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MINIMIZING THE ENERGY SPREAD WITHIN A SINGLE BUNCH BY SHAPING ITS CHARGE DISTRIBUTION*

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Introduction

When electrons or positrons in a bunch pass through the periodic structure of a linear accelerator, they leave behind them energy in the form of longitudinal wake fields. The longitudinal fields left behind by early particles in a bunch decrease† the energy of later particles. For a linear collider, the energy spread introduced within the bunches by this beam loading effect must be minimized because it limits the degree to which the particles can be focused to a small spot due to chromatic effects in the final focus system.‡ For example, for the SLC, the allowable energy spread is ±0.5%.

It has been known for some time that partial compensation of the longitudinal wake field effects can be obtained for any bunch by placing it ahead of the accelerating crest (in space), thereby letting the positive rising sinusoidal field offset the negative beam loading field.[1] The work presented in this report shows that it is possible to obtain complete compensation, i.e., to reduce the energy spread essentially to zero by properly shaping the longitudinal charge distribution of the bunch and by placing it at the correct position on the wave.

Optimizing the Bunch Shape

The energy gained by a single particle riding at an angle θ_1 with respect to the crest of a traveling wave of accelerating gradient E_0 over a length L is

$$V = E_0 L \cos\theta_1 \ . \qquad (1)$$

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†Actually, if one waits long enough, the wake fields change sign and produce acceleration.

‡There may also be measurements of particle resonances which would benefit from extremely narrow energy spreads.

In the case of a bunch consisting of many particles, this energy is modified by the presence of the wake fields left by particles ahead of θ_1. For the examples worked out in this report, we will use the SLAC constant-gradient structure although the technique should be applicable to any structure for which the longitudinal wake function is known. This wake function, $w_L(\theta)$, is defined as the voltage excited by a unit charge traversing the structure. It is shown in Fig. 1 as calculated for a single average cavity of length d ($d = 3.5$ cm) of the SLAC structure.[2] To obtain the function W_L for the entire accelerator, one simply has to multiply $w_L(\theta)$ by N, the number of cavities (L/d). With these definitions and a bunch charge distribution $f(\theta')$ as illustrated in Fig. 2, Eq. (1) now becomes:

$$V(\theta_1) = V_0 \cos\theta_1 - \int_0^{(\theta_0-\theta_1)} f(\theta') W_L(\theta_0 - \theta_1 - \theta')\, d\theta' \qquad (2)$$

where $V_0 = E_0 L$, θ_0 is the position of the head of the bunch with respect to the wave and θ', the coordinate within the bunch, is made to vary from 0 (the head of the bunch) to $\theta_0 - \theta_1$ (the position where we want to know the net energy).

In order to reduce the energy spread within the bunch to zero, we must make $V(\theta_1)$ independent of θ_1. This requires that

$$\frac{\partial V(\theta_1)}{\partial \theta_1} = 0 \ . \qquad (3)$$

By taking the partial derivative of Eq. (2) with respect to θ_1 and setting it to zero, we get:

$$-V_0 \sin\theta_1 - \int_0^{(\theta_0-\theta_1)} f(\theta') \frac{\partial W_L}{\partial \theta_1}(\theta_0 - \theta_1 - \theta')\, d\theta' + f(\theta_0 - \theta_1) W_L(0) = 0 \ ,$$

or

$$f(\theta_0 - \theta_1) = \frac{V_0}{W_L(0)} \sin\theta_1 - \int_0^{(\theta_0-\theta_1)} \frac{f(\theta') \frac{\partial W_L}{\partial \theta_1}(\theta_0 - \theta_1 - \theta')}{W_L(0)}\, d\theta' \ . \qquad (4)$$

Letting $\theta_0 - \theta_1 = x$ where $x \geq \theta'$, Eq. (4) becomes:

$$f(x) = \frac{V_0}{W_L(0)} \sin(\theta_0 - x) - \int_0^{x} \frac{\frac{\partial W_L}{\partial x}(x - \theta') f(\theta')}{W_L(0)}\, d\theta' \qquad (5)$$

