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THE TWELVE COLOURFUL STONES

by

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Abstract

A dynamics with twelve colourful stones is created based on the concepts of gauge and colour. It is associated different gauge fields to the same group. A group of gauge invariant Lagrangians is established. A gauge invariant mass term is introduced. The colourful stone physical insight is to be building blocks for quarks and leptons. (Author)

1. INTRODUCTION

It seems that a major consequence of the development of quark physics was the appearance of the colour concept. It is based on experimental results.

The consequence is that it is a new fundamental parameter to analyse the nature process with. Such dynamics has previously been built in terms of phenomenological entities such as space, time, mass and charge. Yet, the surprising fact is that while this new parameter influences experimental results, it cannot be observed. We thus suppose that the Physics of Galileo, where all the parameters of a given theory are directly measured, falls apart.

The idea that we are motivated to develop is that the colour and gauge concepts are enough to build up a dynamics. A Lagrangian based only on it, as the massless QCD, can predict properties of matter like for instance, the Asymptotic Freedom-Confinement, Decay Rates, Anomalous Dimension, etc. Therefore, the first aim of this work is to understand elementary particles made by colour only. These particles will be called the Colourful Stones. They will yield two consequences. First, a way to create motion. It can be an option for the Big Bang. The other is by observing that the last acts of our age in Physics are in the hands of two sinister actors: the quark and the lepton. Then, these stones will appear as sources for the last chapter. However they differ from the Preon concept.

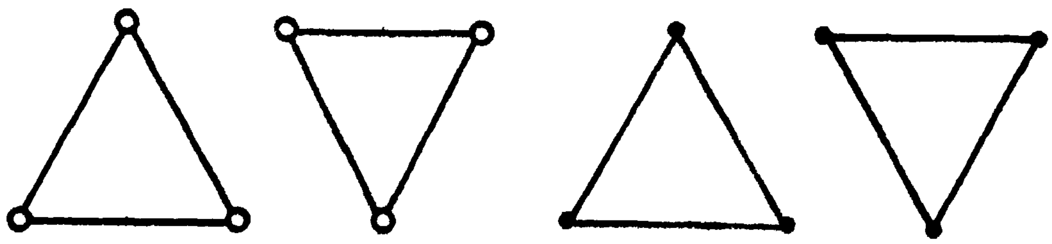
The paper is based on three steps. In Part I, the Colourful Stones are defined, and some types of possible Lagrangians that are gauge invariant are studied. In Part II, the asymptotic free dom property is prevised through the presence of the three gauge boson graph. In Part III, a gauge boson mass term is introduced preserving the gauge invariance. In the conclusion the presence of these stones are justified with the concept of unification. They will mean a unification comming from the back.

PART I - THE COLOURFUL DYNAMICS

2. Searching for the Yang Yin Lagrangian

The colourful stones dynamics will be organized by some Lagrangian. We will call it the Yang Yin Lagrangian. Some facts suggests us to search for a Yang-Mills type [1]. They are the QCD applications in strong interactions, the broken symmetry in weak interactions and the Einstein Gravitation. And also, colour has emerged from a Yang-Mills Lagrangian.

Considering only the concept of colour our first principle is to consider theses stones in two types. They will be called yang and yin stones. Following some experimental sense these stones will have three colours. Thus, it yields the triplets as in Fig. 1. The Yang stones will have three colours and the respective anti colours. The Yin stones will have the same colours and anticolours. The total gives twelve different stones.



Yang Stones Yin Stones

FIGURE 1

The twelve yang-yin stones are arranged now in triplets and antitriplets. They will need a Lagrangian to establish their dynamics.

The other principle to be adopted is that fundamental fields (in the case they are the Yang and the Yin) are associated with different gauge fields. It also means that these two fundamental fields can not interact directly, but via gauge bosons. For instance, it will restrict terms as

$$\mathcal{L}_I = g\,\bar{\Psi}\,\Psi\,\phi$$

if Ψ and ϕ are fundamental fields. This principle implies consequences in the renormalizability of the theory. It can avoid the necessity of the potential concept.

3. Matter Interaction

Associating the index i=1,2 to the Yang and Yin stones respectively yields in the space time structure the expression

$$A^i_\mu\;\rho^j\;\delta_{ij} \tag{1}$$

where A^i_μ and ρ^j are the gauge fields and Yang-Yin field stones.

Following QED we can suppose the interacting part of the colourful stones Lagrangian to have the following form

$$\mathcal{L}_I = \bar{\rho}_1\left[i\gamma^\mu(\partial_\mu - ig\,A^{1,a}_\mu t_a)\right]\rho_1 + \rho_2^+\left[(\partial_\mu - ig\,A^{2,a}_\mu t_a)(\partial_\mu - ig\,A^{2,a}_\mu t_a)\right]\rho_2 \tag{2}$$

Observe that it depends only on the concept of colour through the index a. It must be invariant under some compact Lie Group of transformations G. The various fields belong to the representation U of G. U must be chosen unitary. Considering the presence of three colours and anticolours the group to be chosen can be SU(3). Then L_I must be invariant under the field transformation

$$\rho^i \longrightarrow U(\theta)\ \rho^i \quad , \ i = 1,2$$

where (3)

$$U(\theta) = e^{i\, t^a \theta_a(x)} \quad , \ a = 1,2,\ldots,8$$

Introducing the covariant derivative,

$$D^j_\mu\ \rho^i\ \delta_{ij} = (\partial^j_\mu - ig\, A^{j,a}_\mu\, t_a)\ \rho^i\ \delta_{ij}$$

demanding that this type of derivative has the same transformation property as ρ^i itself,

$$(D^j_\mu\, \rho^i)' = U(\theta)\ (D^j_\mu\, \rho^i) \tag{4}$$

and using the following notation for the fields

$$\rho_1 \equiv \Psi \quad , \quad A^{1,a}_\mu \equiv A^a_\mu \quad ; \quad \rho_2 \equiv \phi \quad , \quad A^{2,a}_\mu \equiv B^a_\mu$$

gives from (4)

$$(t_a\, A^a_\mu)' = U(t_a\, A^a_\mu)\ U^{-1} - \frac{i}{g}\left[\partial_\mu U\right] U^{-1} \tag{5}$$

and a similar transformation for B_μ^a.

