CONF-8405193--39

ANL-HEP-CP-84-45

# PROBLEMS WITH QUANTIZING THE SKYRMION: A CRITICAL REVIEW*

John P. Ralston
High Energy Physics Division
Argonne National Laboratory
Argonne, IL 60439

ANL-HEP-CP--84-45

DE84 015826

ABSTRACT

We review the motivation and construction of the chiral soliton picture of baryons. We discuss the semi-classical quantization procedure of Adkins, Nappi and Witten and the stability of the semi-classical solution under the collective coordinate quantization. By studying the behavior in the chiral limit and specific numerical predictions, we conclude that the collective coordinate procedure is inadequate.

## THE APPROACH

For many years people have discussed the concept of an effective action description of low-energy hadron dynamics. For years people have also known that low energy pion dynamics is significantly constrained by chiral symmetry.[1] Skyrme's old suggestion[2] that baryon number is associated with topological solitons, and the fact that such solitons are present in chiral models, was perhaps not so well known. Recently, however, there has been such a renaissance of interest and activity in these topics that some explanation, and a partly pedagogical review, is called for.

Although Pak and Tze[3] had attempted to revive interest in chiral solitons in 1979, a significant contribution was made by Rosenzweig et al. and diVecchia et al.[4] in discussions of the large $N_c$ limit of QCD. These workers argued that a kind of non-linear sigma model is a natural outcome of low energy hadron dynamics in this limit. Although interest at first concentrated on incorporating anomalies (the Wess-Zumino problem) and the related problem of the coupling of the $\eta'$, Balachandran et al.[4] soon realized that the models naturally embodied the conjecture that baryons could be solitons.[5] This prompted the work of Adkins, Nappi and Witten[6] which sparked a great deal of activity through its appealing and simple semi-classical quantization of the same soliton found by Skyrme in 1960.

In the 24 year gap between Skyrme's initial insight and his work's present state of popularity, physicists discovered QCD. We also discovered that QCD is difficult, and we (as a community) discovered enough complexity to understand that QCD is unlikely to provide a simple picture of low energy hadron dynamics in terms of quark and gluon fields. With this background it makes sense to return to the effective action approach, although all our knowledge of QCD has not yet provided a huge amount of guidance.

The approach[1,4] is to create a hadronic Lagrangian incorporating the flavor symmetries of QCD; color, of course, is

---

* (presented at Conference on Intersections of Nuclear and Particle Physics, Steamboat Springs, Colorado, May 1984).

hidden. For SU(2) flavor, the quantum symmetry should be $SU(2)_L \times SU(2)_R \times U(1)$, with the axial symmetry spontaneously broken. A convenient way to implement this is to write down the most general Lagrangian in terms of

$$L_\mu = U^+ \partial_\mu U \tag{1}$$

where U(x,t) is an SU(2) matrix. $U_{ij}$ can be related to pions ($\vec{\pi}$) via some expansion, e.g. $U = 1 + 2i\vec{\pi}\cdot\vec{\tau}/F_\pi + \ldots$, and the low energy expansion of the pion Lagrangian is understood to be an expansion in derivatives (powers of $L_\mu$). Since the pion is a Goldstone boson, a mass term which is a function of U (not $L_\mu$) is tolerated only as a correction to the chiral limit $m_\pi \to 0$.

The Lagrangian density for the Skyrme model with pion masses is

$$\mathscr{L} = \frac{F_\pi^2}{16} \mathrm{Tr}\left(\partial_\mu U^+ \partial_\mu U\right) - \frac{m_\pi^2 F_\pi^2}{8} (2 - \mathrm{Tr}\ U)$$
$$- \frac{1}{16e^2} \mathrm{Tr}\left(\partial_\mu U\ \partial_\mu U^+ \partial_\nu U\ \partial_\nu U^+ - \partial_\mu U\ \partial_\nu U^+ \partial_\mu U\ \partial_\nu U^+\right), \tag{2}$$

in which a choice of one of the two possible four-derivative terms has been made. The model has three parameters: the pion decay constant $F_\pi$ = 186 MeV, the pion mass $m_\pi$, and the Skyrme constant e. A configuration with finite energy must satisfy the boundary condition $U(\vec{r},t) \to 1$ as $|\vec{r}| \to \infty$. The configurations then fall into topological sectors which can be labelled by the integer valued index

$$B = \frac{1}{24\pi^2} \varepsilon^{ijk} \int d^3x\ \mathrm{Tr}\left\{U^+ \partial_i U\ \partial_j U^+ \partial_k U\right\}. \tag{3}$$

