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DOMAIN WALLS AT FINITE TEMPERATURE*

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ABSTRACT. We suggest that the phase transition of $\lambda\phi^4$ theory as a function of temperature coincides with the spontaneous appearance of domain walls. Based on one-loop calculations, we estimate $T_c = \frac{4M}{\sqrt{\lambda}}$ as the temperature for these domains to become energetically favored, to be compared with $T_c \simeq 4.9 \frac{M}{\sqrt{\lambda}}$ from effective potential calculations (which we perform directly in the broken phase). Domain walls, as well as other types of fluctuations, disorder the system above T_c, leading to $\langle\phi\rangle = 0$. We also compute the critical exponent for the specific heat above T_c and obtain $\alpha = \frac{2}{3} + O(\sqrt{\lambda})$.

RESUMO. Sustentamos que a transição de fase da teoria $\lambda\phi^4$ como função da temperatura coincide com o aparecimento espontâneo de paredes (separando domínios). Valendo-nos de uma aproximação semiclássica, em 1ª ordem, estimamos $T_c = \frac{4M}{\sqrt{\lambda}}$ como sendo a temperatura em que tais paredes tornam-se energeticamente favoráveis, temperatura a ser comparada com $T_c \simeq 4.9 \frac{M}{\sqrt{\lambda}}$, obtida de cálculos com o potencial efetivo (que realizamos diretamente na fase quebrada). Paredes, bem como outros tipos de flutuações, desordenam o sistema acima de T_c, o que leva a $\langle\phi\rangle = 0$. Também calculamos o expoente crítico do calor específico acima de T_c e obtivemos $\alpha = \frac{2}{3} + O(\sqrt{\lambda})$.

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1. INTRODUCTION

The description of the evolution of the universe in the standard (Big Bang) cosmological model requires studying Field Theories at finite temperature[1]. In addition, heavy-ion collisions provide an adequate experimental setting for testing finite temperature effects up to temperatures of the order of 100 Mev, so far[2]. It is, then, not at all surprising that a lot of progress has been made in the subject, as evidenced by the investigation of such interesting phenomena as pion condensation[3] and deconfinement of quarks[4]. Temperature gives us access to new phases of matter through phase transitions such as the two just mentioned. The investigation of one such transition, in the case of a scalar field theory, will be the main interest of this paper.

The $\lambda\phi^4$ theory has been used extensively[1,5] in studies of finite temperature effects. Being a prototype of the self-interactions of the Higgs sector in the electroweak and in unified theories, as well as a model for pion condensation, its investigation is not devoid of phenomenological interest. We shall concentrate our attention on the phase transition that takes place when, starting from the low-temperature ordered phase, we increase T up to a critical T_c, beyond which the system is in a disordered (high-T) phase. The two phases may be distinguished by an order parameter, $\langle\phi(x)\rangle_T$, the expectation value of the scalar field at finite temperature. For $T \geq T_c$ this parameter vanishes, whereas it is different from zero in the ordered phase $(T<T_c)$.

As far as we know, this phase transition has been studied by means of the effective potential[5]. The potential is a function of the order parameter, obtained under the assumption (equivalent to having translational invariance) that $\bar{\phi}(T)\equiv\langle\phi(x)\rangle_T$ is independent of position. As it is related to the Gibbs free energy of the system per unit volume, its minima yield the picture of the transition - above T_c, $\bar{\phi}(T)=0$; below, $\bar{\phi}(T)\neq 0$. If, however, we break translational invariance explicitly, either through an external field $J(x)$ or through appropriate boundary conditions, we have to calculate a functional[6], usually called the Γ-functional, which depends on the function $\phi_c(x)\equiv\langle\phi(x)\rangle_T$ and is also related to the Gibbs free energy of the system. What we have done was to calculate the functional Γ (actually the free energy per unit area) for a $\phi_c(x)$ given by a solution[8]. This solution would have finite Euclidean action in two dimensions but in four, as we know well from Derrick's theorem, its action diverges. However, the action and free energy per unit area are finite so that we may ask ourselves when will this quantity vanish. At the temperature where this occurs (usually called percolation temperature) it becomes energetically favorable to have a domain wall separating regions of different "vacua" instead of a homogeneous system. This phenomenon has already been discussed by Ventura[7] and by Marques and Ventura[8].

The physical picture that emerges is the following[8]: the vacuum state at T=0, which corresponds to $\bar{\phi}(0) = \frac{6M}{\sqrt{\lambda}}$ as we shall see, will be replaced by new minima at finite temperature, such that $|\bar{\phi}(T)|<|\bar{\phi}(0)|$. This indicates that configurations that tend to disorder the system are less suppressed in the functional integral. There exists a temperature at which the system finally goes into a completely disordered phase, where correlations cease to be long range. We compute such a temperature by identifying the point where a domain wall has Gibbs free energy equal to that of the homogeneous situation. It should be clear that this coincides with Peierl's argument to estimate the critical temperature. In the Ising model of classical Statistical Mechanics, this type of reasoning leads to finding the value of T where the surface tension of a bubble of reversed spins vanishes, thus favoring the appearance of interfaces.

The purpose of this paper is fourfold: (i) we present a new method for computing the high T (small λ) behavior of the soliton free energy, which is simpler than that of references (7), (8) and (9) and enables us to get the temperature of spontaneous generation of solitons quite easily; (ii) we compute the effective potential directly in the phase with broken symmetry using an appropriate renormalization procedure; (iii) we exhibit a detailed comparison between the effective potential and the soliton method and call attention to the fact that the former becomes imaginary just at the temperature where solitons appear; (iv) we calculate the critical exponent $\alpha = \frac{2}{3} + O(\sqrt{\lambda})$ in agreement with Fisher's inequality.

The paper is divided as follows: Section 2 exhibits the calculation of the free energy per unit area of a domain wall; Section 3 shows the calculation of the effective potential directly in the broken phase and the evaluation of effective couplings as functions of temperature; Section 4 discusses the phase transition and gives our estimate of T_c; Section 5 presents a proposal for computing the critical exponent α of the specific heat above T_c which leads to a value of 2/3; Section 6 comments on the limitations of the effective potential approach and compares it to our method. Details of the renormalization procedure are left for Appendices A, B and C.

2. THE SOLITON FREE ENERGY

We shall consider $\lambda\phi^4$ theory at finite temperature. Its Hamiltonian density is given by:

$$\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 \qquad (2.1)$$

We may obtain the partition function, $Z(\beta)$, by taking the trace:

$$Z(\beta) = \mathrm{Tr}(e^{-\beta H}) \quad ; \quad H = \int d^3x \, \mathcal{H} \qquad (2.2)$$

$Z(\beta)$ may expressed[5] as a functional integral over the fields and their canonical momenta, defined in Euclidean four-space:

$$Z(\beta) = \mathcal{N}^{-1}\oint[D\phi]\int[D\pi] \exp\left\{-\int_0^\beta d\tau \int d^3x \left[\pi \frac{\partial\phi}{i\partial\tau} + \mathcal{H}\right]\right\} \qquad (2.3)$$

where $\tau = it$ is the Euclidean time, the integral over momenta is unrestricted whereas the integral over fields only includes those which satisfy periodic (since we are dealing with bosons) boundary conditions in τ:

$$\phi(\vec{x},\beta) = \phi(\vec{x},0) \qquad (2.4)$$

The normalizing constant $\mathcal{N}$ may be chosen so that $Z(\infty)=1$. Performing the (quadratic) momentum integral yields:

$$Z(\beta) = N^{-1}(\beta)\oint[D\phi]\exp\left\{-\frac{1}{2}\int_0^\beta d\tau\int d^3x\left[(\partial_\mu\phi)^2 + m^2\phi^2 + \frac{\lambda}{12}\phi^4\right]\right\} \qquad (2.5)$$