Considering the infinitesimal rotation

$$U(\theta) \approx 1 - i\, t^a\, \theta_a(x)$$

yields

$$B_\mu^{a'} = B_\mu^a + C^a{}_{bc}\, \theta^b B_\mu^c - \frac{1}{g}\, \partial_\mu \theta^a \tag{6}$$

and a similar expression for A_μ^a.

4. The Possible Gauge Fields Covariant Contributions

Inspired in the Yang-Mills Lagrangian type [1] and observing the gauge invariance (5) we can build up the following tensors: (The notation is $t_a X^a \equiv X$).

(i) the tensor $G_{\mu\nu}^a$. Define

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu B_\mu^a + C^a{}_{bc} B_\mu^b A_\nu^c \tag{7}$$

Considering the infinitesimal rotation (6),

$$\begin{aligned} \delta A_\mu^a &= C^a{}_{bc}\theta^b A_\mu^c - \frac{1}{g}\, \partial_\mu \theta^a \\ \delta B_\mu^b &= C^a{}_{bc}\theta^b B_\mu^c - \frac{1}{g}\, \partial_\mu \theta^a \end{aligned} \tag{8}$$

Using that

$$f_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu B_\mu^a \tag{9}$$

and replacing (8) in (9) gives

$$\delta f^a_{\mu\nu} = C^a{}_{bc}\theta^b f^c_{\mu\nu} + C^a{}_{bc}(\partial_\mu\theta^b)\, A^c_\nu - C^a{}_{bc}(\partial_\nu\theta^b)\, B^c_\mu \tag{10}$$

The second term in (7) transform like [1]

$$g\, c^a{}_{bc}\,\delta\left[B^b_\mu A^c_\nu\right] = C^a{}_{\ell\delta}\, C_{sbm}\,\theta^\ell B^b_\mu A^m_\nu - C^a{}_{bc}(\partial_\mu\theta^b)\, A^c_\nu + C^a{}_{bc}(\partial_\nu\theta^b)\, B^c_\mu \tag{11}$$

Adding (10) and (11)

$$\delta G^a_{\mu\nu} = C^a{}_{bc}\,\theta^b\, G^c_{\mu\nu} \tag{12}$$

that gives

$$t_a G^a_{\mu\nu} \longrightarrow U\, t_a G^a_{\mu\nu}\, U^{-1} \tag{13}$$

Observe that the tensor $G^a_{\mu\nu}$ is rot antisymmetric.

(ii) The tensor $H^a_{\mu\nu}$. Define

$$H^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu A^a_\mu + C^a{}_{bc} A^b_\mu B^c_\nu \tag{14}$$

Similarly to (7)

$$t_a H^a_{\mu\nu} \longrightarrow U\, t_a H^a_{\mu\nu}\, U^{-1} \tag{15}$$

$H_{\mu\nu}$ is not antisymmetric either. It obeys the relation

$$H^a_{\mu\nu} = -G^a_{\nu\mu} \tag{16}$$

Another way to obtain the relations (13) and (15) is by defining the following covariant derivatives

$$D_\mu(A) \equiv \partial_\mu - i\,g\,A_\mu \;,\; D_\nu(B) = \partial_\nu - i\,g\,B_\nu \qquad (17)$$

giving

$$[D_\mu(A)\,,\,D_\nu(B)] = -i\,g\,H_{\mu\nu} \qquad (18)$$

$$[D_\mu(B)\,,\,D_\nu(A)] = -i\,g\,G_{\mu\nu} \qquad (19)$$

(iii) Similarly to QCD define

$$^{A}F^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + C^a{}_{bc} A^b_\mu A^c_\nu \qquad (20)$$

$$^{B}F^{a}_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + C^a{}_{bc} B^b_\mu B^c_\nu \qquad (21)$$

Considering (8)

$$\begin{aligned} {}^{A}F_{\mu\nu} &\longrightarrow U\, {}^{A}F_{\mu\nu}\, U^{-1} \\ {}^{B}F_{\mu\nu} &\longrightarrow U\, {}^{B}F_{\mu\nu}\, U^{-1} \end{aligned} \qquad (22)$$

(iv) Define the tensors

$$^{Y}F^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + C^a{}_{bc} B^b_\mu B^c_\nu \qquad (23)$$

$$^{y}F^{a}_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + C^a{}_{bc} A^b_\mu A^c_\nu \qquad (24)$$

they yield

$$ {}^{Y}F_{\mu\nu} + {}^{y}F_{\mu\nu} \longrightarrow U({}^{Y}F^{a}_{\mu\nu} + {}^{y}F^{a}_{\mu\nu})\, U^{-1} \tag{25} $$

(v) Consider (23), (24) and define the relation

$$ C^{a}{}_{bc} A^{b}_{\mu} A^{c}_{\nu} = C^{a}{}_{bc} B^{b}_{\mu} B^{c}_{\nu} + \Lambda^{a}_{\mu\nu} \tag{26} $$

where $\Lambda^{a}_{\mu\nu}$ is something that transform like

$$ \Lambda_{\mu\nu} \longrightarrow U\, \Lambda_{\mu\nu}\, U^{-1} \tag{27} $$

Observe that the antisymmetric matrices whose coefficients are numbers can satisfy (27) directly.

Thus,

$$ \begin{aligned} {}^{Y}F_{\mu\nu} &\longrightarrow U\, {}^{Y}F_{\mu\nu}\, U^{-1} \\ {}^{y}F_{\mu\nu} &\longrightarrow U\, {}^{y}F_{\mu\nu}\, U^{-1} \end{aligned} \tag{28} $$

(vi) Relate the Yang and Yin structures with α and β planes respectively. The tensor which represents the kinetic part of the fields A^{a}_{μ} and B^{a}_{ν} will be attached to the planes α and β. In order to make such a study we have first to define some operations.

