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STOSCHASTIC ANALYTIC REGULARIZATION

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STOCHASTIC ANALYTIC REGULARIZATION*

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ABSTRACT

Stochastic regularization is reexamined, pointing out a restriction on its use due to a new type of divergences which is not present in the unregulated theory. Furthermore, we introduce a new form of stochastic regularization which permits the use of a minimal substraction scheme to define the renormalized Green functions.

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1. INTRODUCTION

The stochastic quantization method of Parisi and Wu [1] has already found a large set of applications, ranging from quantization of gauge theories without gauge fixing [1] to studies of the large N reduction of field theories [2] and numerical simulations of quantum systems [3].

Last year, a new application of the stochastic quantization method was exhibited: A non-perturbative regularization of the infinities of Q.F.T. that preserves all the symmetries of the Langevin equation (Stochastic Regularization) [4] S.R. The main idea is as follows: The initial unregulated theory is obtained as the equilibrium limit of a Markovian stochastic process described by a Langevin equation. Stochastic regularization modifies the Markovian nature of the process in a well defined manner that renders the formerly divergent graphs finite.

One purpose of the present work is to analyze stochastic regularization from the point of view of the finite fictitious time theory [5]. We believe this is necessary because stochastic quantization assumes the existence of the finite fictitious time theory and thus we obtain the standard formulation when the fictitious time tends to infinity. Following this line of thought, we discover a restriction of stochastic regularization that was unnoticed in the original work [4]: The class of theories which are rendered finite by stochastic regularization is slightly larger than the class of renormalizable theories. Outside this class the stochastic regularization does not work.

A second objective of the work is to present a form of stochastic regularization that makes direct contact with Speer's analytic regularization [6] and therefore permits the introduction of a minimal subtraction scheme to define the divergent part of One-Particle Irreducible Graphs.

Since the new regularization is a form of stochastic regularization it preserves automatically all the symmetries of the Langevin equation. We illustrate its use with an example and develop the similarities with analytic regularization in a future paper.[7]

A word of caution: we assume that the reader is familiar with the rules and notation of the perturbative expansion of stochastic quantization [8].

2. STOCHASTIC REGULARIZATION

In this section we review the rules of Stochastic Quantization [9] in order to introduce the concept of Stochastic Regularization [4].

We are interested in the evaluation of (Euclidean) Green's functions defined by:

$$\langle \phi_{\ell_1}(x_1) \ldots \phi_{\ell_n}(x_n) \rangle = \frac{\int \mathcal{D}\phi \, \phi_{\ell_1}(x_1) \ldots \phi_{\ell_n}(x_n) \, e^{-S}}{\int \mathcal{D}\phi \, e^{-S}} \tag{2.1}$$

$\phi_\ell(x)$ is a boson field defined on the space time point x; ℓ denotes internal as well as Lorentz indices. S is the Euclidean classical action of the system.

We shall say that $\eta_\ell(x,t)$ is a random variable with Gaussian distribution if the following property is satisfied:

$$\langle \eta_{\ell_1}(x_1,t_1)\, \eta_{\ell_2}(x_2,t_2) \ldots \eta_{\ell_{2n}}(x_{2n},t_{2n}) \rangle_\eta$$

$$= \sum_{\substack{\text{possible} \\ \text{combinations}}} \prod_{\text{pair}} \langle \eta_{\ell_i}(x_i,t_i)\, \eta_{\ell_j}(x_j,t_j) \rangle_\eta \quad . \tag{2.2}$$

We shall refer to (2.2) as "Wick's decomposition property".

Moreover we assume that

$$\langle \eta_\ell(x,t)\, \eta_{\ell'}(x',t') \rangle_\eta = 2\, \delta_{\ell\ell'}\, \delta(x-x')\, \delta(t-t') \quad . \tag{2.3}$$

Below we shall list the steps which should be taken in order to calculate Green's function (2.1) by means of the stochastic quantization.

Firstly, we solve the following stochastic evolution equation, called Langevin equation;

$$\frac{\partial}{\partial t} \phi_\ell(x,t) = - \frac{\delta S}{\delta \phi_\ell(x,t)} + \eta_\ell(x,t) \tag{2.4a}$$

$$\phi_\ell(x,0) = 0 \quad . \tag{2.4b}$$

Next we calculate the η-average of

$$\langle \phi_{\ell_1}(x_1,t) \ldots \phi_{\ell_n}(x_n,t) \rangle_\eta \quad , \tag{2.5}$$

where $\phi_\ell(x,t)$ is the solution of (2.4) for η arbitrary.