The expression inside the square brackets in just the Euclidean Lagrangian, $\mathcal{L}_E$, and it leads to the equation of motion:

$$\Box\phi - m^2\phi - \frac{\lambda}{6}\phi^3 = 0 \; ; \; \Box \equiv \partial_\mu\partial_\mu = \frac{\partial^2}{\partial t^2} + \vec{\nabla}^2 \qquad (2.6)$$

If we take $m^2<0$, the preceding equation possesses nontrivial solutions, independent of τ, which correspond to one-dimensional solitons. If we introduce the notation $\vec{x}\equiv(x_L,\vec{x}_T)$ for longitudinal and transverse components, respectively, one such solution (located at the origin for simplicity) will be:

$$\phi_s(x_L) \equiv \sqrt{6}\,\frac{|m|}{\sqrt{\lambda}}\tanh\left(\frac{|m|\,x_L}{\sqrt{2}}\right) \qquad (2.7)$$

The situation $m^2<0$, typical of the broken symmetry phase, leads to an effective potential, at tree level, which has degenerate minima at $\phi_v = \pm\sqrt{6}|m|/\sqrt{\lambda}$, if we take the temperature to the zero. If we compute the classical action of ϕ_s minus that of one of the minima, in just one dimension (x_L), we obtain a finite result:

$$\Delta S_{cl} \equiv S_{cl}(\text{soliton}) - S_{cl}(\text{minimum}) = \int_{-\infty}^{\infty} dx_L\{\mathcal{L}_E(\phi_s) - \mathcal{L}_E(\phi_v)\} =$$

$$= 4\sqrt{2}\,\frac{|m|^3}{\lambda} \qquad (2.8)$$

However, in three spatial dimensions the analogous quantity will diverge like an area. We shall regard such a structure as being a two-dimensional domain wall, immersed in three-space, that separates, along the longitudinal direction, the two distinct vacua. Being independent of τ, this solution satisfies the periodicity condition trivially. In fact, a simple rescaling $\tau\to\tau/\beta$ shows that the kinectic term in $\mathcal{L}_E$ is more and more suppressed as we increase the temperature, so that static solutions are most relevant.

We shall compute, in semiclassical approximation[10], the contribution to the partition function coming from the soliton sector, normalized by that of the vacuum sector (T=0). This amounts to taking as background classical field, $\phi_c(\vec{x},\tau)$, the soliton solution ($\phi_s(x_L)$) and one of the (position independent) degenerate minima (ϕ_v), respectively. We then write:

$$\phi(\vec{x},\tau) = \begin{cases} \phi_s(x_L)+\eta(\vec{x},\tau) \\ \phi_v + \eta(\vec{x},\tau) \end{cases} \qquad (2.9)$$

Where $\eta(\vec{x},\tau)$ denotes quantum fluctuations around the classical background. We expand the action up to terms quadratic in η which corresponds to keeping only one-loop contributions in our calculation. If m and λ are to be taken as renormalized parameters we shall have to introduce comuterterms to obtain physical quantities (since we work only up to one-loop order no wavefunction renormalization will be needed)[6]. These will remove the ultraviolet infinities that appear in the calculation. Thus, performing the expansion in η we obtain:

$$\frac{Z_s(\beta)}{Z_v(\beta)} = \frac{\exp\left\{-\int_0^\beta d\tau \int d^2x_T \int_{-\infty}^{\infty} dx_L \mathcal{L}_E(\phi_s(x_L))\right\}\left(\det\left[-\Box+m^2+\frac{\lambda}{2}\phi_s^2(x_L)\right]_\beta\right)^{-1/2}}{\exp\left\{-\int_0^\beta d\tau \int d^2x_T \int_{-\infty}^{\infty} dx_L \mathcal{L}_E(\phi_v)\right\}\left(\det\left[-\Box+m^2+\frac{\lambda}{2}\phi^2_v\right]_\beta\right)^{-1/2}} \qquad (2.10)$$

where $(\det[M]_\beta)^{-1/2} \equiv \oint [D\eta]\exp\left\{-\int_0^\beta d\tau \int d^3x \ \eta[M]\eta\right\}$

The integral above is over functions η which are periodic in τ, of period β. Therefore we may write:

$$\eta(\vec{x},\tau) = \frac{1}{\sqrt{\beta}} \sum_{n=-\infty}^{\infty} a_n(\vec{x}) e^{i\omega_n\tau} \quad ; \ \omega_n \equiv \frac{2\pi n}{\beta} \qquad (2.11)$$

Using this, the logarithm of the ratio Z_s/Z_v may be written:

$$\ln(Z_s/Z_v) = -\beta\lambda\,\Delta S_{c\ell} - \frac{1}{2}\ln\left\{\frac{\prod_{n=-\infty}^{\infty}\det[M_s(n)]}{\prod_{n=-\infty}^{\infty}\det[M_v(n)]}\right\}. \qquad (2.12)$$

where we have taken $\int d^2x_T = A$ to be the transverse area and:

$$M_s(n) = \delta(\vec{x}-\vec{x}')\left[\omega_n^2-\vec{\nabla}^2+m^2+\frac{\lambda}{2}\phi_s^2(x_L)\right]$$

$$M_v(n) = \delta(\vec{x}-\vec{x}')\left[\omega_n^2-\vec{\nabla}^2+m^2+\frac{\lambda}{2}\phi_v^2\right] \tag{2.13}$$

The eigenvalues of these operators may be obtained by solving:

$$\left[-\vec{\nabla}^2+m^2+\frac{\lambda}{2}\phi_c^2(\vec{x})\right]v_j(\vec{x}) = E^2(j)v_j(\vec{x}) \tag{2.14}$$

where $\phi_{cl}(\vec{x})$ may be either $\phi_s(x_L)$ or ϕ_v. Then:

$$\ln(Z_s/Z_v) = -\beta A\,\Delta S_{cl} - \frac{1}{2}\ln\left[\frac{\prod_{n=-\infty}^{\infty}\prod_{js}(\omega_n^2+E_s^2(js))}{\prod_{n=-\infty}^{\infty}\prod_{jv}(\omega_n^2+E_v^2(jv))}\right] \tag{2.15}$$

If we invert the order of the products and use the identity:

$$\prod_{n=1}^{\infty}\left(1+\frac{z^2}{n^2}\right) = \frac{sh(\pi z)}{\pi z} \tag{2.16}$$

we obtain:

$$\ln(Zs/Zv) = -\beta A\,\Delta S_{cl} - \sum_{js}\ln\left[sh(\beta E_s(js)/2)\right] + \sum_{jv}\ln\left[sh(\beta E_v(jv)/2)\right] \tag{2.17}$$

The sums extend over discrete as well as continuous eigenvalues. Finally we may reexpress (2.17) as:

$$\ln(Z_s/Z_v) = -\beta A\,\Delta S_{cl} - \left\{\sum_{js}\left[\frac{\beta E_s(js)}{2}+\ln\left(1-e^{-\beta E_s(js)}\right)\right]\right.$$

$$\left. - \sum_{jv}\left[\frac{\beta E_v(jv)}{2}+\ln\left(1-e^{-\beta E_v(jv)}\right)\right]\right\} \tag{2.18}$$

Clearly, the first term in each sum refers to the zero print energy contribution and the second is a Bose-Einstein term.