(a) The representation of the fields in terms of planes

$$ A^{a}_{\mu} \longrightarrow A^{a}_{\mu}\, \hat{\alpha} \quad ; \quad B^{a}_{\mu} \longrightarrow B^{a}_{\mu}\, \hat{\beta} \tag{29} $$

(b) Product-we can build up two kinds of products between such vectors, the internal and the external. The first one is defined by

$$A^a_\mu . A^a_\nu = A^a_\mu A^a_\nu \quad \hat{\alpha}.\hat{\alpha} \tag{30}$$

and the other is

$$A^a_\mu \wedge A^a_\nu = A^a_\mu A^a_\nu \quad \hat{\alpha} \wedge \hat{\alpha} \tag{31}$$

Defining that

$$\begin{aligned} \hat{\alpha}.\hat{\alpha} &= \hat{\alpha} \quad , \quad \hat{\beta}.\hat{\beta} = \hat{\beta} \\ \hat{\alpha}\wedge\hat{\alpha} &= \hat{\beta} \quad , \quad \hat{\beta}\wedge\hat{\beta} \quad \hat{\alpha} \end{aligned} \tag{32}$$

gives that the product of two vectors in one plane can generate a tensor is the same plane or in the other plane. This creates the following possibilities

$${}^{A}F^a_{\mu\nu}\ \hat{\alpha} = \partial_\mu A^a_\nu \hat{\alpha} - \partial_\nu A^a_\mu \hat{\alpha} + [A_\mu, A_\nu]^a \hat{\alpha} \tag{33}$$

$${}^{Y}F^a_{\mu\nu}\ \hat{\alpha} = \partial_\mu A^a_\nu \hat{\alpha} - \partial_\nu A^a_\mu \hat{\alpha} + [B_\mu, B_\nu]^a \hat{\alpha} \tag{34}$$

observe that the tensor in (33) is only in the α plane, while in (34) there is a term with fields in the β plane. Considering (26) they also transform like

$$ {}^{A}F^{a}_{\mu\nu}\ \hat{\alpha} \longrightarrow U\ {}^{A}F^{a}_{\mu\nu}\ U^{-1}\ \hat{\alpha} \qquad (35) $$

Similarly there, will appear such tensors in the β plane. The importance in introducing the tensors with $\hat{\alpha}$ or $\hat{\beta}$ is that it will restrict the equations of motion.

(vii) If two tensors X and Y transform covariantly, the addition and product will also have such property

$$ \begin{aligned} X \pm Y &\longrightarrow U(X \pm Y)\ U^{-1} \\ XY &\longrightarrow U(XY)\ U^{-1} \end{aligned} \qquad (36) $$

5. A Toy Model - The Shower

Considering that with two gauge fields appears differents Lagrangians let us interpret them with a model. It is based on fluid dynamics. There we have

$$ \frac{D}{Dt} f(x,y,z,t) \equiv \left(\frac{\partial}{\partial t} + \vec{v}.\vec{\nabla}\right) f(x,y,z,t) \qquad (37) $$

where $\underline{v}$ is the river velocity. The relation (37) tell us that the derivative of a function f which depends on the river coordinates is in true D_t f. Thus,

$$ \partial_t\ f \longrightarrow D_t\ f \qquad (38) $$

Similarly, we can write for quantum field theory that

$$\partial_\mu \Psi \longrightarrow D_\mu \Psi \qquad (39)$$

with

$$D_\mu = \partial_\mu + v_{\mu\nu} \nabla_\nu \qquad (40)$$

the first term in (40) represents the local variation of the field and the second is a variation correlated with the "river" as in Fig. 2. Thus we are going to define this river made out of the fields A^a_μ such that

$$V_{\mu\nu} \nabla_\nu \equiv ig A^a_\mu t_a \qquad (41)$$

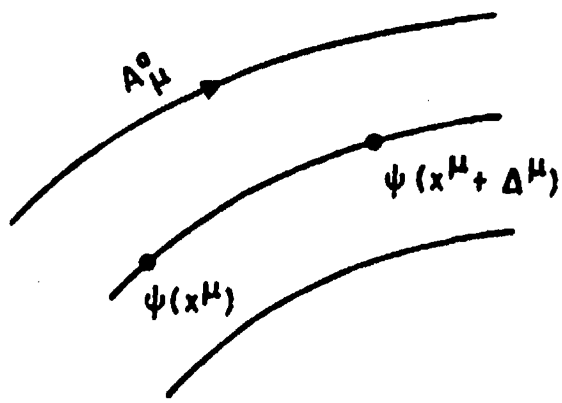


Fig. 2. The variation of the field is given by

$$\delta \Psi = (D_\mu \Psi) \delta x^\mu$$

The basic idea of these two structures called by Yang-Yin is to create two different dynamics but that are related. This is expressed through the terms $\delta\phi$ and $\delta\Psi$ which are connected with different "rivers". Comparing the gauge fields as the "river waters" and the fields as the ships let us try to understand intuitively the Lagrangian,

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_I$$

the first term corresponds to the river energy and the other to the navigation of the ship. The difficulty in section 4 was to define L_G uniquely. In order to understand it, let us remember from fluid dynamics that to observe a flow of a river there are two operators: the divergent and curl

RIVER TENSOR ≡ DIVERGENT + CURL

In QED this tensor, $F_{\mu\nu}$, is given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

as the divergent part $\partial_\mu A^\mu$ is considered zero, only the curl $F_{\mu\nu}$ represents such river tensor. Then the question is for the Yang-Mills case where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + C^a{}_{bc} A^b_\mu A^c_\nu$$

and so, it is necessary an explanation for the second term.

Let us observe the tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + C^a{}_{bc} B^b_\mu B^c_\nu$$

and interpret it as in Figure 3. Then, the term $C_{abc}\ B^b_\mu\ B^c_\nu$ plays the role of a shower from the plane β on α. The dynamics in the α plane will depend on the river on the β plane and vice versa. We expect to understand this dependence as being the origin of Asymptotic Freedom-Confinement properties: (23), (24), (33) and (34) are tensors with such shower.

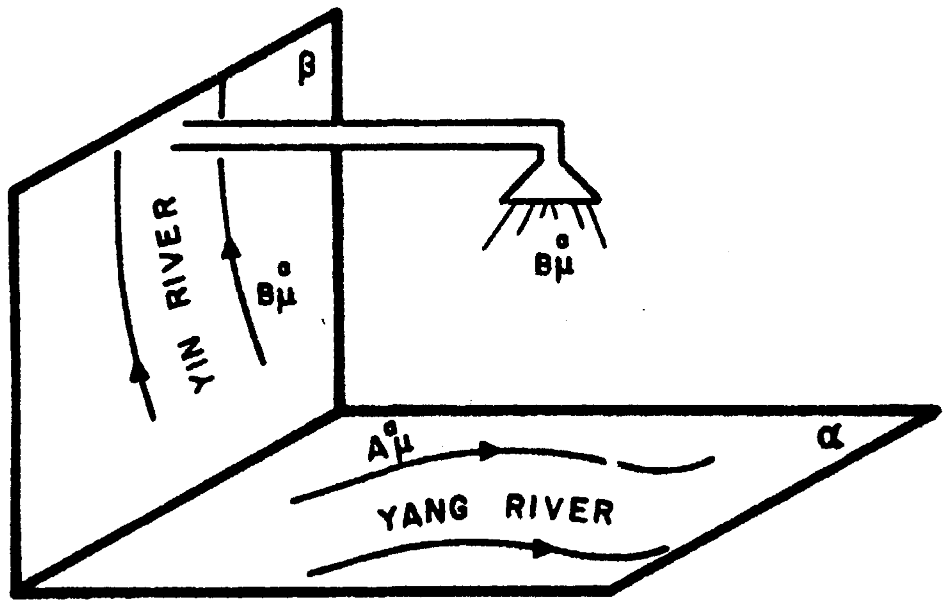


Fig. 3. B_μ^a fields dropping in the α plane through a shower in β.

