE

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International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

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EFFECTIVE LAGRANGIANS FOR SUSY QCD WITH PROPERTIES SEEN IN PERTURBATION THEORY *

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ABSTRACT

We construct effective Lagrangians for supersymmetric QCD which properly incorporate the relevant Ward identities and possess features encountered in perturbation theory. This shows that the unusual scenarios, proposed for SUSY QCD, are not necessary.

MIRAMARE - TRIESTE
June 1984

* To be submitted for publication.

Our understanding of the spectrum and symmetry realizations in supersymmetric gauge theories is very uncertain [1-12]. One approach [4,5,6] has been to write manifestly supersymmetric effective Lagrangians in terms of some simple gauge invariant composite fields. However, unusual scenarios have been claimed to follow uniquely from this approach [4,5]. It has been pointed out elsewhere [6] that the way in which the anomalous conservation laws are incorporated in the approach of Veneziano et al. [4], is too restrictive and has been partly responsible for such scenarios. Here we will see that this trouble is compounded by some apparently obvious assumptions made in writing the effective Lagrangians.

In this letter, we construct effective Lagrangians for supersymmetric QCD, properly incorporating the symmetries of the original theory., These effective Lagrangians possess features encountered in perturbation theory. Therefore, there is no ground for unusual scenarios proposed on the basis of effective Lagrangians.

We will see that the D-terms can be crucial for calculating the vacuum expectation values, even when the vacuum is supersymmetric. We will consider a simple model, with close analogies to the case under consideration, to justify the class of effective Lagrangians that we propose.

We will consider the effective Lagrangians written in terms of chiral superfields $T_i^{\ j} = \sum_\alpha \phi_{Li}^{\ \alpha} \bar{\phi}_{R\alpha}^{\ \ j}$. Here, $\phi_{Li}^{\ \alpha}$ and $\bar{\Phi}_{R\alpha}^{\ \ j}$ ($\alpha = 1,2,\dots N$) are chiral (quark) superfields in N and N^* representations of the colour SU(N). $i,j = 1,2,\dots n$ are the flavour labels. The superfields $T_i^{\ j}$ were first used by Peskin [5] in the context of non-linear realization of the global symmetries. However, it has since been realized [13] that the linear and non-linear realizations are equivalent for the present case. We will, therefore, use the former.

Now there is an argument, which apparently implies, that the conclusions drawn in Refs.[5] are essentially unique. With massless (s)quarks, there is a non-anomalous R-invariance under which

$$T_i^{\ j}(x,\theta) \to e^{2i\alpha(N-n)}\, T_i^{\ j}(x,\theta e^{i\alpha n}), \quad (1)$$

Only possible F-term invariant under this transformation is,

$$\int d^4x \int d^2\theta \quad (\det T)^{-\frac{1}{N-n}} . \quad (2)$$

This term need not be present in the effective Lagrangian. If one assumes that it is present, then the usual method of obtaining supersymmetric vacua by an extremization of the F-terms gives,

$$\begin{aligned} \langle T \rangle &= \infty , \quad n < N \\ &= 0 , \quad n > N \end{aligned} \quad . \tag{3}$$

When the (s)quarks have a mass m, we would have an additional F-term,

$$\int d^4x \int d^2\theta \;\; m \, \Sigma_i \, T_i^{\;i} \tag{4}$$

Assuming a proper definition of renormalized composite operators, Ward identities for the symmetries broken by the mass terms give (4) as the only symmetry breaking term in the effective Lagrangian [6]. We now see why the F-term like in (2) seems to be forced upon us. Witten's index arguments require that there should be a supersymmetric ground state, at least for $m \neq 0$. This is apparently not possible, with only the F-term of (4).

It is the F-term of (2) which leads to pathologies. We get, for the associated Goldstone bosons,

$$\begin{aligned} m \to 0 : \; & m_\pi \to \infty \; , \;\; f_\pi \to 0 \; ; \quad n < N \\ & m_\pi \to 0 \; , \;\; f_\pi \to \infty \; ; \quad n \geq N . \end{aligned} \tag{5}$$

(This means that for $n > N$ there is no chiral symmetry breaking.)

Notice that the proposed solution does not work for $n = N$. A F-term of the type in (2) does not exist and therefore a mass term would apparently break supersymmetry, contradicting the index arguments.