The free energy difference per unit area between solitón and vacuum (T=0) sectors is given by:

$$\Delta f \equiv f_s - f_v = -\frac{1}{\beta A} \ln(Z_s/Z_v) \tag{2.19}$$

Using $f=e-T\mathscr{s}$, where e and $\mathscr{s}$ denote internal energy per unit area and entropy per unit area, respectively, we may write:

$$\Delta e = \Delta S_{cl} = \frac{4\sqrt{2}|m|^3}{\lambda} \tag{2.20}$$

$$T\Delta\mathscr{s} = -\frac{1}{2\beta A}\left\{\left[\mathrm{Tr}\ \ln M_s\right]_\beta - \left[\mathrm{Tr}\ \ln M_v\right]_\beta\right\} \tag{2.21}$$

where we have made use of det $M \equiv \exp \mathrm{Tr}\ \ln M$, while M_s, M_v refer to the arguments of the determinants in (2.10). Thus far, we have associated the classical action of the soliton with the internal energy and the fluctuations around the soliton background with the entropy. It will be very useful, later on, to note that the full free energy is connected to the Γ-functional of Field Theory, ie, the generator of one-particle irreducible graphs, via:

$$\Delta f = +\frac{1}{\beta A}\left\{\Gamma(\phi_s(x_\perp)) - \Gamma(\phi_v)\right\} = +\frac{1}{\beta A}\Delta\Gamma \tag{2.22}$$

In fact, we shall adopt the convention $\Gamma(\phi_v)\equiv 0$ so as to measure deviations from the zero-temperature vacuum. Thus:

$$\Delta\Gamma(\phi_c(\vec{x},t)) \equiv \Gamma(\phi_c(\vec{x},\tau)) \tag{2.23}$$

At this stage, all we need to arrive at the free energy per unit area is to exhibit the eigenvalues for both soliton and vacuum (T=0) sectors and perform the sums in (2.18).

Since neither classical background depends on $\vec{x}_T$, we put:

$$v_j(\vec{x}) = e^{i\vec{k}_T.\vec{x}_T}\, v_{jL}(x_L) \qquad (8.24)$$

The resulting eigenvalue equations are well-known[10]. The sums over the eigenvalues require some caution and they are performed in detail in Appendix A. Here we just quote the result (with $M\equiv|m|$):

$$T\Delta\beta = -\{\zeta_1 + T\zeta_2(T)\} \qquad (8.25)$$

$$\zeta_1 = -\frac{1}{2}\int \frac{d^2k_T}{(2\pi)^2}\left\{\sqrt{k_T^2} + \sqrt{k_T^2+\frac{3M^2}{2}} - \frac{3\sqrt{2}}{\pi}M - \frac{1}{M}\int_{-\infty}^{\infty}\frac{dk_L}{2\pi}\, g_M(k_L)\sqrt{k_L^2+k_T^2+2M^2}\right\} \qquad (2.26)$$

$$\zeta_2(T) = -\frac{1}{2}\int \frac{d^2k_T}{(2\pi)^2}\left\{\ln\left(1-e^{-\beta\sqrt{k_T^2}}\right) + \ln\left(1-e^{-\beta\sqrt{k_T^2+\frac{3M^2}{2}}}\right) - \frac{1}{M}\int_{-\infty}^{\infty}\frac{dk_L}{(2\pi)}\, g_M(k_L)\cdot\ln\left(1-e^{-\beta\sqrt{k_L^2+k_T^2+2M^2}}\right)\right\} \qquad (2.27)$$

$$g_M(x) = 2\sqrt{2}\, M^2\left\{\frac{1}{2x^2+M^2} + \frac{1}{x^2+2M^2}\right\} \qquad (2.28)$$

As we had already indicated, this result is meaningless as it stands, since it diverges in the ultraviolet. This was to be expected as we have not yet made use of the counterterms. These are the same that appear in the theory at T=0 because the temperature just modifies the infrared, not the ultraviolet, structure of the problem. Thus, in practice, only ζ_1 is affected. Once the counterterms are identified and their contribution subtracted from (2.26) we obtain a finite result:

$$T\Delta\beta = -\{\zeta_3 + T\zeta_2(T)\} \qquad (2.29)$$

$$\zeta_3 = -\sqrt{\frac{3}{2}}\left(\frac{1}{48\pi}\right)M^3 \tag{2.30}$$

We have used renormalization conditions at zero momentum and the steps leading to (2.29) are shown in Appendix B. The free energy difference per unit area in the one-loop approximation, with respect to that of the vacuum at $T=0$, is finally given by,

$$\Delta f = \Delta e - T\Delta s = 4\sqrt{2}\,\frac{M^3}{\lambda} - \sqrt{\frac{3}{2}}\,\left(\frac{1}{48\pi}\right)M^3 + T\zeta_2(T) \tag{2.31}$$

3. THE EFFECTIVE POTENTIAL

We shall now compute the effective potential for the theory at finite temperature, in the one-loop approximation, directly in the broken phase. This allows for a comparison with the results of the preceding section and will lead to the effective (temperature dependent) couplings mentioned in the Introduction. Calculating the potential amounts to finding the generating functional $\Gamma(\phi_c(\vec{x},\tau))$ in the special case where $\phi_c=\tilde{\phi}$ is independent of the coordinates $(\vec{x},\tau)$. Indeed, the effective potential, U, is just an ordinary function of $\tilde{\phi}$ obtained from Γ by simply dividing it by βV(V is the spatial volume). Thus:

$$U_\beta(\tilde{\phi}) = \frac{1}{\beta V}\,\Gamma(\tilde{\phi}) \tag{3.1}$$

Starting from a functional Taylor expansion for Γ:

$$\Gamma(\phi_c(\vec{x},\tau)) = \sum_{n=1}^{\infty}\frac{1}{n!}\int_0^\beta d\tau_1\int d^3x_1\ldots\int_0^\beta d\tau_n\int d^3x_n\,\Gamma^{(n)}(\vec{x}_1,\tau_1;\ldots;\vec{x}_n\tau_n)\,*$$

$$*\,\left[\phi_c(\vec{x}_1,\tau_1)-\phi_v\right]\ldots\left[\phi_c(\vec{x}_n,\tau_n)-\phi_v\right] \tag{3.2}$$

where we have $\Gamma(\phi_v)=0$ (as we had done in Section 2) we arrive at:

$$U_\beta(\tilde{\phi}) = \sum_{n=1}^{\infty}\frac{1}{n!}\,\tilde{\Gamma}^{(n)}(\vec{0},0;\ldots;\vec{0},0)\left[\tilde{\phi}-\phi_v\right]^n \tag{3.3}$$

$\tilde{\Gamma}^{(n)}$ denotes the Fourier transform of $\Gamma^{(n)}$:

$$\Gamma^{(n)}(\vec{x}_1,\tau_1;\ldots;\vec{x}_n,\tau_n) =$$

$$=\frac{1}{\beta^n}\sum_{N_1,\ldots,N_n=-\infty}^{\infty}\int\frac{d^3k_1}{(2\pi)^3}\cdots\frac{d^3k_n}{(2\pi)^3}\,\tilde{\Gamma}^{(n)}(\vec{k}_1,\omega_1;\ldots;\vec{k}_n,\omega_n)\,e^{-i\sum_{j=1}^{n}(\vec{k}_j\vec{x}_j+\omega_j\tau_j)} \tag{3.4}$$

where $\omega_j = \frac{2\pi N_j}{\beta}$. The $\Gamma^{(n)}$ are $n^{\underline{th}}$ order functional derivatives of Γ. In perturbation theory they correspond to all the one-particle irreducible graphs with their n external legs amputated. In fact, the effective potential defined in (3.1) relates to the f e energy per unit volume, $\Delta\Omega$, of the system, for situations where one has translational invariance:

$$\Delta\Omega = \min_{\bar{\phi}} \left[U_\beta(\bar{\phi}) - U_\beta(\phi_v)\right] \qquad (3.5)$$

We may, then, characterize the minima of (3.5) which correspond to the expectation value $<\phi(x)>_T$ at finite temperature. These minima will be denote by $\phi_v(T)$ so that $\phi_v(0)=\phi_v$ is the zero temperature vacuum.