The interpretation of (7) and (14) is in figure 4.

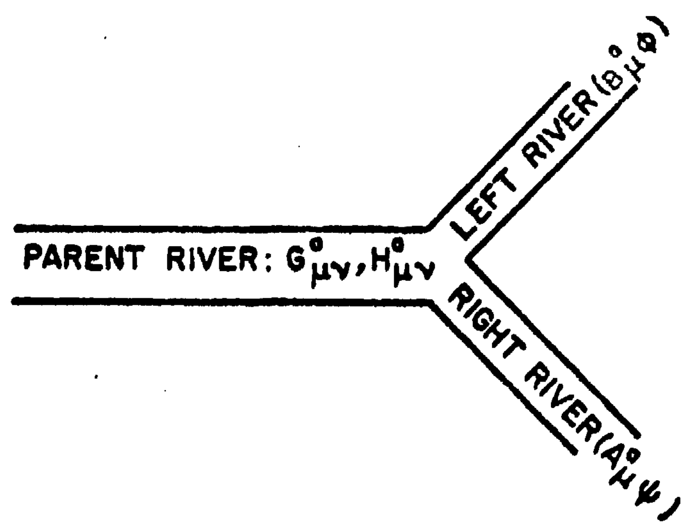


Fig. 4. The fields A_μ^a, B_μ^a are mixed in the parent river.

6. The Possible Lagrangians for the Gauge Fields

Considering (13), (15), (25), (27), (34), (35), (36) there will appear a large number of possibilities to build up gauge invariant Lagrangians. For instance (13) and (15) yield

$$\begin{aligned}
\mathcal{L}_G &= G^a_{\mu\nu} G^{\mu\nu}_a \\
\mathcal{L}_G &= H^a_{\mu\nu} H^{\mu\nu}_a \\
\mathcal{L}_G &= G^a_{\mu\nu} H^{\mu\nu}_a \\
\mathcal{L}_G &= \pm G^a_{\mu\nu} G^a_{\mu\nu} \pm H^a_{\mu\nu} H^{\mu\nu}_a \quad , \text{ etc}
\end{aligned} \tag{42}$$

It would be nice to have only one Lagrangian. We could avoid (25) with the argument that is does not minimize the Lagrangian effects. We expect that others necessities will enable us to reduce the selection.

7. Motion Equations

The variation of the fields Ψ and ϕ gives

$$\left[i\,\gamma^\mu(\partial_\mu - ig\,A^a_\mu\,t_a)\right]\Psi = 0 \tag{43}$$

$$\left[(\partial_\mu - ig\,B^a_\mu\,t_a)(\partial^\mu - ig\,B^{\mu,a}\,t_a)\right]\phi = 0 \tag{44}$$

For $\mathcal{L}_G = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$ the variation in relation to A^a_μ yields

$$G^a_{\mu\nu,\mu} = -C^a{}_{bc}\,B^b_\mu\,G^c_{\mu\nu} \tag{45}$$

and the variation in relation to B^a_μ

$$G^a_{\nu\mu,\mu} = -C^a{}_{bc}\, A^b_\mu\, G^c_{\nu\mu} \qquad (46)$$

For $\mathcal{L}_G = -\frac{1}{4} H^a_{\mu\nu} H^{\mu\nu}_a$ similarly yield,

$$H^a_{\nu\mu,\mu} = -C^a{}_{bc}\, B^b_\mu\, H^c_{\nu\mu} \qquad (47)$$

$$H^a_{\mu\nu,\mu} = C^a{}_{bc}\, A^b_\mu\, H^c_{\mu\nu} \qquad (48)$$

For $\mathcal{L}_G = -\frac{1}{4} G^a_{\mu\nu} H^{\mu\nu}_a$,

$$2\, G^a_{\mu\nu,\mu} = C^a{}_{bc}\, B^b_\mu\, H^c_{\mu\nu} + C^a{}_{bc}\, B^b_\mu\, G^c_{\nu\mu} \qquad (49)$$

$$2\, H^a_{\mu\nu,\mu} = C^a{}_{bc}\, A^b_\mu\, H^c_{\nu\mu} - C^a{}_{bc}\, A^b_\mu\, G^c_{\mu\nu} \qquad (50)$$

For $\mathcal{L}_G = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a \pm \frac{1}{4} H^{\mu\nu}_a H^a_{\mu\nu}$,

$$G^a_{\mu\nu,\mu} \pm H^a_{\nu\mu,\mu} = \mp C^a{}_{bc}\, B^b_\mu\, G^c_{\mu\nu} \mp C^a{}_{bc}\, B^b_\mu\, H^c_{\nu\mu} \qquad (51)$$

$$G^a_{\nu\mu,\mu} \pm H^a_{\mu\nu,\mu} = \mp C^a{}_{bc}\, A^b_\mu\, G^c_{\nu\mu} \mp C^a{}_{bc}\, A^b_\mu\, H^c_{\mu\nu} \qquad (52)$$

For $\mathcal{L}_G = -\frac{1}{4} {}^A F^a_{\mu\nu}\, {}^A F^{\mu\nu}_a - \frac{1}{4} {}^B F^a_{\mu\nu}\, {}^B F^{\mu\nu}_a$,

$${}^A F^a_{\mu\nu,\mu} = -C^a{}_{bc}\, A^b_\mu\, {}^A F^c_{\mu\nu} \qquad (53)$$

$${}^B F^a_{\mu\nu,\mu} = -C^a{}_{bc}\, B^b_\mu\, {}^B F^c_{\mu\nu} \qquad (54)$$