Notice also the following: the squark potential for SUSY QCD is

$$\begin{aligned} & \tfrac{1}{2} g^2 \sum_a \Big[\sum_i \big(\varphi_L^{*i} T^a \varphi_{Li} - \varphi_R^{*i} T^a \varphi_R \big) \Big]^2 \\ & + m^2 \Sigma_i \big(\varphi_L^{*i} \varphi_{Li} + \varphi_R^{*i} \varphi_{Ri} \big) , \end{aligned} \tag{6}$$

where T^a are the generators of the colour group.

When $m = 0$, we have a continuum of vacua corresponding to any φ_L and φ_R such that $\sum_i (\varphi_L^{*\,i} T^a \varphi_{Li} = \varphi_R^{*\,i} T^a \varphi_{Ri}) = 0$, $\forall a$. This

corresponds [13] to an arbitrary VEV for $T_i^{\;j}$. When $m \neq 0$, there is a unique vacuum, $\varphi_L = \varphi_R = 0$, which classically corresponds to $T_i^{\;j} = 0$.

These results are valid to any order in renormalized perturbation theory because of the non-renormalization theorem [14]. Obviously these features are not being exhibited by the above effective Lagrangians. If the F-term of (2) were absent, $T_i^{\;j}$ would have an arbitrary VEV when $m = 0$. This is because, the D-terms would give only derivative terms in the scalar component of the superfield $T_i^{\;j}$. The F-term of (2) lifts this degeneracy, causing a breakdown of the non-renormalization theorem and can only come from non-perturbative effects.

While such non-perturbative effects cannot be ruled out a priori, it is also inconceivable that it is impossible to write effective Lagrangians possessing features seen in the perturbation theory. Therefore, the arguments which led to Eqs.(3) and (5) are to be suspected.

We will now consider a model which has close analogies to the case under consideration and where it is not possible to invoke non-perturbative effects. Consider free massive supersymmetric theory corresponding to one chiral superfield! The Lagrangian is

$$\begin{aligned} \mathcal{L} &= \bar{\Phi} \Phi \big|_D + \tfrac{1}{2} m \left(\Phi^2 + \bar{\Phi}^2 \right) \big|_F \\ &= -\partial_\mu A^* \partial_\mu A + i \partial_\mu \bar{\psi} \sigma^m \psi + F^* F \\ &\quad + m \left(A F + A^* F^* - \tfrac{1}{2} \psi \psi - \tfrac{1}{2} \bar{\psi} \bar{\psi} \right) . \end{aligned} \tag{7}$$

We want to write effective Lagrangian *) for the composite superfield $\chi \equiv \phi^2$. (We will presume some supersymmetric regularization.) The Lagrangian in (7) has an R-invariance, $\phi(x,\theta) \to e^{-i\alpha} \phi(x, \theta e^{i\alpha})$. Any R-transformation with a different R-charge is broken explicitly by the mass term. The corresponding Ward identity then implies that the only symmetry breaking term in the effective Lagrangian is $m(\chi + \bar{\chi})\big|_F$. This would also be the only F-term in

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*) There are no poles but only cuts in the channels of χ. Generally, effective Lagrangians are written for fields which are presumed to correspond to the particles in the theory. But this is not necessary. For us the effective Lagrangian is an object, which when used in the tree approximation, incorporates the basic features of the exact Green's functions. More important, it should give correct vacua on minimization.

the effective Lagrangian. It would appear that supersymmetry must be necessarily broken. But we know that it is not.

When $m = 0$, the vacuum expectation value of φ and hence of χ may be regarded as arbitrary. We have a continuum of vacua. When $m \neq 0$, we have $\langle\phi\rangle = 0$ and also $\langle\chi\rangle = 0$. How are these results reproduced by an effective Lagrangian for χ?

An effective Lagrangian should be regarded as a truncation [6] of the generating functional Γ of 1PI vertices [15]. This is all the more so, when the effective Lagrangian is used to calcualte the vacuum expectation values. This is because the exact VEV's are given by the extrema of Γ, equivalently of the effective potential. (The effective potential is obtained from Γ by dropping the space-time dependence of the fields and factoring out the space-time volume.)