The strategy for cumputing the effective potential in the broken phase, up to one-loop order, is rather simple. It amounts to replacing $\phi_s(x_L)$ with a constant $\bar{\phi}$ in (2.10), taking a logarithm and performing a Legendre transform. The calculation of the determinants in (2.10) can be done either by computing eigenvalues (as in Section 2).or, more conventionally, by making use of a graphical expansion. To see how this goes we identify:

$$T\Delta\sigma = -\frac{1}{2\beta V} \ln \left[\frac{\det(-\Box+m^2+\frac{\lambda}{2}\bar{\phi}^2)}{\det(-\Box+m^2+\frac{\lambda}{2}\phi_v^2)}\right] \qquad (3.6)$$

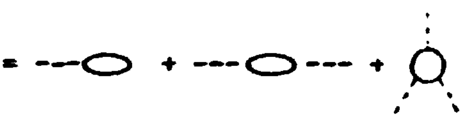
σ denotes entropy per unit volume. Using the value of $\phi_v^2 = \frac{6m^2}{\lambda}$, we may rewrite (3.6) as:

$$T\Delta\sigma = -\frac{1}{2\beta V} \ln \left[\frac{\det(-\Box+2M^2+\frac{\lambda}{2}(\bar{\phi}^2-\phi_v^2))_\beta}{\det(-\Box+2M^2)}\right] \qquad (3.7)$$

where we have made use of $M^2 = |m^2|$. Then:

$$T\Delta\sigma = -\frac{1}{2\beta V}\,\mathrm{Tr}\,\ln\left(1 + G_\beta\left[\frac{\lambda}{2}(\bar{\phi}^2-\phi_v^2)\right]\right) \qquad (3.8)$$

The operator $G_\beta \equiv (-\Box+2M^2)_\beta^{-1}$ is just a free propagator at finite temperature, with mass $\sqrt{2M^2}$. This mass is associated to the excitations around the minima, $\pm\phi_v$, of the broken phase. If we use $B = \frac{\lambda}{2}\phi^2$, then:

$\mathrm{Tr}\,\ln(1 + G_\beta(\bar{B}-B_v)) =$  $+ \ldots$ (3.9)

The dashed lines correspond to the "background field" $(\bar{B}-B_v)$, whereas the internal lines denote propagators G_β whose Fourier transform is:

$$\tilde{G}_\beta = \frac{1}{\omega_j^2+\vec{p}^{\,2}} \quad ; \quad \omega_j = \frac{2\pi j}{\beta} \qquad (3.10)$$

The momentum integrals in Feynman graphs involve discrete sums, $\frac{1}{\beta}\sum_j \int \frac{d^3p}{(2\pi)^3}$. Thus, the expansion in (3.9) is just:

$$\mathrm{Tr}\,\ln(1 + G_\beta(\bar{B}-B_v)) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(\bar{B}-B_v)^n\,\frac{1}{\beta}\sum_{j=-\infty}^{\infty}\int\frac{d^3p}{(2\pi)^3}\,\frac{1}{\left[\omega_j^2+\vec{p}^{\,2}+2M^2\right]^n} \qquad (3.11)$$

since the background carries no momentum. Expression (3.6) yields the one-loop contribution to the effective potential. The zero-loop term is just the classical action for $\bar{\phi}$ minus that of ϕ_v, which can be regarded as an internal energy difference per unit volume:

$$\Delta\epsilon = \frac{1}{\beta V}\left[S_{cl}(\bar{\phi})-S_{cl}(\phi_v)\right] = \frac{\lambda}{4!}(\bar{\phi}^2-\phi_v^2)^2 = \frac{1}{6\lambda}(\bar{B}-B_v)^2 \qquad (3.12)$$

Adding (3.11) and (3.12) gives $U_\beta(\bar{\phi})$. Although it is expressed in terms of $(\bar{B}-B_v)$ it is a trivial matter to rewrite it as a series expansion in $(\bar{\phi}-\phi_v)$, as in (3.3). We can thus identify the $\Gamma^{(n)}$ up to one-loop order.

The sum in (3.11) is still divergent. The problem comes from the two first graphs in (3.9). Formally, we may sum (3.11) to obtain:

$$U_\beta(\bar{\phi}) = \frac{\lambda}{4!}(\bar{\phi}^2-\phi_v^2)^2 + \frac{1}{2\beta}\sum_{j=-\infty}^{\infty} \frac{d^3p}{(2\pi)^3}\, \ln\left(1 + \frac{\frac{\lambda}{2}(\bar{\phi}^2-\phi_v^2)}{\omega_j^2+\vec{p}^2+2M^2}\right) \quad (3.13)$$

If we now subtract the contribution of the graphs just mentioned, calculated at <u>zero temperature</u>, the result is finite and given by:

$$U_\beta(\bar{\phi}) = \frac{\lambda}{4!}(\bar{\phi}^2-\phi_v^2)^2 + \frac{1}{2\beta}\sum_{j=-\infty}^{\infty}\int \frac{d^3p}{(2\pi)^3}\, \ln\left(1 + \frac{\frac{\lambda}{2}(\bar{\phi}^2-\phi_v^2)}{\omega_j^2+\vec{p}^2+2M^2}\right) -$$

$$- \frac{\lambda}{2}(\bar{\phi}^2-\phi_v^2)\int\frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2+2M^2} + \frac{\lambda^2}{8}(\bar{\phi}^2-\phi_v^2)^2\int\frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2+2M^2} \;;\quad p^2 \equiv \vec{p}^2+p_4^2 \quad (3.14)$$