Skyrme interpreted the index B as baryon number, in which case the lowest energy state in the B = 1 sector should be identified with the nucleon. The lowest <u>classical</u> energy $E_0$ is attained by the static "spherically-symmetric" Skyrme solution, which has the form

$$U_0(\vec{r}) = e^{iF_0(r)\hat{r}\cdot\vec{\tau}} \tag{4}$$

with boundary conditions $F_0(0) = \pi$ and $F_0(\infty) = 0$. The function $F_0(r)$ and the energy $E_0$ must be determined numerically. This solution has classical angular momentum $\vec{J} = 0$ and isospin $\vec{I} = 0$. Some work must be done to relate it to the quantized nucleon, which has spin and isospin 1/2.

## QUANTIZATION

Adkins, Nappi and Witten[6] (ANW) chose to quantize the B=1 sector by using the collective coordinate method. The motivation for this is to incorporate the dynamical degrees of freedom associated with zero frequency modes. (In traditional soliton physics[7], such modes usually lead to infrared divergences when quantum corrections are added unless they are explicitly treated in this manner). In the case of the ansatz (4), the rotational and isospin symmetries imply that $AU_0(r,t)A^+$, where A is a constant SU(2) matrix, has the same energy as the Skyrme solution $U_0$. The matrix A parametrizes the zero modes; promoting A to a dynamical variable A(t) and quantizing it amounts to trading the infinite number of quantum variables U(x,t) for the three quantum degrees of freedom describing A. In addition, U is also described by an infinite number of classical coordinates--$F_0(r)$ in the solution (4)--which would be quantized in a fuller treatment. The quantum Hamiltonian of the A variables is then diagonalized. This results in a complicated non-local effective Hamiltonian for the remaining classical degrees of freedom, and of course, disregards vibrational modes that would otherwise be associated with fluctuations of U(x,t).

To understand the physics of this approximation one can use a physical analogy.[8] The replacement

$$U(x,t) \rightarrow A(t)\ U(x,t)\ A^+(t) \tag{5}$$

corresponds to a global isospin rotation. For general configurations in the B=1 sector, the rotation $A\exp(i\hat{n}\cdot\vec{\tau}G(x,t))A^+ = \exp(iA\hat{n}\cdot\vec{\tau}A^+G(x,t))$ allows the $\hat{n}$ degree of freedom to vary on the surface of a sphere ($\hat{n}^2 = 1$). Diagonalizing the energy associated with the collective motion of $\hat{n}(x)$ is analogous to separating out the spherical harmonics in the non-relativistic quantum mechanics of a particle in a central potential. There is, in fact, a useful parallel with the semiclassical quantization of the hydrogen atom by Bohr, which quantized the rotational degree of freedom.

A classical (delta function) approximation can be made for the distribution of the remaining radial degrees of freedom if the rotational energy is large; such concepts are familiar from, e.g., molecular physics. Similarly, in the soliton problem one hopes the analogous semiclassical treatment (which formally keeps terms from rotation of order $\hbar^2$ while discarding vibrational modes of order $\hbar$) would be sufficient.

If one limits oneself to rotations of the Skyrme solution $U_0$, the collective coordinates yield the Lagrangian

$$L = -M(F_0) + \Lambda(F_0)\mathrm{Tr}[\dot{A}\dot{A}^+] \tag{6}$$

where $M(F_0)$ is the original Skyrme energy functional, and $\Lambda$ is given by

$$\Lambda(F) = \int 4\pi r^2 dr \{\frac{F_\pi^2}{6} \sin^2(F) + \frac{2}{3e^2} \sin^2(F)[(\frac{\partial F}{\partial r})^2 + \frac{\sin^2 F}{r^2}]\} .$$