Usually, the true vacuum corresponds to the absolute minimum [15] of the effective potential. However, because of the presence of auxiliary fields, the effective potential in supersymmetric theories must be handled with more care. This is seen even in the free Wess-Zumino model, Eq.(7). The effective potential is,

$$V(A,F) = -(F^{*}+mA)(F+mA^{*}) + m^{2}A^{*}A\ . \qquad (8)$$

This has the absolute minimum at $|F^{*} + mA| = \infty$, $A = 0$. However, the correct vacuum corresponds to $F^{*} + mA = 0$, $A = 0$ which is only a saddle point of $V(A,F)$. This illustrates the correct method of calculating the vacuum expectation values from supersymmetric effective Lagrangians Γ. Find the simultaneous solution of the equations

$$\frac{\partial\Gamma}{\partial A_i} = 0\ , \quad \frac{\partial\Gamma}{\partial F_i} = 0\ , \qquad (9)$$

for space-time independent A_i and F_i. Here A_i and F_i are the A and F components of the chiral superfields ϕ_i used in Γ. Only in (effective) Lagrangians with simple D-terms this will be equivalent to extremizing the F-terms *).

*) We may use the second of Eqs.(9) to eliminate F in favour of A in Γ. The true vacuum is then given by the extrema of $\Gamma[A] \equiv \Gamma[A,F(A)]$, for space-time independent A. This follows from Eqs.(9).

When using effective Lagrangians written in terms of superfields, two more facts have to be kept in mind. (1) D-terms, involving the superfields $D^2\chi, \bar{D}^2\bar{\chi}$, are to be expected, in general. (2) D-terms which are non-analytic in superfields are to be expected, more often than not. These, apparently unphysical features are present because of the use of auxiliary fields. Once auxiliary fields are eliminated, these features disappear too.

We will illustrate these features for the case of $\chi \equiv \phi^2$ in the model, Eq.(7). It will be clear later that the features are not special to this model.

To calculate Γ, one must first couple χ to an external source J via the F-term,

$$\phi^2 J\big|_F + h.c. = j\left(AF - \tfrac{1}{2}\psi\psi\right) - \eta\psi A + \tfrac{1}{2}sA^2 + h.c. \qquad (10)$$

where J is a chiral superfield (j,η,s). One then integrates over the quantum fields to get $W[J]$, the generating functional of connected Green's functions of composite superfield $\chi = (c,\xi,f)$. With

$$\chi[J] = \frac{\delta W}{\delta J}\ , \qquad (11)$$

Γ is defined as,

$$\Gamma[\chi] = W[J] - \left(\int d^4x\ J\chi + h.c.\right)\Big|_F\ . \qquad (12)$$

Here J on the right hand side is presumed to be eliminated in favour of χ using Eq.(11).

An explicit inversion of Eq.(11) is not possible even for the case of free field theory. Therefore we will content ourselves with a calculation of the effective potential, $V(c,\xi = 0,f)$. For this it suffices *) to use a space-time independent source $J = (j,0,s)$. Then, for $\mathcal{E} = (\int d^4x)^{-1}\, W(j,s)$, we get

$$\mathcal{E}(M,S) = \int \frac{d^4p}{(2\pi)^4}\ \frac{1}{2}\ \ln\left|\frac{(p^2+MM^{*})^2 - SS^{*}}{(p^2+MM^{*})^2}\right| \qquad (13)$$

*) This procedure. of using constant external sources is correct. However, when used for obtaining the effective potential for the basic fields of the theory, it is important to handle the infinite volume limit carefully.

where $M = m + j$ and p is the Euclidean momentum. In the argument of the logarithmic function, the numerator is the contribution from the bosons and the denominator is that of the fermions.

For $s = 0$, the effective potential vanishes. This is to be expected. The effect of j is to simply shift the mass and therefore supersymmetry is intact. The vacuum energy density stays at zero. When the F-component s of the source is non-zero, supersymmetry is explicitly broken and $\varepsilon \neq 0$ anymore *).

The expectation values of $f \equiv AF - 1/2\ \psi\psi$ and $c \equiv 1/2\ A^2$, in presence of the external sources are,

$$c = \frac{\partial \varepsilon}{\partial s} = -s^* \int \frac{d^4p}{(2\pi)^4}\, \frac{1}{2}\, \frac{1}{(p^2 + MM^*)^2 - ss^*} ,$$

$$f = \frac{\partial \varepsilon}{\partial M} = M^* ss^* \int \frac{d^4p}{(2\pi)^4}\, \frac{1}{(p^2 + MM^*)\left((p^2+MM^*)^2 - ss^*\right)} . \qquad (14)$$

Notice that $c = 0$, $f = 0$ when the external sources are switched off, as we expect.