The two terms that render (3.13) finite are the same one would use at zero temperature since the temperature does not effect the ultraviolet structure of the theory. In Appendix C it is shown that they correspond to the contribution of all the counterterms that are required, <u>in the broken phase</u>, to extract divergences up to one-loop order. Subtractions were performed at zero external momenta, in view of the (zero-temperature) renormalization conditions:

$$\Gamma^{(2)}_{T=0}(\omega_i=0;\vec{p}_i=\vec{0}) = 2M^2 \quad (3.15)$$

$$\bar{\Gamma}^{(4)}_{T=0}(\omega_i=0;\vec{p}_i=0) = \lambda \tag{3.16}$$

$$\left.\frac{\partial\bar{\Gamma}^{(2)}_{T=0}}{\partial p^2}\right|_{\omega_i=0;\vec{p}_i=0} = 1 \tag{3.17}$$

$$\bar{\Gamma}^{(2,1)}_{T=0}(\omega_i=0;\vec{p}_i=\vec{0}) = 1 \tag{3.18}$$

Just as we had done before, we may isolate the temperature independent term in the effective potential. We obtain:

$$U_\beta(\bar{\phi}) = \frac{\lambda}{4!}(\bar{\phi}^2-\phi_v^2)^2 + \int\frac{d^3p}{(2\pi)^3}\left(\sqrt{\vec{p}^2+2M^2+\frac{\lambda}{2}(\bar{\phi}^2-\phi_v^2)} - \sqrt{\vec{p}^2+2M^2} - \right.$$

$$- \frac{\lambda}{2}(\bar{\phi}^2-\phi_v^2)\int\frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2+2M^2} + \frac{\lambda^2}{8}(\bar{\phi}^2-\phi_v^2)^2\,\frac{d^4p}{(2\pi)^4}\,\frac{1}{(\vec{p}^2+2M^2)^2} +$$

$$+ \frac{2}{\beta}\int\frac{d^3p}{(2\pi)^3}\ln\left(1-e^{-\beta\sqrt{\vec{p}^2+2M^2+\frac{\lambda}{2}(\bar{\phi}^2-\phi_v^2)}}\right) - \frac{2}{\beta}\int\frac{d^3p}{(2\pi)^3}\ln(1-e^{-\beta\sqrt{\vec{p}^2+2M^2}}) \tag{3.19}$$

It is easy to check that the second, third and fifth terms add up to a finite result (they are T-independent). The T-dependent part has no divergences. From eq.(3.19) one sees that if $\bar{\phi}^2 \leq \phi_v^2 - \frac{4M^2}{\lambda}$ the effective potential acquires an imaginary part. In the next section we discuss this in connection with the appearance of solitons.

Knowledge of the effective potential leads to a natural identification of the effect of temperature on the couplings of the theory. We may define effective (temperature dependent) mass and coupling constant via:

$$M^2(T) \equiv \left.\frac{d^2U_\beta}{d\bar{\phi}^2}\right|_{\bar{\phi}=\phi_v(T)} \tag{3.20}$$

$$\lambda(T) \equiv \left.\frac{d^4U_\beta}{d\bar{\phi}^4}\right|_{\bar{\phi}=\phi_v(T)} \tag{3.21}$$

At zero temperature ($\phi_v=\phi_v(0)$), we recover $M^2=M^2(0)$ and $\lambda=\lambda(0)$. Equations (3.20) and (3.21) may be inverted to yield M^2 and λ as functions of $M^2(T)$ and $\lambda(T)$. This may then be used to reexpress the results of sections 2 and 3 in terms of temperature dependent couplings.

Before we move on to discuss the phase transition let us make one brief comment. Normally, in the literature, the effective potential in the broken phase is obtained by just continuing the result of the symmetric phase to negative values of m^2. This is, in principle, perfectly valid because the potential should be the same no matter which phase is used. However, a one-loop calculation is just the first term in a semiclasssical expansion. Thus, the background field around which we expand should better be a minimum of the action or else we will have to face negative eigenvalues for the determinant of the fluctuations. Although the calculation of the effective potential can be done in either phase, the approximation used has its validity closely tied to the particular phase (minima) we are considering. Our calculation avoids some of the problems encountered previosly[11] by taking this directly into account. It turns out that the replacement $m^2\to-m^2$ in the effective potential of the symmetric phase <u>does</u> lead to our result and if one is careful to take derivatives at the appropriate minima ($\bar{\phi}=0$ or $\bar{\phi}=\pm\phi_v$, respectively) the effective couplings are also identical.

4. THE PHASE TRANSITION

The results of this section will always refer to the high-temperature limit, T>>M, in which the thermal wavelength is much smaller than the Compton wavelength of the theory. The leading terms in such a limit are rather easy to obtain if we look at the graphical expansion described previously. Let us begin by examining the effective potential and the usual description of the transition. The first graph of (3.9) involves:

$$\bigcirc = \frac{1}{\beta} \sum_{j=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \frac{1}{(\frac{2\pi j}{\beta})^2 + \vec{p}^{\,2} + 2M^2} \tag{4.1}$$

Performing the sum over j we obtain:

$$\bigcirc = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\sqrt{\vec{p}^{\,2}+2M^2}} + \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{\vec{p}^{\,2}+2M^2}\,(e^{\beta\sqrt{\vec{p}^{\,2}+2M^2}}-1)} \tag{4.2}$$

The first term is the zero-temperature contribution which is cancelled by counterterms. The temperature dependence comes from the second term, R(T), which we rewrite as:

$$R(T) = \frac{1}{2\pi^2\beta^2} \int_0^\infty \frac{dx \;\; x^2}{\sqrt{x^2+2\beta^2M^2}\,(e^{\sqrt{x^2+2\beta^2M^2}}-1)} \tag{4.3}$$

We may safely take the limit $\beta M \ll 1$ in (4.3) to arrive at:

$$R(T) \xrightarrow{T\gg M} \frac{T^2}{12} \tag{4.4}$$

One can show[12] that the leading behavior of this graph dominates the expansion for T>>M. The other graphs contribute to higher powers of $(\frac{M}{T})$. Therefore, to leading order:

$$U_\beta(\bar\phi) = \frac{\lambda}{4!}(\bar\phi^2-\phi_v^2)^2 + \frac{1}{2}\left[\frac{\lambda}{2}(\bar\phi^2-\phi_v^2)\right]\left(\frac{T^2}{12}\right) = \frac{\lambda}{4!}(\bar\phi^2-\phi_v^2)\left[(\bar\phi^2-\phi_v^2)+\frac{T^2}{2}\right] \tag{4.5}$$

Clearly the extrema of this expression occur at:

$$\bar\phi^2 = \begin{cases} 0 & \text{(local maximum)} \\ \phi_v^2(T) = \phi_v^2 - \frac{T^2}{4} & \text{(minima at } \pm\phi_v(T)) \end{cases}$$

Since $\phi_v^2 = \phi_v^2(0) = \frac{6M^2}{\lambda}$, the three extrema coincide at a temperature $\bar T_c$ given by:

$$\bar T_c^2 = \frac{24M^2}{\lambda} \tag{4.7}$$

Figure 1 shows the picture of how the effective potential changes with temperature up to $\bar T_c$. We note that, by construction, $U_\beta(\phi_v(0)) \equiv 0$, since we chose ϕ_v to be the zero of the free energy.