Then, the large t limit of (2.5) gives the Euclidean Green function (2.1),

$$\langle \phi_{\ell_1}(x_1,t) \ldots \phi_{\ell_n}(x_n,t) \rangle_\eta \xrightarrow[t\to\infty]{} \langle \phi_{\ell_1}(x_1) \ldots \phi_{\ell_n}(x_n) \rangle \quad . \tag{2.6}$$

In general the perturbative (2.6) (See Secs. 3 and 4) is plagued with ultraviolet divergences. Stochastic Regularization modifies (2.3) while it keeps the Gaussian property of the noise (2.2) to regulate these divergences. More specifically, it assumes

$$\langle \eta_\ell(x,t)\, \eta_{\ell'}(x',t') \rangle_\eta = 2\, \delta_{\ell\ell'}\, \delta(x-x')\, B_\Lambda(t-t') \quad . \tag{2.7}$$

Since this modification of the η-correlation does not touch the x-dependent part of it, it will preserve all the symmetries of the action S [10].

If one chooses

$$B_\Lambda(t-t') \underset{t\to t'}{\sim} (t-t')^n \qquad n = 0,1,2 \ldots \tag{2.8}$$

with n sufficiently large, it is possible to regularize the ultraviolet divergences of most theories, according to the authors of Ref. 4).

The unregulated theory is obtained formally from (2.7) because it is assumed that

$$\lim_{A\to\infty} B_A(\tau) = \delta(\tau) \quad . \tag{2.9}$$

An example of this class of regulators is:

$$B_A(\tau) = \frac{1}{2} A^2 |\tau| e^{-A|\tau|} \quad . \tag{2.10}$$

3. ANALYSIS OF DIVERGENCES AT FINITE t

In this section we study the idea of stochastic regularization focusing on the finite fictitious time theory[5]. It has been shown in Ref.5) that the existence of the finite fictitious time theory imposses severe constraints on the model. Mainly, only renormalizable models can be described consistently in a stochastic (Langevin) formulation. We want to see at present if the same approach can be combined with stochastic regularization.

We are going to show that S.R. regulates certain ultraviolet divergences (First Class), but introduces new ones (Second Class). Second class divergences are not regulated by S.R. although they also contribute to the infinite fictitious time limit. Happily enough, renormalizable theories and a slightly larger set do not have second class divergences.

Let us calculate the crossed propagator in momentum space[4], according to (2.7). We get

$$\langle \phi(k,t_1)\,\phi(-k,t_2)\rangle_\eta = \int_0^{t_1} d\tau_1 \int_0^{t_2} d\tau_2\; e^{-(k^2+m^2)(t_1-\tau_1+t_2-\tau_2)} \times B_A(\tau_1-\tau_2) \quad . \tag{3.1}$$

Possible ultraviolet divergences appear when the exponent vanishes (later k will be the momentum of one of the lines in a given loop). That is

$$t_1-\tau_1+t_2-\tau_2 = 0 \quad . \tag{3.2}$$

But

$$t_1-\tau_1 \geq 0 \tag{3.3a}$$

and

$$t_2-\tau_2 \geq 0 \quad . \tag{3.3b}$$

Therefore the possible singularity is multiplied by

$$B_A(t_1-t_2) \quad . \tag{3.4}$$

Now assume that the vertices t_1 and t_2 are connected by an uncrossed propagator. To this line we associate the factor (see below)

$$e^{-(\bar{k}^2+m^2)|t_1-t_2|} \quad . \tag{3.5}$$

The ultraviolet singularity is possible only if, in addition to (3.2), we have

$$t_1-t_2 = 0 \quad . \tag{3.6}$$

In this case the would-be singularity is multiplied by (See (3.4))

$$B_A(0) \quad .$$

Choosing $B_A(0) = 0$, according to (2.8), we can regulate this type of singularity. We call it First Class divergence.