For $\mathcal{L}_G = -\frac{1}{4}({}^{Y}F^a_{\mu\nu} + {}^{y}F^a_{\mu\nu})^2$,

$$ {}^{Y}F^a_{\mu\nu,\mu} + {}^{y}F^a_{\mu\nu\ \mu} = C^a{}_{bc}\, A^b_\mu\, ({}^{Y}F^c_{\mu\nu} + {}^{y}F^c_{\mu\nu}) \qquad (55) $$

$$ {}^{Y}F^a_{\mu\nu,\mu} + {}^{y}F^a_{\mu\nu,\mu} = C^a{}_{bc}\, B^b_\mu\, ({}^{Y}F^c_{\mu\nu} + {}^{y}F^c_{\mu\nu}) \qquad (56) $$

For $\mathcal{L}_G = -\frac{1}{4}\, {}^{Y}F^a_{\mu\nu}\, {}^{Y}F^{\mu\nu}_a - \frac{1}{4}\, {}^{y}F^a_{\mu\nu}\, {}^{y}F^a_{\mu\nu} + \lambda^{\mu\nu,a}\left[C_{abc}\,(A^b_\mu A^c_\nu - B^b_\mu B^c_\nu) + \Lambda^a_{\mu\nu}\right]$

with $\lambda^{\mu\nu,a}$ being a Lagrangian multiplier. It is a symmetric tensor. Similarly gives

$$ {}^{Y}F^a_{\mu\nu,\mu} = C^a{}_{bc}\, A^b_\mu\, F^c_{\nu\mu} \qquad (57) $$

$$ {}^{y}F^a_{\mu\nu,\mu} = C^a{}_{bc}\, B^b_\mu\, F^c_{\nu\mu} \qquad (58) $$

For $\mathcal{L}_G = -\frac{1}{4}\, {}^{Y}F^a_{\mu\nu}\, {}^{Y}F^{\mu\nu}_a\, \hat{\alpha} - \frac{1}{4}\, {}^{y}F^a_{\mu\nu}\, {}^{y}F^{\mu\nu}_a\, \hat{\beta}$ + the Lagrangian multiplier in $\hat{\alpha}$ and $\hat{\beta}$. Calculating the equations of motion for the α and β planes give respectively

$$ {}^{Y}F^a_{\mu\nu,\mu}\, \hat{\alpha} = C^a{}_{bc}\, B^b_\mu\, {}^{Y}F^c_{\nu\mu}\, \hat{\alpha} $$

$$ {}^{y}F^a_{\mu\nu,\mu}\, \hat{\beta} = C^a{}_{bc}\, A^b_\mu\, {}^{y}F^c_{\nu\mu}\, \hat{\beta} \qquad (60) $$

We suppose that equations (57) and (58) are hard to interpret. However in (59) and (60) things can start to become more clear. For instance, consider the case where the plane α does not depende on the shower from β. This means $B^a_\mu = 0$ on that plane. In this limit (59) is

$$(\partial_\nu A^a_\mu - \partial_\mu A^a_\nu)_{,\mu} = 0 \qquad (61)$$

that would be just like a QED with colour.

8. Conserved Currents

Any continuous transformation which leaves the action invariant will give a conserved current. In order to simplify the calculations, the currents corresponding to the Lagrangians in section 6 will be obtained with SU(2) algebra. They will be invariant under the global symmetry

$$\Psi' \longrightarrow e^{i\vec{L}.\vec{\theta}}\,\Psi \;;\quad g\,\vec{L}.\vec{A}'_\mu = e^{i\vec{L}.\vec{\theta}}\,g\,\vec{L}.\vec{A}_\mu\,e^{-i\vec{L}.\vec{\theta}} \qquad (62)$$

and similarly for the fields ϕ and $\vec{B}_\mu$.

The expression for the conserved current is

$$j^\mu = \frac{\partial \mathcal{L}}{\partial \phi_{i,\mu}}\,\delta\phi_i \qquad (63)$$

Consider (7),

$$\mathcal{L} = -\frac{1}{4}\vec{G}_{\mu\nu}.\vec{G}^{\mu\nu} + i\,\overline{\Psi}D_\mu(A)\,\Psi + |\not{D}_\mu(B)\,\phi|^2 \qquad (64)$$

(62) and (63) yield

$$j^\mu = -{}^{Y}j^\mu + {}^{y}j^\mu - \frac{1}{2}G_{\mu\nu}\wedge A_\nu + G_{\nu\mu}\wedge B_\nu \qquad (65)$$

Using the Euler Lagrange equations

$$\partial_\mu (G_{\mu\nu} \wedge A_\nu) = -(B_\mu \wedge G_{\mu\nu}) \wedge A_\nu + G_{\mu\nu} \wedge A_{\nu,\mu}$$

$$\partial_\mu (G_{\nu\mu} \wedge B_\nu) = -(A_\mu \wedge G_{\nu\mu}) \wedge B_\nu + G_{\nu\mu} \wedge B_{\nu,\mu} \qquad (66)$$

Applying the Jacobi identity in (66)

$$\partial_\mu j^\mu = \partial_\mu {}^Y j^\mu + \partial_\mu {}^y j^\mu = 0 \qquad (67)$$

where ${}^Y j^\mu = \bar{\Psi} \gamma^\mu L^i \Psi$ and ${}^y j^\mu = i \phi^\dagger L^i \overleftrightarrow{\partial}_\mu \phi$ are the Yang and Yin currents respectively.

The Lagrangian (14) gives the same expression as (67). The others from (20), (21), (23), (24) give

$$D_\mu (A) {}^Y j_\mu + D_\mu (B) {}^y j_\mu = 0 \qquad (68)$$

(32) and (33) yield

$$D_\mu (A) {}^Y j^\mu \hat{\alpha} + D_\mu (B) {}^y j^\mu \hat{\beta} = 0 \qquad (69)$$

Observe that two fields Ψ and ϕ can rotate under the same group

$$\phi \longrightarrow e^{i\alpha} \phi \quad ; \quad \Psi \longrightarrow e^{i\alpha} \Psi$$

but that does not mean they are equal. In a forthcaming work we will show the conditions for $A_\mu^a \neq B_\mu^a$ through current conservation.

PART II - THE RUNNING COUPLING CONSTANT

9. The Asymptotic Freedom - AF

It is sensible to consider the colour matter in Confinement. Therefore it is essential for the Lagrangian (s) here to have this property. However the calculations through perturbation theory allow us to observe only the region where AF can exist. Thus we will assume our belief in a correspondence between such properties. Then it will be enough to calculate the beta function only for small g.