We now go on to examine our alternative method by looking at the soliton sector. One could just take the high-T limit of (2.31), ie, examine the limit of $\zeta_2(T)$. However, our graphical expansion in (3.9) can also be extended to a situation where $\langle\phi(x)\rangle_T$ is position dependent. In fact this is used in Appendix B to take into account the counterterms that render (2.25) finite. All we have to do is to replace $\bar\phi$ with $\phi_s(x_\perp)$ in the graphs of (3.9). In the high-T limit the leading behavior is given by the first graph and can be easily calculated:

$$= \left\{\int_0^\beta d\tau \int d^3x\left[B_s(x_\perp)-B_v\right]\right\}\left\{\frac{1}{\beta}\sum_{j=-\infty}^{\infty}\int\frac{d^3p}{(2\pi)^3}\frac{1}{\omega_j^2+\vec p^2+2M^2}\right\} = \frac{-\sqrt{2}}{2}\beta M T^2 \tag{4.8}$$

Therefore the contribution to the free energy per unit area for $T \gg M$ is:

$$\Delta f = \Delta e - T\Delta\varphi = \left[4\sqrt{2}\,\frac{M^3}{\lambda} + 0\,(1)\right] - \left[\frac{\sqrt{2}}{4}\,MT^2\left\{1 + 0\,(M/T)\right\}\right] \quad (4.9)$$

This quantity will vanish at a temperature T_c given by:

$$T_c^2 = 16\,\frac{M^2}{\lambda} \qquad (4.10)$$

If we go back to the effective potential in the one-loop approximation we will see that it becomes complex for a $\bar{\phi}_{cr}^2 = 16M^2$. This means that even within a one-loop calculation results obtained from the effective potential cannot be trusted for $|\bar{\phi}|<|\bar{\phi}_{cr}|$. The critical temperature estimated from the effective potential violates this condition since $\tilde{T}_c = 4.9\, M/\sqrt{\lambda} > \bar{\phi}_{cr} = 4M/\sqrt{\lambda}$. It is interesting to note that the free energy of the domain walls vanishes at exactly $\bar{\phi}_{cr} = T_c = 4M/\sqrt{\lambda}$! We think that these results are related. For $T > \frac{4M}{\sqrt{\lambda}}$ solitons can appear spontaneously in our system. The effective potential, in turn, gains an imaginary part at exactly the same temperature so as to indicate that a uniform background field is no longer stable: The system in fact prefers to go to a soup of solitons.

One should remark that the temperature calculated using the domain wall and the temperature where the order parameter vanishes are not necessarily the same[13]. In d=3, for the Ising model, they differ slightly. The former is called the percolation temperature (T_p), whereas the latter corresponds to the usual critical temperature (T_c). Nevertheless, for d=2 and d≥4 they coincide (There exists also the roughening temperature, T_R, which has to do with fluctuations of the interface. One has $T_R \le T_p \le T_c$: in d=2, $T_R=0$ and $T_p=T_c$; in d=3, $T_R<T_p<T_c$ with $T_p=0.95\ T_c$ and $T_R=0.57\ T_c$; in d=4 one expects $T_R=T_p=T_c$; all the results quoted are for the Ising model). We shall assume that in d=3 the two temperatures are close enough that we can identify them.

5. THE CRITICAL EXPONENT α

We shall now present our calculation of the critical exponent, α , for the specific heat above T_c. We shall have to assume that in the symmetric phase ($\langle\phi\rangle_T=0$) the fluctuations that disorder the system (we shall only consider domain walls) can be characterized by the quantity $a(T)$, which is the number of domains per unit length. Since we know the free energy per unit area of these domains, the total free energy of the system per unit volume (neglecting interactions among the domains) will be taken to be $(a\Delta f)$. Working at fixed volume:

$$\frac{d}{dT}(a\Delta f) = \frac{\partial}{\partial T}(a\Delta f)\Big|_V = - a\Delta s \qquad (5.1)$$

where Δs is the entropy per unit area. From (5.1) we obtain:

$$(\Delta f)\frac{da}{dT} + a\,\frac{d}{dT}(\Delta f) = - a\Delta s \qquad (5.2)$$

Using the results of section 3 we have:

$$\frac{d(\Delta f)}{dT} = \frac{\partial(\Delta f)}{\partial T} + \frac{\partial(\Delta f)}{\partial M^2(T)}\,\frac{dM^2(T)}{dT} + \frac{\partial(\Delta f)}{\partial\lambda(T)}\,\frac{d\lambda(T)}{dT} \qquad (5.3)$$

where $M^2(T)$ and $\lambda(T)$ are effective couplings and, physically, represent the reaction of the medium (fluctuations) on the domain walls. If we denote these couplings generically by $\chi_i(T)$, $i=1,2$, and use (5.2):

$$\frac{da}{dT} = - \frac{\left[\frac{\partial(\Delta f)}{\partial\chi_i}\,\frac{d\chi_i}{dT}\right]}{(\Delta f)} \qquad (5.4)$$

Near the critical temperature we have:

$$\Delta f(T) \underset{T\to T_c}{=} \left[\frac{d(\Delta f)}{dT}\right]_{T=T_c} (T-T_c) \qquad (5.5)$$

with $\left[\frac{d(\Delta f)}{dT}\right]_{T=T_c} < 0$. Let us define $K_c > 0$ as:

$$K_c \equiv - \frac{\left[\frac{\partial(\Delta f)}{\partial \chi_i} \frac{d\chi_i}{dT}\right]}{\left[\frac{d}{dT}(\Delta f)\right]_{T=T_c}} \qquad (5.6)$$

Using (5.4) we obtain:

$$(T-T_c)\frac{da}{dT} + K_c a = 0 \qquad (5.7)$$

Whose solution is:

$$a(T) = A(T-T_c)^{K_c} \qquad (5.8)$$

Since $\not{s}$ is finite at T_c (as we have shown in section 2), we obtain:

$$a\not{s} \sim (T-T_c)^{K_c} \qquad (5.9)$$

This is the entropy per unit volume, whose derivative with respect to T yields the specific heat at constant volume, C_v. Thus:

$$C_v \sim (T-T_c)^{K_c - 1} \qquad (5.10)$$

We may then identify the critical exponent α of the specific heat:

$$\alpha = 1 - K_c \qquad (5.11)$$

The high-temperature behavior of $M^2(T)$ and $\lambda(T)$ is given by (see eqs.(3.20) and (3.21)):

$$M^2(T) = 2M^2 + \frac{\lambda T^2}{24} \qquad (5.12)$$

$$\lambda(T) = \lambda + \lambda^2\left[\frac{T}{M} + \ln\left(\frac{M}{T}\right)\right] \qquad (5.13)$$

This can be inverted to yield M^2 and λ as functions of $M^2(T)$ and $\lambda(T)$. To leading order in T we have:

$$2M^2 \approx M^2(T) - \frac{\lambda T^2}{24} \qquad (5.14)$$

$$\lambda \approx \lambda(T) \qquad (5.15)$$

where we have neglected the variation of $\lambda(T)$ with temperature compared to that of $M^2(T)$. Rewriting the free energy in terms of these effective couplings and performing the calculations described previously, we obtain:

$$\frac{d}{dT}(\Delta f)\Big|_{T=T_c} = -2\sqrt{2}\,\frac{M^2}{\sqrt{\lambda}} \qquad (5.16)$$

$$K_c = \frac{1}{3} + O(\sqrt{\lambda}) \qquad (5.17)$$

$$\nu = \frac{2}{3} + O(\sqrt{\lambda}) \qquad (5.18)$$

The value obtained for ν obeys Fisher's inequality, $d\nu \geq 2-\eta$ and, if we neglect the $O(\sqrt{\lambda})$ corrections, it saturates the bound ($d=3$). The fact that our calculation respects a rigorous bound such as the one above is further indication that our method, based on domain walls, is a sensible way of viewing the phase transition.

6. CONCLUSIONS

We have presented an alternative way of computing the transition temperature in $\lambda\phi^4$ theory. Our method is based on a one-loop calculation around a soliton background. Recent results[14] draw attention to the fact that a one-loop effective potential, besides being complex for $|\vec{\phi}|<\phi_{cr}$, is also nonconvex in the region between minima. Although also restricted to one-loop order, our calculation incorporates a novel aspect, typically nonperturbative, that might improve upon the previous result (some of whose problems we have already mentioned).