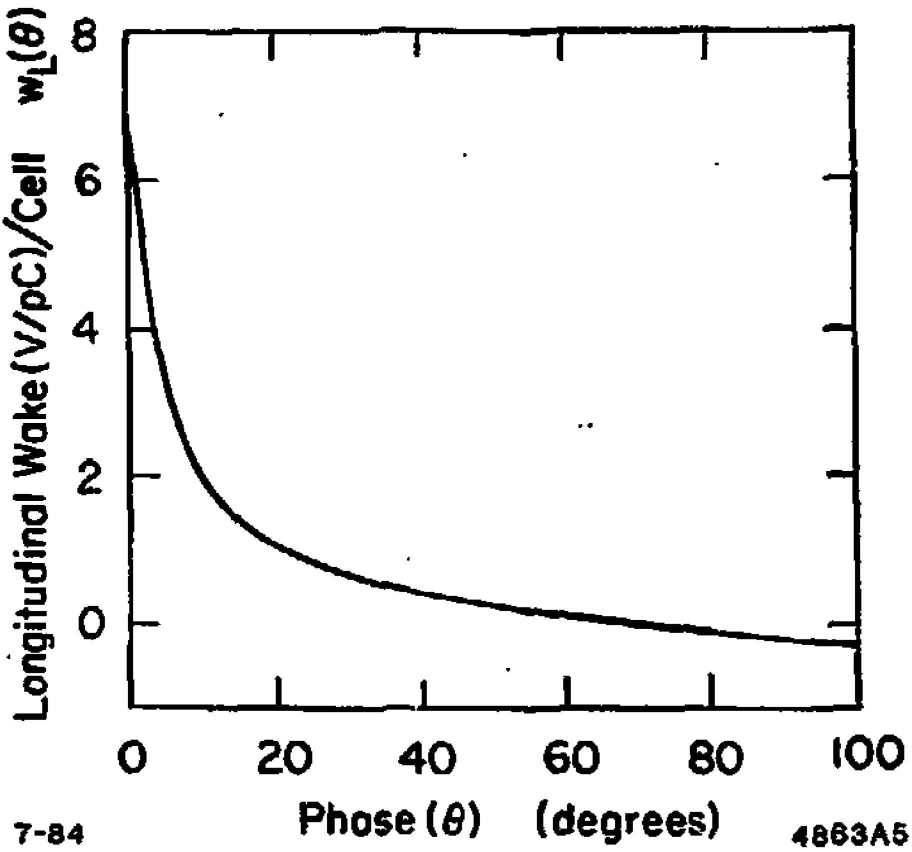

Fig. 1. Longitudinal wake function as a function of phase angle for SLAC constant-g‾adient structure.

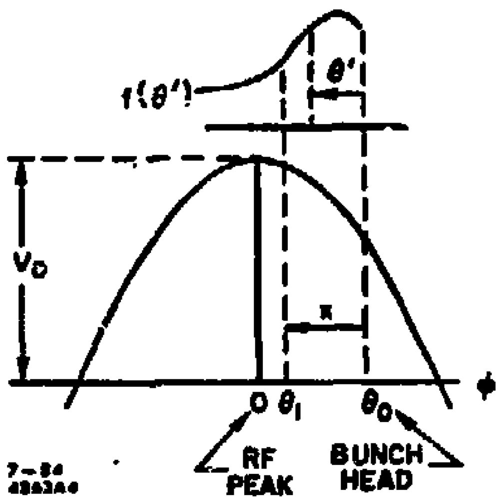

Fig. 2. Definitions of phase angles showing position of bunch with respect to accelerating wave. The charge distribution is $f(\theta')$ and the maximum energy gain is V_0.

which is a Volterra integral equation of the second kind. This equation can be solved digitally through a multi-step method using Day's starting procedure in conjunction with Simpson's rule and the three-eighth rule. The wake function can be fitted with a polynomial so as to be represented by an analytical expression.