We know from perturbative QCD that the three gluon vertex is responsible for the appearance of AF [2]. In the case of the Lagrangians in (22) it is similar to QCD. The three gauge bosons graphs will be as Fig. 5. The same kind of

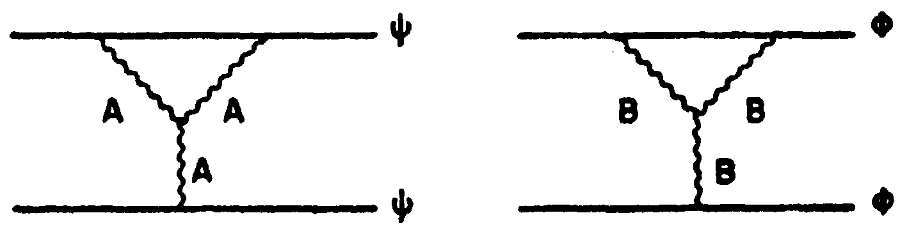


Fig. 5. Three gauge bosons graphs generated by (22). The fields Ψ and ϕ have an independent dynamics.

graphs will appear from (13), (15), (21), (34), however qualitatively different. This means that the yang yin stones are interacting through Fig. 6.

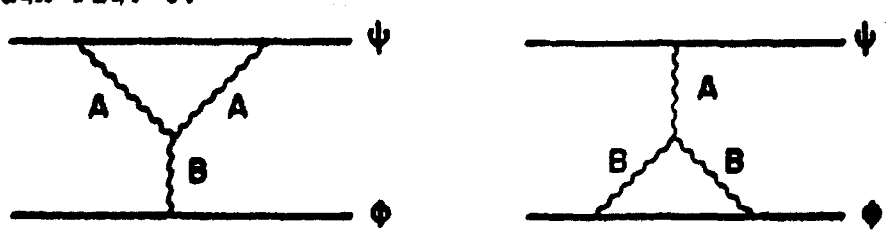


Fig. 6. Three gauge vertex generated by (13), (15), (21), (34).

A Yang Mills Lagrangian has the asymptotic freedom property. In our case it is extended with the presence of two gauge fields in the same group. In order to generate the first numbers we prefered to choose the Lagrangian most directly connected with the shower model in Fig. 3. However it is not gauge invariant. It is just a mathematical exercise to show the appearance of a negative sign in the beta function. In Appendix A we have calculated the running coupling constant for

$$\mathcal{L} = -\frac{1}{4}\, {}^{Y}F^{a}_{\mu\nu}\, {}^{Y}F^{\mu\nu}_{a} - \frac{1}{4}\, {}^{y}F^{a}_{\mu\nu}\, F^{\mu\nu}_{a} + i\,\bar{\Psi}\, \not{D}_{\mu}(A)\,\Psi + |D_{\mu}(B)\,\phi|^{2} \tag{70}$$

that yields for the not gauge dependent part,

$$\frac{\bar{g}^{2}}{16\pi^{2}} = \frac{1}{\beta_{0}\,\ell n\frac{Q^{2}}{\Lambda^{2}}} \tag{71}$$

where

$$\beta_{0} = \frac{27}{2} - \frac{5}{6}\,N_{f} \quad ;$$

We symbolically would interpret the exercise in (71) as a motivation for a cake recipe. In order to prepare it we need basically wheat and butter. After, some fruits can be added. An accelerator can separate these fruits, but not the yang butter from the yin wheat. Physically, the yang yin stones interaction as in Fig. 6 intend to generate the AF property. The confinement will be prevised by the appearance of a mass term for the gauge boson.

PART III - A GAUGE INVARIANT MASS TERM

9. The Gauge Field Mass

A problem for the gauge formalism is how to obtain gauge fields with mass preserving the gauge invariance. The gauge symmetry for just one field does not yield a massive field. Normally there is just three dynamical variables. Observe that in order to realize such symmetry it is necessary to choose some gauge reference system (gauge fixing). It will reduce another degree of freedom. However a massive field must have three degrees of freedom. The transformations (5) allows us to obtain a covariant expression by subtracting the fields

$$A'_\mu - B'_\mu \longrightarrow U(A_\mu - B_\mu)\, U^{-1} \tag{72}$$

that gives a gauge invariant mass term $m^2(A^a_\mu - B^a_\mu)^2$. (72) could represent a mass term with colour indices. It would be a NxN matrix (where N is the dimension of the adjoint representation) such that

$$[U, m^2] = 0 \tag{73}$$

(73) brings a mass term written in terms of Casemir operators. This fact creates a gauge boson mass depending on the matrices of the colour group.

Another procedure is by redefining the fields

$$C_\mu = A_\mu - B_\mu \tag{74}$$

$$D_\mu = A_\mu + B_\mu \tag{75}$$

obtaining

$$\mathcal{L} = -\frac{1}{4} C_{\mu\nu} C^{\mu\nu} + \frac{1}{2} m^2 C_\mu C^\mu - \frac{1}{4} D_{\mu\nu} D^{\mu\nu} - \frac{1}{2\alpha} (\partial_\mu D^\mu)^2 + \lambda C_{\mu\nu} D^{\mu\nu} \tag{76}$$

that is a massive and gauge invariant Lagrangian. The parameter λ is arbitrary and can be considered equal one. The strenght tensor $D_{\mu\nu}$ is Yang Mills type, while $C_{\mu\nu}$ not necessarily. For instance,

$$C_{\mu\nu} = D_\mu(D)\, C_\nu - D_\nu(D)\, C_\mu + g[C_\mu, C_\nu]$$

where

$$D_\mu(D) = \partial_\mu - g\, D_\mu \tag{77}$$

10. CONCLUSION

The general idea is to start nature dynamics with twelve colourful stones. They would represent a type of unified theory. Observing that the measurable world has a fermion-boson structure, motivate us to postulate the stones nature also in two families. They were called by yang and yin and with the same three colours.

Our belief is that colour is a general property of nature. It does not belong only to the quarks. Thus the colourful stones would appear as building blocks for quarks and leptons. A consequence is that the Asymptotic Freedom for the stones would mean that quarks and leptons are also interacting weakly for short dis tances. The quarks wave function can be defined as $\chi_i = f_{ijk}\phi^j\psi^k$, and the lepton by $\eta = \phi^i\psi_i$. A constant f_{ijk} is used for to balance the colour indices.