We have not bothered about the strong evidence for the triviality of the continuum limit of lattice $\lambda\phi^4$ in four dimensions at $T=0$[15]. To begin with, such evidence was gathered in the symmetric phase, nothing being said about the case of broken symmetry. Secondly, even if this result holds for the broken phase, continuum $\lambda\phi^4$ (if we ever make sense of its perturbation theory) might well be a good approximation to an effective low-energy theory[16,17]. Clearly, the results we have obtained can be extended to D=2 and D=3 (D-1 of these are spatial dimensions) where nontrivial limits do exist. Nonetheless, it is the case we have discussed that will be relevant for Particle Physics and Cosmology.

The renormalization procedure in the soliton sector, up to one loop, has been carried out recently using dimensional regularization[18]. It enables one to have very compact and elegant expressions, valid for arbitrary dimension.

A final check on the existing estimates of T_c can be provided by the calculation of the effective potential at finite temperature by a Monte Carlo simulation. This would be an extension of the results of reference 14. Work is already in progress in this direction as well as towards computing the exponents ν and η.

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APPENDIX A

We shall outline the steps leading to equations (2.25)-(2.28) of Section 2. The eigenvalue equation (2.14), for both soliton and vacuum sectors, is given by:

$$\left[-\frac{d^2}{dz^2}+V(z)\right]v_{jL}(z) = \varepsilon(j)v_{jL}(z) \qquad (A.1)$$

where we have used the dimensionless variable $z \equiv \frac{Mx_L}{\sqrt{2}}$, $\varepsilon(j) \equiv \frac{E(j)-\vec{p}_T^{\,2}}{M} - 2$ ($\vec{p}_T$ for soliton and $\vec{k}_T$ for vacuum) and V(z) defined as:

$$V(z) = \begin{cases} 3(\tanh^2 z-1) = \dfrac{-3}{\cosh^2 z} & \text{(soliton)} \\ \text{zero} & \text{(vacuum)} \end{cases} \qquad (A.2)$$

The soliton equation is the Schröedinger equation for the Posch-Teller potential[10]. It has two bound-states:

$$\varepsilon_0 = -2 \;;\; v^s_{0L}(z) = \frac{1}{\cosh^2 z} \rightarrow E^2_s(0) = \vec{p}_T^{\,2} \qquad (A.3)$$

$$\varepsilon_1 = -\frac{1}{2} \;;\; v^s_{1L}(z) = \frac{\sinh z}{\cosh^2 z} \rightarrow E^2_s(1) = \vec{p}_T^{\,2} + \frac{3}{2}M^2 \qquad (A.4)$$

and a continuum:

$$\varepsilon(\tilde{p}_L) = \frac{1}{2}\tilde{p}_L^2 \;;\; v^s_{p_L}(z) = e^{i\tilde{p}_L z}(3\tanh^2 z - 1 - \tilde{p}_L^2 - 3i\tilde{p}_L\tanh z) \qquad (A.5)$$

$$\rightarrow E^2_s(p_L) = \vec{p}_T^{\,2} + p_L^2 + 2M^2$$

with $\tilde{p}_L = \frac{\sqrt{2}\,p_L}{M}$, p_L the longitudinal momentum. The vacuum equation is just a free Schröedinger equation:

$$\varepsilon(\tilde{k}_L) = \frac{1}{2}\tilde{k}_L^2 \;;\; v^v_{k_L}(z) = e^{i\tilde{k}_L z} \rightarrow E^2_v(k_L) = \vec{k}_T^{\,2} + k_L^2 + 2M^2 \qquad (A.6)$$

The asymptotic behavior of the continuum eigenvectors in the soliton sector is given by:

$$v^{s}_{p_L}(z) \xrightarrow{z\to\pm\infty} e^{i(\bar{p}_L z \pm \frac{\delta}{2})} \qquad (4.7)$$

$$\delta(\bar{p}_L) = -2\arctan\left(\frac{3\bar{p}_L}{2-\bar{p}_L^2}\right) \qquad (4.8)$$

If we take a box of side L in the longitudinal direction, periodic boundary conditions yield:

$$\bar{p}_{L,n}\left(\frac{ML}{\sqrt{2}}\right) + \delta(\bar{p}_{L,n}) = 2\pi n \qquad (A.9)$$

$$\bar{k}_{L,n}\left(\frac{ML}{\sqrt{2}}\right) = 2\pi n \qquad (A.10)$$

for soliton and vacuum sectors, respectively. Form that we derive:

$$k_L = p_L + \frac{\delta(p_L)}{L} \qquad (A.11)$$

Clearly, $\vec{k}_T = \vec{p}_T$. We can now relate the sums over the continuum eigenstates for soliton and vacuum sectors. Taking the box to infinity, we may replace $\frac{1}{L}\sum_{n=-\infty}^{\infty}$ with $\int_{-\infty}^{\infty}\frac{dk_L}{2\pi}$ and use (A.11) to express the sums of (2.18) in terms of k_L and $\vec{k}_T$. Leaving out the classical action and the contribution of bound states, we will have:

$$(T\Delta\rho)_{cont} = \frac{1}{2}\int\frac{d^2k_T}{(2\pi)^2}\int_{-\infty}^{\infty}\frac{L\,dk_L}{2\pi}\left\{\sqrt{\left(k_L+\frac{\delta}{L}\right)^2+\vec{k}_T^2+2M^2} - \sqrt{k_L^2+\vec{k}_T^2+2M^2}\right\} +$$

$$+\int\frac{d^2k_T}{(2\pi)^2}\int_{-\infty}^{\infty}\frac{L\,dk_L}{2\pi}\left\{\ln\left(1-e^{-\beta\sqrt{(k_L+\frac{\delta}{L})^2+\vec{k}_T^2+2M^2}}\right) - \ln\left(1-e^{-\beta\sqrt{k_L^2+\vec{k}_T^2+2M^2}}\right)\right\}$$

(A.12)

If we expand $(k_L + \frac{\delta}{L})^2$ for $L\to\infty$ this becomes:

$$(T\phi\phi)_{cont} = \int \frac{d^2k_T}{(2\pi)^2} \int_{-\infty}^{\infty} \frac{dk_L}{2\pi}\, \delta(k_L) \frac{\partial}{\partial k_L} \left\{ \frac{1}{2} \sqrt{k_L^2 + \vec{k}_T^2 + 2M^2} + \right.$$

$$\left. + \frac{1}{\beta} \ln\left(1 - e^{-\beta\sqrt{k_L^2 + \vec{k}_T^2 + 2M^2}}\right) \right\} \qquad (A.13)$$

Integrating by parts, using (A.8) and adding the contribution of the two bound-states will lead to the desired expressions.

APPENDIX B

Let us go through the procedure used in eliminating the divergences of (2.25)-(2.28) to arrive at (2.29)-(2.31). We shall concentrate on the T=0 terms contributing to T_{44}, which we have named ξ_1 in the text. If we use $D=\sqrt{k_L^2+\vec{k}_T^2+2M^2}$, the integrand of the last term in (2.26) may be rewritten as:

$$\frac{q\,D}{2\sqrt{2}\,M^2} = \frac{q}{2\sqrt{2}\,M^2 D} + \frac{(3/2)M^2\vec{k}_T^2}{D^3(2k_L^2+M^2)} + \frac{(3/2)}{D} + \frac{(3/4)M^2}{D^3} + \frac{(3/2)^2 M^4}{D^3(2k_L^2+M^2)} \tag{B.1}$$

This is the result of a series of manipulations aimed at splitting it into several pieces, each with its particular ultraviolet behavior in $\vec{k}_T$ and k_L . All one has to do is divide and multiply by D and rearrange the expression to finally obtain (B.1).