Figures 3 and 4 give results for several examples. These examples were all worked out for a no-load energy V_0 of 54.75 GeV and an accelerator length L of 960 sections, each with 86 cavities (i.e., $L = 2890$ m, $N = 82560$). The value of V_0 was chosen so as to yield a final beam energy just over 50 GeV. Figure 3 shows five different bunch shapes with the corresponding θ_0's (positions of the head with respect to the wave) required to give essentially zero energy spread. The head of the bunch is on the left (zero-abscissa) and the tail defined as the point where an integrated charge of $5 \times 10^{10} e$ is reached, is at the abscissa corresponding to the letter "T" on each curve. An interesting aspect of these curves is that if the bunches are extended beyond the "T" points as shown, the energy spread continues to be zero even though the charge in the extended bunch is greater than $5 \times 10^{10} e$. The end points on the individual curves give the limits of how far one can go. Note that the shapes of the curves with $\theta_0 = 26°$ and 20° are not too "physical" in the sense that they are not likely to be obtained from a straightforward injector or damping ring. On the other hand, the $\theta_0 = 15°$ and 14° cases are more symmetrical and more likely to be realizable. The curve for $\theta_0 = 13°$ has no T because the integral under it does not quite reach 5×10^{10} particles: its charge is $4.9 \times 10^{10} e$. Figure 4 gives the respective energies of the bunches of Fig. 3 (except for the $\theta_0 = 13°$ case) as a function of angular position. The slight curvature, i.e., deviation from perfect flatness, is real but is believed to be due to an accumulated error in the computation which causes the bunch shapes to be slightly off.

Table 1 gives a summary of the average energies ($\bar{E}$) and spectral qualities [$(E_{max} - E_{min})/\bar{E}$ and $\sigma_E/\bar{E}$] for the cases shown in Figs. 3 and 4. The definitions of $\bar{E}$ and σ_E are:

$$\bar{E} = \frac{\int E(\theta)\, n(\theta)\, d\theta}{\int n(\theta)\, d\theta} \tag{6}$$

$$\sigma_E = \left[\frac{\int (E(\theta) - \bar{E})^2\, n(\theta)\, d\theta}{\int n(\theta)\, d\theta}\right]^{\frac{1}{2}} \tag{7}$$