It is easy to understand that the collective motion has added a rotational energy analogous to $I\omega^2/2$, where I = moment of inertia $\propto \Lambda$ and $Tr[\dot{A}\dot{A}^+]$ corresponds to $\omega^2$. Using the parameterization $A = a_0 1 + i\vec{a}\cdot\vec{\tau}$, ANW canonically quantize (6) by writing

$$p_i = \frac{\partial L}{\partial \dot{a}_i} = 4\Lambda(F_0)\dot{a}_i ; \quad \sum_i a_i^2 = 1 ,$$

and diagonalize H, the effective Hamiltonian to give

$$H = M(F_0) + \ell(\ell + 1)/2\Lambda(F_0) \qquad \ell = 1/2, 3/2... \tag{7}$$

when operating on states of definite isospin I and spin J with $\vec{I}^2 = \vec{J}^2 = \ell^2$. Quantization of the operator A also yields expressions for various operators in the theory.[6] For example the isovector charge operator is given by

$$\begin{aligned} I^k &= \int d^3x \, V^{\hat{a},0} \\ &= 2i\Lambda Tr(\dot{A}A^{-1}\tau^k) . \end{aligned} \tag{8}$$

In these expressions $F_0$ is proportional to the operator 1; the semi-classical approximation is to consider the quantum wave functional of this degree of freedom in a baryon state to be sharply peaked about a classical value at each point in space.

Using this strategy ANW went on to calculate a number of static properties of baryons, identifying the proton and delta with the states of $\ell = 1/2$ and $3/2$, respectively. Several results[6] for isoscalar and isovector quantities such as charge radii, magnetic moments, etc. are given; the typical accuracy is about 30%.

Stimulated by this work, a number of authors have gone on to investigate several aspects of the Skyrme model. For example, elaborations for SU(3) versus SU(2) have been studied by Guadagnini, by Bijnens, et al., and by Manohar[9]. Schnitzer[10] has studied strong CP effects such as the neutron dipole moment, and a beginning on the problem of the two-nucleon potential has been made by several authors including Jackson et al. and Hlousek[11]. Callan and Witten[12] have shown how changing the boundary conditions of the Skyrmion in the presence of a monopole can translate a grand-unified picture of nucleon decay into the topological language of the model, and interesting relations for large $N_c$ (the number of colors) have been given by Bardakci[13] and Manohar.[9c] Finally, π-nucleon scattering has been considered from the viewpoint of small oscillations about the

Skyrme solution by several groups.[14] A consistent treatment of small oscillations, however, involves consideration of the stability of the classical Skyrme configuration for the wave function under the quantum corrections, and this raises new issues.

## PROBLEMS

There are problems. As pointed out in Ref.(8), there is good reason to expect the shape of the Skyrmion to change because of the quantum corrections of the rotational collective mode. There is a precedent for this. In the related treatment of translational collective modes, Gervais and Sakita[15] discussed the feedback of the quantum corrections into the classical effective action for solitons. In this case solving for the shape of the soliton (after diagonalizing the collective translations of arbitrary configurations) restores the proper Lorentz transformation properties of solitons; it gives the expected kinematic contraction from boosts.

In the case of rotational collective motion the distortion is not kinematic. The procedure of solving for the classical shape after the quantum corrections have been made is quite similar, however. One notes that the Skyrme solution ($F_0$) is not an extremum of the effective Hamiltonian (7): it minimizes $M_0(F_0)$, not $M_0(F_0) + \ell(\ell+1)/\Lambda(F_0)$. It follows that a small oscillation treatment starting with (4) and (7) will not be stable, but will reveal modes that grow rather than oscillate harmonically. This was also discussed by Bander and Hayot[16] and in recent work of Biedenharn et al.[16] To minimize the sum of the terms in (7), one obviously can increase $\Lambda$ at the expense of M. That is, the soliton should undergo "centrifugal stretching" to compensate the additional centrifugal potential energy induced by the quantum rotation in (6) and (7). In the language of the Bohr atom, the radial wavefunction should be concentrated in the configuration where the sum of the normal potential and the centrifugal potential is minimized.

It is not difficult to study this effect if one employs a variational principle for the baryon wavefunction.[8] Granting that the contribution from zero-point (vibrational) energies be considered small, one can choose an ansatz for the operator U given by

$$U(r,t) = A(t)e^{i\hat{r}\cdot\vec{\tau}F_\ell(r)}A^+(t) \tag{9}$$

in which the coordinates $F_\ell(r)$ are to be determined, and A(t) is the collective mode. In a variational treatment one fixes $F_\ell(r)$ by minimizing the expectation value of the energy

$$\left.\frac{\delta M_0(F)}{\delta F} - \frac{\ell(\ell+1)}{2\Lambda^2}\frac{\delta\Lambda(F)}{\delta F}\right|_{F=F_\ell} = 0 \tag{10}$$

which implements the centrifugal stretching through the second term. We have studied (10) using numerical techniques, but valuable analytic information can be gained by looking at large r where the boundary conditions require $F_\ell(r) \ll 1$.