Another point of view is about a massive gauge boson. The gauge principle is being considered as the most useful way to generate interactions. A difficulty with this formalism is how to create conditions for the gauge boson mass. Therefore, the strong interactions through QCD has to be mediated by a massless gluon. However, if by the study of the ultraviolet divergence it brought the Asymptotic Freedom property [2], the infrared side gave a disease [3]. In our opinion with the failure of the factorization theorems in the quark gluon scattering [4] either the concept of impulse approximation in a hadron scattering must be revised or the gluons are massive. This fact represents an internal criticism to QCD. In (72) appears a massive field.

An option is to interpret (76) just as a field theory. Thus we would adopt the matter fields being fermionic and bosonic quarks. They would have spin half and zero. Consider scalar quarks included as in Fig. 7 and without charge. Thus we suppose that will become hard to find an experimental counter example for their presence. It is because phenomenologically they will influence only the mass.

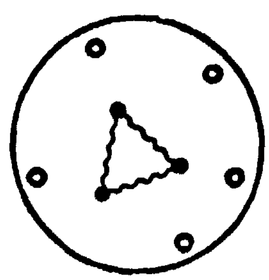


Fig. 7. A hadron model. The valence quarks are described by fermionic structures. Instead of quark-antiquark pairs forming the "sea", we consider a scalar colourful matter.

Basing on the basic gauge fields we can build up different mediating particles. For instance, the tensor $A_{\mu}B_{\nu}$ would correspond to a massless particle with spin two. At this way is possible to generate anothers kinds of interactions. The principle of unification would be coming from behind. This means that the task will be from colourful stones to understand how to create the four basic interactions.

Another attitude is to interpret these two gauge fields as interacting with the same matter field as in Fig. 8.

Although colour cannot be measured our intention is to explore how much it generates the matter process.

Fig. 8. The same matter multiplet interacting with different gauge fields.

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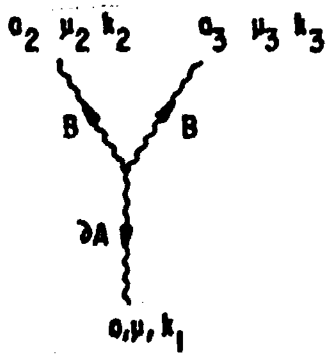
APPENDIX A

It is a study basing on Perturbation theory of (70). We are going to calculate the beta function.

Feynman Graphs

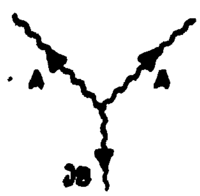
Similarly to QCD it is introduced the gauge fixing term and the ghost term. Rewriting the Lagrangian in terms of a free and an interacting part will yield the Feynman rules. The propagators will be like QCD but with two different fields. The gauge fixing term is $\frac{1}{2\alpha}[\partial_\mu(A^\mu + B^\mu)]^2$. The three gauge vertices will bring two different types. The first one is



$$\mathcal{L}_I = g C_{a_1 a_2 a_3} g_{\mu_1 \mu_3} (\partial_{\mu_2} A_{\mu_1}) B^{a_2}_{\mu_2} B^{a_3}_{\mu_3}$$

$$i\Gamma^{\mu_1\mu_2\mu_3}_{a_1a_2a_3} = -g C_{a_1a_2a_3}[g^{\mu_1\mu_2}(k_1)_{\mu_3} - g^{\mu_1\mu_3}(k_1)_{\mu_2}]$$

Similarly there is the case



The four gauge vertice are similar to QCD. The matter fields Feynman rules are similar to the usual scalar and fermionic fields. We are going to study up to one loop. The basic integral $I_{ab,N}$ to solve the graphs can be expressed by the relation,

$$I_{ab,N} = \int \frac{d^n k}{(2\pi)^n} \frac{k_\mu k_\nu \ldots k_\gamma}{[k^2+i\epsilon]^a [(k+p)^2+i\epsilon]^b} =$$

$$= \frac{(-)^{n-1+N} \; i^{n-1}}{(4\pi)^{n/2}\,\Gamma(a)\,\Gamma(b)} (p^2)^{\frac{n}{2}-a-b} \, p_{\mu_1} \ldots p_{\mu_N} \, \Gamma(a+b-\tfrac{n}{2}) \; B(\tfrac{n}{2}-b,\, N+\tfrac{n}{2}-a) +$$

$$-\frac{1}{4} \frac{(-)^{n+1+N} \; i^{n+1} (N-1)}{(4\pi)^{n/2}\,\Gamma(a)\,\Gamma(b)} (p^2)^{\frac{n}{2}+1-a-b} \, g_{\mu_1\mu_2} p_{\mu_3} \ldots p_{\mu\nu} \, \Gamma(a+b-\tfrac{n}{2}-1) B(\tfrac{n}{2}+1-b,\, N+\tfrac{n}{2}-a-1)$$

where N means the number of terms in the numerator and

$$B(x,y) = \frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)}$$

In order to obtain the beta function it is necessary to calculate the yang renormalization constants Z_3^Y, Z_3^A, Z_1^Y and the yin case Z_3^y, Z_3^B, Z_1^y. Considering our motivation, we are going to approximate the calculation by not considering the mixing propagator between the gauge fields. It is valid because it would only contribute to improve the negative sign. In a further work this presence will be considered

1. **Yang Field Renormalization constant: Z_3^Y**

Calculating the one loop contributions

$$= \alpha g^2 C_2(R)\, \frac{n-2}{2}\, \not p\, I_{11,0}$$

$$Z_3^Y = 1-g^2 \alpha C_2(R) \left[\frac{2}{\varepsilon} + L-1 + \sigma(\varepsilon)\right]$$

where

$$L = -\ell n\left(-\frac{p^2}{\mu^2}\right) + \ell n\, 4\pi + 2-\gamma \Big|_{p^2=-\mu^2}$$

2. **Yin field renormalization constant: Z_3^y**

$$= 0$$

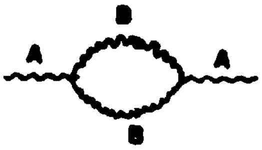
$$= -g^2 C_2(R)\, p^2 \left[2-(1-\alpha)(3-n)\right] I_{11,0}$$

yielding

$$Z_3^y = 1 + \frac{g^2}{(4\pi)^2} C_2(R) \left[\frac{2}{\varepsilon}(3-\alpha) + 2L + (1-\alpha)(L-2) + \sigma(\varepsilon)\right]$$