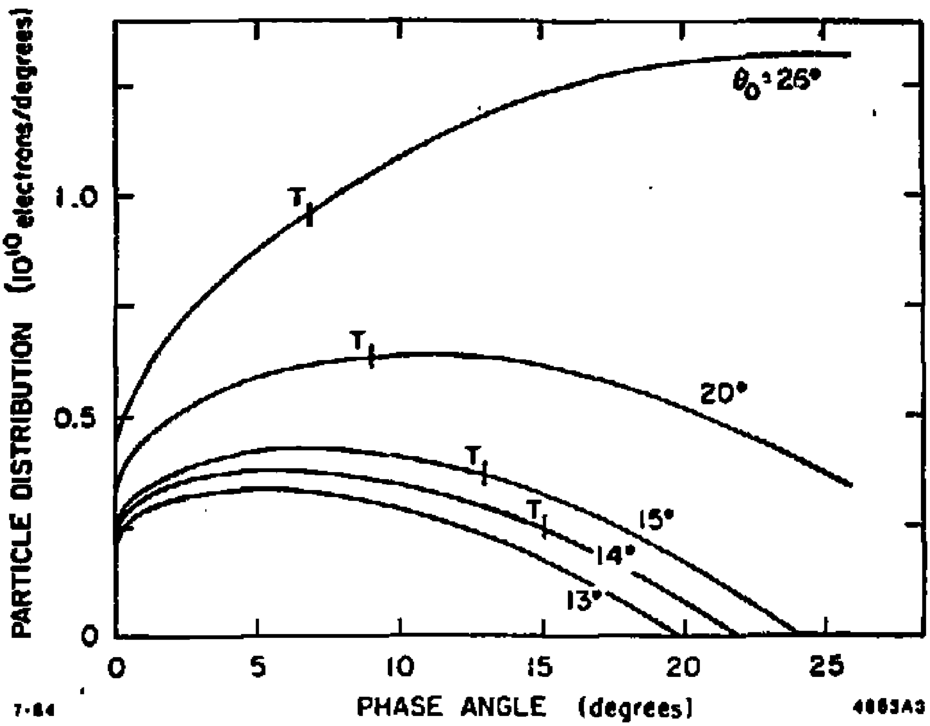

Fig. 3. Bunch shape, i.e., particle distribution as a function of phase angle for various values of θ_0. The point marked "*T*" indicates where the integrated charge in the bunch reaches $5 \times 10^{10}\,e$.

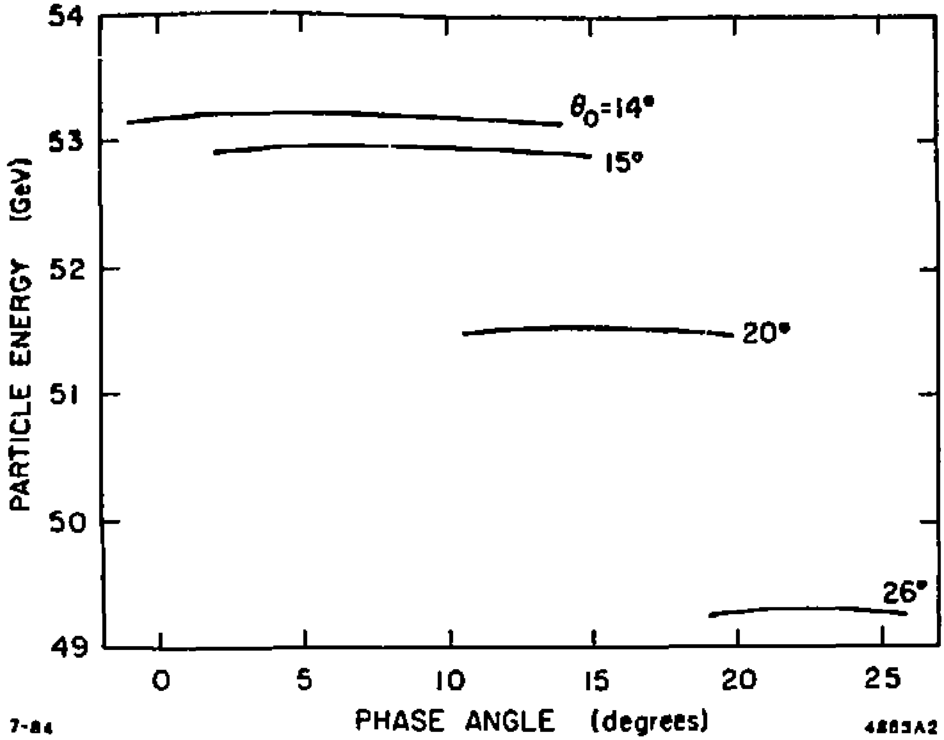

Fig. 4. Particle energy along bunches of Fig. 3 as a function of phase angle.

Table 1

(All cases for 5×10^{10} particles.)

θ_0 (degrees)	E (GeV)	$(E_{max} - E_{min})/E$ (%)	σ_E/E (%)
26	49.285	0.08	0.053
20	51.526	0.135	0.048
15	52.958	0.12	0.045
14	53.196	0.15	0.049
Truncated Gaussian 14	53.233	0.26	0.086

The fifth example, shown also for $\theta_0 = 14°$, is that of a truncated Gaussian fitted to the shape of the "ideal" $\theta_0 = 14°$ case. It has a σ_z of 8.3° but is truncated at ±7.5°. This compares with a σ_E/E of 0.3% for a Gaussian bunch of total length $6\sigma_z$ with a σ_z of 4° and a θ_0 of 19°, which is the best example used for the SLC.