As $r \to \infty$, there are two competing effects on the baryon wavefunction: a pion mass term makes $F_\ell(r)$ fall to zero quickly, while the centrifugal energy is decreased if $F_\ell(r)$ can fall slowly. These lead us to the following bound[8]

$$\Lambda(F_\ell) > \sqrt{2\ell(\ell + 1)/3}\ \frac{1}{m_\pi}\ , \tag{11}$$

from the asymptotic analysis of (10). Note that we do not expect radial symmetry $\left((9)\right)$ to give the exact form for the stretched Skyrmion; it follows that there may be a more stringent bound on $\Lambda$ than (11). In fact, configurations without radial symmetry have been considered;[17] one should consider (7) as a generic rotational collective coordinate result. Different moments of inertia may enter, but always with a tendency to increase in size to lower the energy.

To complete this discussion, we should estimate the characteristic time for a normal Skyrmion to change shape under the centrifugal influence. If the characteristic time of growth is very much longer than a rotational period, one might attempt to justify keeping the Skyrme solution as a sort of Born-Oppenheimer approximation. To check this, one can find time dependent solutions starting with Skyrme's solution at $t = 0$,

$$F_\ell(r,t) \sim F_0(r)\exp(\Gamma t)$$

giving $\Gamma = \sqrt{2\ell(\ell + 1)/3\Lambda^2}$, while the semi-classical rotational frequency is

$$\nu = \frac{\sqrt{\ell(\ell + 1)}}{\Lambda}\ .$$

Treating the Skyrme solution as quasi-static would be valid for $\nu \gg \Gamma$. Since $\nu/\Gamma = \sqrt{3/2}$ is not large, the Skyrme solution changes shape significantly within a rotational period.

That (11) unfortunately leads to significant changes in the predictions of the model is obvious; some of the effects in the chiral limit ($m_\pi \to 0$) are catastrophic. We can list a few obvious consequences:

a) The radial ansatz proton ($\ell = 1/2$) expands until its asymptotic structure is decribed by $F_{1/2} \sim \exp(-\mu r)/r$, where $0 < \mu < m_\pi$; computer calculations for realistic parameters give $\mu \cong .5m_\pi$. This is unacceptable, and will lead, e.g., to the wrong classical two-nucleon interaction.

b) Various static properties can be bounded by (11). The bounds have sensitive chiral limits, e.g. the isoscalar g factor must obey

$$g_p - g_n > 4M_N/3\sqrt{2}\, m_\pi \ .$$

c) If one proposes that the delta ($\ell = 3/2$) has a semi-classical description as a metastable extremum of H with quantized $\vec{I}$ and $\vec{J}$, one finds[8]

$$M_\Delta - M_N < 3(\sqrt{5} - 1)m_\pi/\sqrt{8} \ ,$$

a relation that is violated by the experimental values.

The properties (a–c) above are consequences of (11), a bound independent of e and $F_\pi$. One cannot repair the damage by adjusting these parameters. Although we believe the properties above are physically unacceptable, certain features of the chiral limit are mathematically well defined. For example, the isospin generators (9) have a sensible limit for any $\Lambda$.

How can these problems be reconciled with the large $N_c$ limit which, in some sense, was responsible for the renewed interest in chiral models? As discussed by Bardakci,[13] the soliton's existence at large $N_c$ is consistent with $F_\pi$ and 1/e being of order $N_c^{1/2}$. The Skyrme Hamiltonian is then of order $N_c$, while its quantum corrections $\left(\ell(\ell + 1)/2\Lambda\right)$ scale like $N_c^{-1}$. As $N_c \to \infty$ the problem is circumvented but the predictions of ANW are lost; the baby is thrown out with the bath. Of course, for fixed $N_c$ but $m_\pi \to 0$ (the limit traditionally of interest) the problem remains. This subtlety in the interchange of limits might have been anticipated by examining the $N_c$ scaling properties of the original Lagrangian (2), but we do not believe it is of practical relevance in the comparison with data. This is because in the phenomenological problem of the real proton at fixed $m_\pi = 138$ MeV and $N_c = 3$ the numerical size of the rotational corrections is quite large, and not a small perturbation. In this light, the parallels of the present treatment of the Skyrme model with static strong coupling models of years ago[18] (which also experienced centrifugal stretching) is worth studying.