The divergences of the third and fourth terms of (B.1) are extracted by graphs coming from the comuterterms. These graphs are:

$$-\!\bigcirc \equiv \left\{\int_0^{\infty} d\tau \int d^3x \left[B_s(x_L)-B_v\right]\right\}\left\{\frac{1}{\beta}\sum_{j=-\infty}^{\infty}\int\frac{d^3p}{(2\pi)^3}\,\frac{1}{\omega_j^2+\vec{p}^2+2M^2}\right\} \tag{B.2}$$

$$-\!\bigcirc\!- \equiv \int_0^{\beta} d\tau_x \int d^3x \int_0^{\beta} d\tau_y \int d^3y \left[B_s(x_L)-B_v\right] K(\tau_x,x_L,\vec{x}_T;\tau_y,y_L,\vec{y}_T) \left[B_s(y_L)-B_v\right] \tag{B.3}$$

with K denoting the Fourier transform of $\tilde{K}$:

$$\tilde{K}(a_j,\vec{q}) = \frac{1}{\beta}\sum_{j=-\infty}^{\infty}\int\frac{d^3p}{(2\pi)^3}\,\frac{1}{(\omega_j+a_j)^2+(\vec{p}+\vec{q})^2+2M^2} \tag{B.4}$$

In fact, we have taken $\omega_j=0$, $\vec{q}=\vec{0}$ (subtraction at zero momentum). Thus:

$$\Big[\;\cdots\;\Big]_{\substack{\omega_j=0\\ \vec{q}=0}} = \left\{\int_0^\beta d\tau \int d^3x \left[B_s(x_\perp)-B_v\right]^2\right\}\left\{\frac{1}{\beta}\sum_{j=-\infty}^{\infty}\int\frac{d^3p}{(2\pi)^3}\,\frac{1}{(\omega_j^2+\vec{p}^{\,2}+2M^2)^2}\right\} \quad (B.5)$$

The results (B.2) and (B.5) will cancel the third and fourth term exactly if we just take the $T=0$ contributions to the second curly brackets in both expressions (we could have written directly $\int\frac{d^4p}{(2\pi)^4}$).

The fifth term is convergent, whereas the two remaining terms, when added to the contribution of the bound states, give a finite result which is quoted in (2.31).

The subtraction procedure that we have adopted corresponds to the renormalization conditions shown in Section 3, equations (3.15) to (3.18).

APPENDIX C

The Γ-functional may be written as a functional Taylor expansion. At T=0, in D Euclidean dimensions, we may write:

$$\Gamma(\phi_c(x)) = \sum_{n=1}^{\infty} \frac{1}{n!} \int d^Dx_1 \ldots d^Dx_n \Gamma^{(n)}(x_1,\ldots x_n)[\phi_{c\ell}(x_1)-\phi_v]\ldots[\phi_{c\ell}(x_n)-\phi_v] \tag{C.1}$$

Alternatively we may use $B(x) = \frac{\lambda}{2}\phi_{c\ell}^2(x)$ to write:

$$\Gamma(B(x)) = \sum_{n=1}^{\infty} \frac{1}{n!} \int d^Dx_1 \ldots d^Dx_n \Gamma_B^{(n)}(x_1 \ldots x_n)[B(x_1)-B_v]\ldots[B(x_n)-B_v] \tag{C.2}$$

This last form appears naturally in a graphical expansion analogous to (3.9) for the case of a coordinate dependent background field $\phi_{c\ell}(x)$. The $\Gamma_B^{(n)}(x_1 \ldots x_n)$ can be explicitly evaluated in one-loop order and we may use them to obtain the $\Gamma^{(n)}(x_1 \ldots x_n)$, by using the chain rule of functional differentiation. As an example we may show that:

$$\frac{\delta\Gamma}{\delta\phi_{c\ell}(x)} = \lambda\phi_{c\ell}(x)\frac{\delta\Gamma}{\delta B(x)} \tag{C.3}$$

$$\frac{\delta^2\Gamma}{\delta\phi_{c\ell}(x_1)\delta\phi_{c\ell}(x_2)} = \lambda\delta(x_1-x_2)\frac{\delta\Gamma}{\delta B(x_1)} + \lambda^2\phi_{c\ell}(x_1)\phi_{c\ell}(x_2)\frac{\delta^2\Gamma}{\delta B(x_1)\delta B(x_2)} \tag{C.4}$$

Taking the derivatives at $\phi_{c\ell}=\phi_v$ $(B=B_v)$, we obtain:

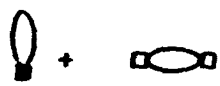
$$\Gamma^{(1)}(x) = \lambda\phi_v\Gamma_B^{(1)}(x) = 6M/\sqrt{\lambda}\,\Gamma_B^{(1)}(x) \tag{C.5}$$

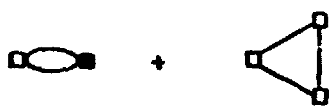
$$\Gamma^{(2)}(x_1,x_2) = \lambda\delta(x_1-x_2)\Gamma_B^{(1)}(x_1) + \lambda^2\phi_v^2\Gamma_B^{(2)}(x_1,x_2) =$$

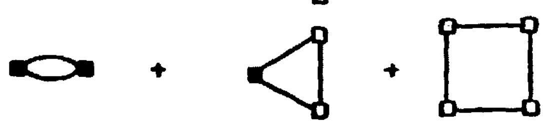
$$= \lambda[\delta(x_1-x_2)\Gamma_B^{(1)}(x_1) + 6M^2\Gamma_B^{(2)}(x_1,x_2)] \tag{C.6}$$

The two preceding equations, as well as the analogous ones for $\Gamma^{(3)}$ and $\Gamma^{(4)}$, can be viewed graphically as:

$\Gamma^{(1)} =$  $- O(\lambda^{1/2})$ (C.7)

$\Gamma^{(2)} =$  $- O(\lambda)$ (C.8)

$\Gamma^{(3)} =$  $- O(\lambda^{3/2})$ (C.9)

$\Gamma^{(4)} =$  $- O(\lambda^{2})$ (C.10)

Starting with the Euclidean Lagrangian:

$$\mathcal{L}_E = \frac{1}{2}(\partial\mu\phi)^2 + \frac{\lambda}{4!}\,(\phi^2-\phi_v^2)^2 \quad ; \quad \phi_v^2 = \frac{6M^2}{\lambda} \qquad \text{(C.11)}$$

and using $\phi = (\phi-\phi_v)$, we obtain:

$$\mathcal{L}_E = \frac{1}{2}(\partial\mu\phi)^2 + \frac{1}{2}(2M^2)\phi^2 + \frac{1}{3!}(6\sqrt{M\lambda})\phi^3 + \frac{1}{4!}(\lambda)\phi^4 \qquad \text{(C.12)}$$

The contributions to $\Gamma^{(n)}$, n=1,2,3,4, coming from this Lagrangian, are identical to the ones depicted in (C.7)-(C.10). Therefore, the use of the graphical expansion corresponds to including counterterms like ϕ^3, typical of the broken phase, with no more effort than the one involved in calculating in the symmetric phase!

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FIGURE CAPTION

Figure 1 - Effective potential in one-loop for different values of $T \leq \bar{T}_c$.