Discussion

If we rewrite Eq. (5) in terms of the gradient E_0 instead of the total energy V_0, the bunch shape becomes:

$$f(x) = \frac{E_0 d}{w_L(0)} \sin(\theta_0 - x) - \int_0^x \frac{\frac{\partial w_L}{\partial x}(x - \theta')}{w_L(0)} f(\theta') d\theta' \tag{8}$$

We see that for a structure with a given $w_L(\theta)$, once the gradient E_0 and the angular position θ_0 of the head are chosen, the shape is fixed by Eq. (8) and is independent of the total energy V_0 and length L. For a given gradient E_0, $f(\theta)$ starts at a higher value as θ_0 is made larger since

$$f(0) = \frac{E_0 d}{w_L(0)} \sin\theta_0 \tag{9}$$

as shown in Fig. 3. Clearly, the more charge one wants, the higher gradient one needs, or the further ahead of crest one must place the head.

Tolerances

To get a feeling for allowable tolerances, it is interesting to calculate the effect of changes in injection angle (θ_0) or bunch charge on $\bar{E}$ and $\sigma_E/\bar{E}$, assuming constant bunch shape. Table 2 shows the effect of varying θ_0 while keeping the total charge of the bunch equal to $5 \times 10^{10} e$ in the case of the truncated Gaussian bunch discussed earlier.

Table 2
Truncated Gaussian bunch.
($\theta_{total} = 15°$, $\sigma = 8.3°$, $5 \times 10^{10} e$)

θ_0 (degrees)	$\bar{E}$ (GeV)	$(E_{max} - E_{min})/\bar{E}$ (%)	$\sigma_E/\bar{E}$ (%)
10	53.52	1.85	0.54
11	53.47	1.37	0.41
12	53.40	0.94	0.29
13	53.32	0.58	0.17
14	53.22	0.27	0.086
15	53.11	0.50	0.13
16	52.97	0.88	0.25
17	52.82	1.43	0.37
18	52.66	1.80	0.49

Table 3 shows the effect of changing the bunch charge while keeping its shape and θ_0 constant.

Table 3
Truncated Gaussian bunch.
($\theta_{total} = 15°$, $\sigma = 8.3°$, $\theta_0 = 14°$)

Bunch Charge ($\times 10^{10} e$)	$\bar{E}$ (GeV)	$(E_{max} - E_{min})/\bar{E}$ (%)	$\sigma_E/\bar{E}$ (%)
3	53.63	1.11	0.35
3.5	53.53	0.97	0.26
4	53.43	0.63	0.19
4.5	53.32	0.48	0.12
5	53.22	0.27	0.086
5.5	53.12	0.42	0.13
6	53.02	0.61	0.20
6.5	52.91	0.88	0.29
7	52.81	1.19	0.37

We see from Table 2 that excursions away from $\theta_0 = 14°$ by $\pm 1°$ are reasonably forgiving. The same is true for $\pm 10\%$ excursions away from $5 \times 10^{10}\, e$ in Table 3. The variations of σ_E/E in both tables are close to hyperbolic.

Conclusions

We have shown in this note that it is theoretically possible to find bunch shapes for the SLC which yield 5×10^{10} or more particles within negligible energy spread at the end of the linac. As it turns out, these shapes depend only on the linac energy gradient and the average angle* at which the head of the bunch is placed with respect to the accelerating wave, and are independent of the total energy or length of the accelerator. Some of these theoretical bunch shapes are not too different from shapes that ought to be realizable from injectors or damping rings. How to obtain these will be the subject of future work.

References

1. See for example, SLAC Linear Collider, Conceptual Design Report, SLAC-[illegible], pp. 17 and 117.
2. *Ibid*, pp. 112-116.

*Excursions away from this average angle in parts of the linac designed to cause Landau damping of the transverse wake field effect are of course permissible as long as overall "phase closure" to preserve the desired average θ_0 is realized.