If one accepts the problem, an obvious weakness is the collective coordinate approximation. A trivial failure of the treatment is that it does not reproduce free-pion field theory in the asymptotic region $\vec{r} \to \infty$, $U \sim 1 + 2i\vec{\pi}\cdot\vec{\tau}/F_\pi + \ldots$ where free pions are known to live. One might disregard this—one cannot expect the truncation of the field theory by the collective coordinate approximation to reproduce all the degrees of freedom—except that the useful predictions of the model are quite sensitive to the asymptotic region in the chiral limit. This suggests that a hybrid wave function for the baryon, which interpolates between free field theory and a truncated theory in the interior, might be an acceptable compromise.

We will conclude by listing a few questions. Some of these are related to the discussion above, while others are of a more general character:

a) What is the semiclassical description of unstable particles? Can a metastable classical solution be convincingly related to analytic properties of the S-matrix of an appropriate scattering process?
b) Since the effects of quantum corrections are not necessarily local, how does one separate such effects from the non-leading (possibly non-local) terms in the effective Lagrangian expansion?
c) Would lattice techniques be useful for investigating the non-perturbative structure of the Skyrmion?
d) We know that every Green function in the theory possesses Adler zeroes in the chiral limit. Is this damping of infrared propagation related to the apparent failure of the zero modes to dominate the dynamics?
e) Is there a way to restore the vibrational modes that will maintain the attractive simplicity of the Skyrme model itself?

## ACKNOWLEDGMENT

I thank A. Goldhaber and E. Braaten for useful discussions.

## REFERENCES

1. S. Weinberg, Physica 96A (1979) 327.
2. T. H. R. Skyrme, Proc. Roy. Soc. A260 (1961) 127.
3. N. K. Pak and A. C. Tze, Ann. Phys. (NY) 117 (1979) 164.
4. C. Rosenzweig, J. Schechter and C. Trahern, Phys. Rev. D21 (1980) 3388; P. di Vecchia and G. Veneziano, Nuc. Phys. B171 (1980) 253; A. P. Balachandran, V. P. Nair, S. G. Rajeev and A. Stern, Phys. Rev. Lett. 49 (1982) and Phys. Rev. D27 (1983) 1153.
5. E. Witten, Nuc. Phys. B160 (1979) 57.
6. G. S. Adkins, C. R. Nappi and E. Witten, Nuc. Phys. B228 (1983) 552.
7. See, e.g. R. Rajaraman in Solitons and Instantons (North Holland, Amsterdam 1982).
8. E. Braaten and J. P. Ralston, Argonne preprint ANL-HEP-PR-84-12.
9. E. Guagdignini, Princeton preprint; J. Bijnens, H. Sonada and M. Wise, Caltech preprint CALT-68-1096 (March 1984); A. Manohar, Harvard preprint HUTP-84/A030 (May 1984); P. Mazur et al., Jagellonian preprint.
10. H. Schnitzer, Harvard preprint HUTP-83/A082 (December 1983).
11. A. Jackson, A. D. Jackson and V. Pasquier, Stony Brook preprint; Z. Hlousek, Brown preprint BROWN-HET-526.
12. C. Callan and E. Witten, Princeton preprint (November 1983).
13. K. Bardakci, Berkeley preprint UCB-PTH-83/21.
14. Ch. Hajduk and B. Schwesinger, Stony Brook preprints (1984); J. Dey and J. LeTourneux, Montreal preprint (1984); A. Hayashi, G. Eckart, G. Holzwarth and H. Walliser, Siegen preprint (1984); J. D. Breit and C. Nappi, Princeton preprint (April 1984); J.

Parmentola, Pittsburgh preprint; H. Walliser and G. Eckart, Siegen preprint; K. Liu (private communication).

15. J. L. Gervais and B. Sakita, Phys. Rev. D11 (1975) 2943.
16. M. Bander and F. Hayot, Saclay preprint (May 1984) ; L. C. Biedenharn, Y. Dothan and M. Tarlini, Austin preprint DOE-ER-03992-556 (May 1984).
17. Ch. Hajduk and B. S. Schwesinger, Ref.14; E. Braaten (in preparation).
18. J. L. Gervais and B. Sakita, Phys. Rev. Lett. 52 (1984) 87; K. Bardakci, Ref.13; C. Goebel in Quanta, Edited by P. Freund, C. Goebel and Y. Nambu (University of Chicago Press 1970).

## DISCLAIMER