The effective potential $V(c,f)$ is computed from $\varepsilon(M,S)$ as follows:

$$V(c,f) = \varepsilon(M,s) - \chi s - \chi^* s^* - fj - f^* j^*$$

$$= \left[\varepsilon(M,s) - \chi s - \chi^* s^* - fM - f^* M^*\right] + (mf + mf^*) . \qquad (15)$$

We have added and subtracted $mf + mf^*$ in the last step. The expression in square brackets is invariant under the "broken" R-transformation. It is a function of cc^* and ff^* combination only (after expressing MM^* and SS^* in terms of cc^* and ff^*). Therefore, it can only come from the D-terms in a superfield formulation, whereas the last term in Eq.(15) is the unique, symmetry breaking, F-term.

We notice immediately that it is not sufficient to consider the F-terms only to find the vacuum expectation values. V is also not just bilinear in f and f^*. This is possible only when $D^2\chi$ and $\bar{D}^2\bar{\chi}$ terms are present in Γ and moreover these occur with the massless propagators ∂^{-2} in order to be

*) The negative sign in from of SS^* in Eq.(13) is correct and unavoidable. This means that $\varepsilon(M,S)$ has a logarithmic singularity in $(MM^* - SS^*)$ and for $|S| > |M|$, the effective potential becomes complex and multiple-valued.

non-vanishing for constant f and c fields. Also, $V(c,f)$ involves terms like $ff^*/(cc^*)^2$, i.e. it is singular in c.

These features are not special to our example. There is no reason for the effective potential to be bilinear in f and f^*. Also, $\varepsilon \to 0$ as $s \to 0$, independently of j, because j does not break supersymmetry. This is the origin of singularities in c.

We will now consider a puppet model, which has all the important features indicated above and with which explicit manipulations can be carried out. This model shows how we can have a unique vacuum at $c = 0$ when $m \neq 0$ and a continuum of vacua with $\langle c \rangle$ = arbitrary, when $m = 0$; all this with supersymmetry unbroken.

Consider the class of effective potentials of the form,

$$-V(c,f) = \frac{ff^*}{g(cc^*)} + mf + mf^* , \qquad (16)$$

where $g(x)$ is regular at $x = 0$, which means V is singular in cc^*. This effective potential comes from an $\varepsilon(M,S)$ of the type,

$$\varepsilon(M,s) = MM^*\, h\left(\frac{ss^*}{(MM^*)^2}\right) , \qquad (17)$$

where $h(x) \to 0$ as $x \to 0$. For example, we may choose,

$$\varepsilon(M,s) = \frac{ss^*}{\sqrt{ss^*} + MM^*} , \qquad (18)$$

which has the general properties for ε extracted from our model.

Using the "equation of motion" for f,

$$\frac{f}{g(cc^*)} = -m . \qquad (19)$$

We get the effective potential in terms of the physical fields,

$$V = m^2\, g(cc^*) . \qquad (20)$$

For $m \neq 0$, we have a unique vacuum at $c = 0$ (on choosing a suitable g). For $m = 0$, the potential becomes flat and the VEV of c is completely arbitrary. Supersymmetry is unbroken in every case. The effective potential Eq.(16), is singular when written in terms of the superfields, but takes an acceptable form, Eq.(20), on eliminating the auxiliary fields.

We propose that these mechanisms are also operating in the case of supersymmetric QCD. This makes it possible to construct effective Lagrangians for SUSY QCD, which are consistent with the features seen in perturbation theory. Various authors [4,5] have claimed that correct implementation of the Ward identities within the framework of effective Lagrangians imply a break-down of the non-renormalization theorem. It is claimed that the massless limit of supersymmetric QCD is pathological. The considerations of this paper make it clear that such conclusions are drastic. The use of auxiliary fields result in apparent pathologies in the effective Lagrangians rather than in the properties of the theory.

It has been claimed that the instanton calculations [11] and sum rules [12] etc. support the unusual scenarios obtained from the effective Lagrangians. But the considerations of this paper suggest that many subtleties are present in supersymmetric theories and great care must be exercised in drawing conclusions. It is quite probable that a careful calculation will give results consistent with the non-renormalization theorem. It has been shown in Ref.[8] that the continuum of supersymmetric vacua seen in perturbation theory in massless supersymmetric QCD is consistent with some non-perturbative constraints, such as 't Hooft anomaly matching conditions.

ACKNOWLEDGMENTS

The author is grateful to Professor G. Veneziano for discusssions and criticisms which motivated this paper. He thanks Drs. R. Kaul, B. Milewski and K.S. Narain for valuable discussions. He would also like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

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