However, a different situation is possible. That is, there is no uncrossed propagator connecting the vertices t_1 and t_2. In this case the possible overall divergence is multiplied by $[B_A(t_1 - t_2)]^J$, J being the number of crossed propagators connecting the vertices t_1 and t_2. Clearly S.R. is unable to regulate this kind of divergences. We call them Second Class. They could occur when all fictitious times branching from t_1, t_2 in the perturbative expansion of ϕ_η collapse into t_1 and t_2 simultaneously. Fig. (1a) is an example of first class divergences while Fig. (1b) contains a Second Class divergence. Both graphs are assumed to be in 8-dimensional space-time. The Second Class divergence appears in Fig. (1b) when $\tau_1 = \tau_3 = \tau_4$ and $\tau_2 = \bar{\tau}_3 = \bar{\tau}_4$ simultaneously.

Since S.R. does not work for second class divergences, it is crucial to determine under which condition they are absent. To see this, we repeat the power counting done in Ref. 5) with the necessary modifications to include Second Class divergences.

But, first let us introduce some definitions. (For more details see Ref. 5).

Denote by $G(x_1,t_1;\, x_2,t_2)$ the stochastic propagator connecting the vertex $(x_2,\, t_2)$ of a One-Particle Irreducible (1PI) graph G2 with the vertex $(x_1,\, t_1)$ of another 1PI graph G1. We will say that the line $(x_1,\, t_1;\, x_2,\, t_2)$ is crossed (uncrossed) relative to G2 if $t_1 < t_2$ ($t_1 > t_2$).

According to the last definition we can classify 1PI graphs as follows: $S^{(L_0,\, L_c)}$ = 1PI graph with L_0 uncrossed external lines and L_c crossed external lines

$$\Gamma^{(n+1)} \equiv S^{(1,n)}$$

$$S^{(2)} \equiv S^{(2,0)} .$$

The boxes of Fig.1 provide examples of $\Gamma^{(2)}$(a) and $S^{(2)}$(b).

We proceed to find the overall degree of 2nd class divergence of $S^{(L_0,L_c)}$.

Call m the number of contracted crosses inside $S^{(L_0,L_c)}$, $\tilde{N}$ the number of τ - integrations inside the 1PI graph beside those assigned to the L_0 uncrossed legs.

The uncrossed legs enter $S^{(L_0,\, L_c)}$ at fictitious times τ_i, $i = 1 \dots L_0$. As we mentioned above, the second class divergence could occur when all the branching times τ_{ij} evolving from τ_i in the expansion of ϕ_η collapse into τ_i simultaneously. We write the following change of variables;

$$\tau_{ij} = \tau_i - r\,\theta_{ij} \quad . \tag{3.7}$$

Then, when $r \to 0$, $\tau_{ij} \to \tau_i$

The measure of integration is

$$dr\, r^{\tilde{N}-1}\, [d\theta_{ij}] \quad . \tag{3.8}$$

Since the τ_{ij} appears only linearly as coefficient of K_i^2 in the stochastic propagator (K_i is one of the loop momenta to be integrated over), each K_i-integral will bring down a factor of

$$r^{-n/2} \tag{3.9}$$

n is the dimension of space time.

That is, in the neighbourhood of (3.7) we have

$$S^{(L_0,L_c)} \sim \int_0^\varepsilon dr\, r^{\tilde{N}-1-\frac{n}{2}L} \tag{3.10}$$

L is the number of loops of $S^{(L_0,\, L_c)}$.

We define the overall degree of 2nd Class divergence of the stochastically regularized $S^{(L_0,\, L_c)}$ by

$$\tilde{D}_n = nL - 2\tilde{N} \quad . \tag{3.11}$$

If $\tilde{D}_n \geq 0$ the stochastically regularized graph is divergent.

It should be observed that

$$\tilde{N} = N + m - L_0 + 1 \tag{3.12}$$

where N is the number of τ-integration inside $S^{(L_0,\, L_c)}$ in the unregulated theory. The reason being that stochastic regularization introduces additional m τ-integrations inside the 1PI graph. Of these $(L_0 - 1)$ are excluded from $\tilde{N}$ because they correspond to the τ_i's of uncrossed legs.

Therefore

$$\tilde{D}_{n+2} = D_n + 2\,(L - m + L_0 - 1) \tag{3.13}$$

D_n is the naïve degree of divergence of $S^{(L_0,\, L_c)}$ in the unregulated theory.