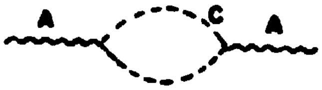
3. **Yang Gauge field renormalization constant: Z_3^Λ**

This number is determined by the following graphs



$$= g^2 C_2(G)\, \delta_{ab} (g_{\mu\nu} p^2 - p_\mu p_\nu) \left[2-(1-\alpha)(3-\frac{n}{2}) + (1-\alpha)^2 (1-\frac{n}{4}) \right] I_{11,0}$$

$$= 0$$



$$= g^2 C_2(G) \left[\frac{n-2}{4(n-1)} p_\mu p_\nu + \frac{1}{4(n-1)} p^2 g_{\mu\nu} \right] I_{11,0}$$

$$= -\frac{1}{2} g^2\, T(R)\, \delta_{ab} \left[p^2 g_{\mu\nu} - p_\mu p_\nu \right] I_{11,0}$$

Adding these graphs and considering the symmetrical factors yield the tensor $i\pi_{\mu\nu}$. The field C represents the ghost. It is interesting to note that with thus concept that mix the A_μ and B_μ fields the longitudinal part of the polarization tensor disappear naturally. It means that it is not necessary to introduce the ghost term as in QCD. The coefficient is

$$Z_3^A = 1 + g^2 \delta_{ab} \Big[C_2(G) \left[\frac{1}{\varepsilon}(2+2\alpha) + 1+L-\alpha(1-L) + \frac{1}{2}(1-\alpha)^2 \right] +$$

$$-T(R) \left[\frac{1}{\varepsilon} \frac{8}{3} - \frac{4}{9} + \frac{4}{3} L + \delta(\varepsilon) \right] \Big]$$

the ghosts were not considered in the expression above.

4. Yin Gauge Field Renormalization Constant: Z_3^B

It is similar to the yang case. The difference is in the graph

$$= -g^2\, T(R)\, \frac{1}{n-1}\left[g_{\mu\nu}\, p^2 - p_\mu p_\nu\right] I_{11,0}$$

then,

$$Z_3^B = 1 + \Big[C_2(G)\Big[\frac{1}{\varepsilon}(2+2\alpha) + 1+L-\alpha(1-L) + \frac{1}{2}(1-\alpha)^2 + \sigma(\varepsilon)\Big]$$

$$-T(R)\Big[\frac{1}{\varepsilon}\,\frac{2}{3} + \frac{1}{3}\Big(\frac{2}{3} + L\Big) + \sigma(\varepsilon)\Big]\Big]$$

without ghosts.

5. Yang Vertex Renormalization Constant: Z_1^Y

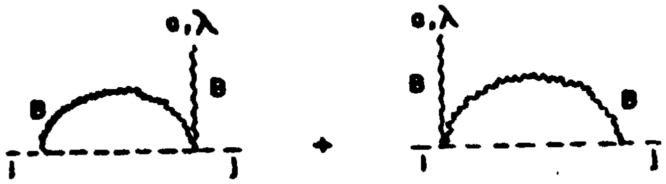
$$= g^3 (t_a)_{ij}\left[C_2(R) - \frac{1}{2}C_2(G)\right] \alpha(2-n)\left\{\left(2-\frac{n}{2}\right)\frac{\not p\, p_\mu}{p^2} - \frac{\gamma_\mu}{2}\right\} I_{11,0}$$

$$= 0$$

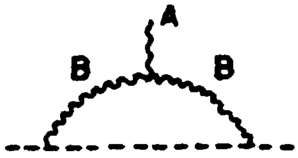
In the second graphs was calculated with the approximation $(k+Q)^2 \sim k^2$.

$$Z_1^Y = 1 - \frac{g^2}{(4\pi)^2}\left[C_2(R) - \frac{1}{2}C_2(G)\right]\left[\frac{3\alpha}{\varepsilon} - \frac{\alpha}{2} - \alpha(1+L) + \sigma(\varepsilon)\right]$$

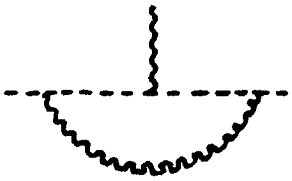
6. Yin Vertex Renormalization: Z_1^y



$$= g^3 \left[2C_2(R) - \frac{1}{2} C_2(G) \right] (t^a)_{ij}\; 2p_\lambda \cdot \left\{ -\frac{3}{2} + (1-\alpha)\left(2-\frac{n}{2}\right) \right\} I_{11,0}$$



$$\equiv g^3 \frac{1}{2} C_2(G)(t^a)\; 2p_\lambda \left[3\left(2-\frac{n}{2}\right) - \alpha\left(\frac{13}{2} - 2n\right) \right] I_{11,0}$$



$$= g^3 \left[C_2(R) - \frac{1}{2} C_2(G) \right](t_a)\; 2p_\lambda \left[3\left(2-\frac{n}{2}\right) + \alpha\left(\frac{n}{2}-1\right) \right] I_{11,0}$$

giving,

$$Z_1^y = 1 - \frac{g^2}{(4\pi)^2} \left[C_2(G) \left[\frac{1}{\varepsilon}\left(\frac{3}{2}-\alpha\right) - 1 + \frac{3}{2} L - 2\alpha + \frac{1}{2}\alpha L \right] + \right.$$

$$\left. + C_2(R) \left[\frac{1}{\varepsilon}(-6+2\alpha) + 5 - 3L - 3\alpha + \alpha L \right] + \sigma(\varepsilon) \right]$$

7. The beta Function: $\beta(g_R)$

The Lagrangian bring studied is not gauge invariant. Therefore the parameter α is meaningless. However our motivation is just to understand the influence of the three gauge vertice (Fig. 6) in the beta function. Then we are going to consider the calculations for $\alpha=0$. It similarly to QCD brings a negative sign.

$$\beta^{Y}(g_R) = -\frac{g_R^3}{(4\pi)^2}\left[2\,C_2(G) - \frac{4}{3}\,T(R)\right]$$

$$\beta^{y}(g_R) = -\frac{g_R^3}{(4\pi)^2}\left[\frac{5}{3}\,C_2(G) - \frac{1}{3}\,T(R)\right]$$

Adding these two contributions yields a beta function less than zero for the case $N_f \leq 16$. Thus with the twelve colourful stones we can have the asymptotic freedom property. It is only a mathematical result.

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