The following identity is true for any local theory.

$$L - m + L_0 - l = 0 \quad . \tag{3.14}$$

That is

$$\tilde{D}_{n+2} = D_n \quad . \tag{3.15}$$

Since a renormalizable theory in $n+2$ - dimensional space is superenormalizable in n-dimensional space, (3.15) reduces the analysis of 2nd Class divergences in this case to the study of a finite number of graphs.

We illustrate the use of (3.15) with some examples (V is the number of vertices of $S^{(L_0,\, L_c)}$)

i) $\lambda\phi^4$ theory : $D_n = V(n-4) - L_0(1+n/2) + L_c(1-n/2) + n + 2$

Then $\tilde{D}_4 = -2V - 3L_0 + 4$.

Only $\Gamma^{(2)}$ to one loop could be divergent, but it is not 2nd Class divergence. Therefore it is regulated by stochastic regularization.

For $n = 5$ only $\Gamma^{(2)}$ in 1 and 2 loop order are potentially divergent, but they are not 2nd class, so they can be regulated by S.R.

Instead for $n \geqslant 6$, there is an infinite set of graphs with second class divergences, an example being $S^{(2)}$ at two loop order. Stochastic Regularization cannot be applied to this theory.

We see that the class of theories without 2nd class divergences is slightly larger than renormalizable theories.

ii) $\lambda\phi^3$ theory:

$D_n = (\frac{n}{2} - 3)V - (1+\frac{n}{2})L_0 + (1-\frac{n}{2})L_c + n + 2$

Then $\tilde{D}_6 = D_4 = -V - 3L_0 - L_c + 6$,

By inspection, we can see that there is no 2nd Class divergences.

Instead for $n \geqslant 8$, Fig. (1b) has a 2nd class divergence and then we cannot use stochastic regularization in this case.

iii) $n = 4$ is a special case in this context because any interaction of

the field ϕ without derivative coupling is superenormalizable in $n = 2$

Therefore these four dimensional theories can be regularized using S.R.

The question arises if the 2nd Class singularities contribute to the equilibrium theory or they are an artifice of the finite t formulation. An example will suffice to settle this point. In fact if one explicitly evaluates Fig.(1b) one finds a divergent contribution for large t and for any $B_A(\tau)$ corresponding to the region of integration $\tau_3 = \tau_4 \sim \tau_1$ and $\bar{\tau}_3 = \bar{\tau}_4 \sim \tau_2$ $(n \geq 8)$

Therefore 2nd Class singularities are present also in the equilibrium theory when they are at finite t invalidating the use of stochastic regularization even in this restricted sense.

4. A NEW REGULATOR

The type of regulator (2.7) has various disadvantages. First it is not easy to characterize the type of divergence for large A we shall encounter in a general stochastic graph. In the second place there is no precise prescription to fix the divergent piece of the 1PI graphs.

The solution of both problems is easy if the singularities of loop integrals appear as poles in the parameters of the regulator. In this case a Minimal Substraction Scheme is the natural substraction procedure. That is we remove the singularity just by removing the pole parts in the Laurent series for the quantity under consideration.

The work of Speer [6] realizes this programme for the usual (non-stochastic) graphs. But it has the disadvantage of not being gauge invariant. A gauge invariant analytic regularization is obtained by introducing the dimension of space time as regulator (dimensional regularization)[11]. However, dimensional regularization faces difficulties when chiral theories are present and also when massless particles are considered [12].

In this section we want to present a form of stochastic regularization that makes direct contact with analytic regularization. It is a gauge invariant regularization for which a Minimal Substraction Scheme is possible. Further, it does not have the limitations of dimensional regularization because it does not involve a change in the dimensionality of space time. In addition, if renormalization is made at finite t the problem with massless particle never appears.

Presently, we content ourselves with the definition of the regulator, leaving for a future publication [7] the detailed analysis of the divergences in arbitrary stochastic graphs. Its use will be illustrated in 2-dimensional U(N) gauge theory.

The authors of Ref. 4) introduced the form (2.9) for the δ-function in t. Because the unregulated theory is obtained when $A \to \infty$, it is not possible to expect analytic behaviour of the theory in the variable A. To remedy this we use a standard method of transforming asymptotic behaviour in a large parameter into an analysis of singularities on the complex plane of a different parameter: The Mellin transform [13].

The Mellin transform of $f(x) \in [0, +\infty]$ is defined by:

$$F(\beta) = \int_0^\infty dx \, x^{-\beta-1} f(x) \quad . \tag{4.1}$$

The crucial point is that the Mellin transform of

$$f(x) = x^{\beta_c} \frac{(\ln x)^{n-1}}{(n-1)!} \tag{4.2}$$

is

$$F(\beta) = \frac{1}{(\beta-\beta_c)^n} \quad . \tag{4.3}$$

That is the asymptotic behaviour of f(x) when $x \to \infty$ is replaced by the behaviour of $F(\beta)$ when $\beta \to \beta_0$.

The Mellin transform of (2.10) considered as a function of A is up to a normalization factor,

$$B_\beta(t) = \frac{1}{2} \beta \, |t|^{\beta-1} . \tag{4.4}$$

It is possible to prove that

$$\lim_{\beta \to 0} B_\beta(t) = \delta(t) \quad . \tag{4.5a}$$

Therefore (4.4) is acceptable as stochastic regulator. (4.4) has various nice properties. First it has a zero of arbitrary order at t=0 (choosing β large enough) which regularize any first class ultraviolet divergence. In contrast the regulator (2.10) cannot render finite a quartic divergence.

Secondly since only for $\beta \to 0$ we get a δ-function (physical limit), this means, according to (4.2), that we keep only logarithmic divergences. The other singularities are not relevant for the physical limit, a characteristic that our regulator shares with analytic and dimensional regularization.

Moreover, the form (4.4) makes direct contact with the analytic renormalization of Speer [6]. In particular it is not difficult to see that the ultraviolet divergences will appear as poles in the complex variable β at $\beta=0$. This last property is expected because of (4.2) and (4.3).

An intuitive way to see that the singularities at $\beta=0$ are poles is to compare our crossed propagator (3.1) and regulator (4.4)

$$\langle \phi(k,t_1) \, \phi(-k,t_2) \rangle_\eta = \int_0^{t_1} d\tau_1 \int_0^{t_2} d\tau_2 \, e^{-(k^2+m^2)(t_1+t_2-\tau_1-\tau_2)} \times \, \beta \, |\tau_1-\tau_2|^{\beta-1} \tag{4.5b}$$

with the Speer regulator in Euclidean momentum space (See equation (2.4) of second reference 6)

$$\int_0^\infty d\alpha \, \alpha^{\lambda-1} e^{-\alpha(k^2+m^2)} \quad . \tag{4.5c}$$

In the Speer method the singularity appears when $\lambda \to 1$, whereas in our case it appears when $\beta \to 0$. The difference being due to the fact that

(4.5b) contains one more integral than (4.5c). But in the neighbourhood of the first class singularity we have $t_1 = t_2 = \tau_1 = \tau_2$ (see section 3). And we can write

$$\begin{aligned} t_2 &= t_1 + \alpha[\theta]_2 \\ \tau_1 &= t_1 + \alpha[\phi]_1 \\ \tau_2 &= t_1 + \alpha[\phi]_2 \end{aligned} \tag{4.5d}$$

[ ] correspond to angular variables.

Then

$$\langle \phi(k,t_1)\,\phi(-k,t_2)\rangle_\eta \underset{\alpha\to 0}{\sim} \int d\alpha\, \alpha^\beta [d\theta_2][d\phi_1][d\phi_2]\, e^{-(k^2+m^2)\alpha([\theta]_2-[\phi]_1-[\phi]_2)} . \tag{4.5e}$$

Therefore in the neighbourhood of the singularity (4.5b) and (4.5c) are identical if we identify $\beta = \lambda - 1$. But Speer has shown that the singularities of canonical Feymann graphs are poles in the variable λ at $\lambda = 1$ [5]. Therefore we should expect that the singularities of stochastic graph are poles at $\beta = 0$. The detail of this analysis will be reported elsewhere [7].

To illustrate the use of (4.4) we calculate the one-loop contribution to $\Gamma^{(2)}$ in two dimensional gauge theory.

This theory is described by:

$$\frac{\partial}{\partial t} A_\mu = D_\nu F_{\nu\mu} + D_\mu \partial_\nu A_\nu + \eta_\mu \tag{4.6}$$

$$F_{\nu\mu} = \partial_\nu A_\mu - \partial_\mu A_\nu - ie[A_\nu, A_\mu] \tag{4.7}$$

$$D_\mu = \partial_\mu - ie[A_\mu, \ ] . \tag{4.8}$$

We have chosen the "Landau gauge" [14].

We assume the following form for the η-correlation

$$\langle \eta_\mu(x,t)\,\eta_\nu(x',t')\rangle_\eta = \beta\, \delta(x-x')\, \delta_{\mu\nu}\, |t-t'|^{\beta-1} . \tag{4.9}$$

The one loop contribution to $\Gamma^{(2)}$ is given in Figs. 2. Both graphs are primitively logarithmically divergent. If gauge invariance is to be respected the divergence of (a) should cancel the divergence of (b)

$$\begin{aligned} \text{(a)} \quad &= -4e^2 \delta_{\mu\nu}[N\delta_{ad}\delta_{bc} - \delta_{ab}\delta_{dc}] \\ &\quad \frac{\beta}{(2\pi)^2}\int_0^{t} d\bar\tau_1 \int_0^{\bar\tau_1} d\bar\tau_2\, (\bar\tau_1-\bar\tau_2)^{\beta-1} e^{-s\tau_1} \int d^2q\, e^{-q^2(2\tau_1-\bar\tau_1-\bar\tau_2)} \\ &= -4e^2 \delta_{\mu\nu}[N\delta_{ad}\delta_{bc} - \delta_{ab}\delta_{dc}]\, \frac{A(\beta)}{(2\pi)^2}\, e^{-s\tau_1} \end{aligned} \tag{4.10}$$

$$A(\beta) = \beta \int_0^{\tau_1} d\bar\tau_1 \int_0^{\bar\tau_1} d\bar\tau_2\, \frac{(\bar\tau_1-\bar\tau_2)^{\beta-1}}{2\tau_1-\bar\tau_1-\bar\tau_2}\, \pi . \tag{4.11}$$

To evaluate $A(\beta)$ we adopt Speer's method [6] to our case. By introducing the following change of variables

$$\begin{aligned} \bar\tau_2 &= \tau_1(1-\alpha_2) \\ \bar\tau_1 &= \tau_1(1-\alpha_1\alpha_2) \end{aligned} \qquad 0 \le \alpha_i \le 1 . \tag{4.12}$$

We have that

$$A(\beta) = \pi\beta\,\tau_1^\beta \int_0^1 d\alpha_1\, d\alpha_2\, \frac{(1-\alpha_1)^{\beta-1}\,\alpha_2^{\beta-1}}{1+\alpha_1} . \tag{4.13}$$

The singularity for $\beta \to 0$ here reflects in the fact that α_2 integral diverges when $\beta \to 0$. We integrate by parts over α_2. Then,

$$A(\beta) = \frac{1}{\beta}\left\{ \pi \beta \tau_1^{\beta} \int_0^1 d\alpha_1 \frac{(1-\alpha_1)^{\beta-1}}{1+\alpha_1} \right\}$$

$$\approx_{\beta \to 0} \frac{\pi}{2\beta} + \text{finite} . \tag{4.14}$$

Notice that the singularity of $A(\beta)$ is a pole at $\beta = 0$. That is

$$PP(a) = -\frac{e^2}{2\pi} \delta_{\mu\nu} [N \delta_{ad} \delta_{bc} - \delta_{ab} \delta_{dc}] \frac{e^{-s\tau_1}}{\beta} \tag{4.15}$$

$$(b) = 8 e^2 [N \delta_{ad} \delta_{bc} - \delta_{ab} \delta_{dc}] \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3 \int_0^{\tau_3} d\bar{\tau}_3 \int \frac{d^2 q}{(2\pi)^2} e^{-q^2(\tau_1 - \tau_2)}$$

$$e^{-(k-q)^2(\tau_1 + \tau_2 - \tau_3 - \bar{\tau}_3)} \; e^{-s\tau_2} \; \frac{\beta}{2} (\tau_3 - \bar{\tau}_3)^{\beta - 1} \tag{4.16}$$

$$\left\{ (k_\mu \delta_{\alpha_1 \alpha_2} - 2 \delta_{\alpha_1 \mu} k_{\alpha_2})(2 p_{\alpha_1} \delta_{\nu \alpha_2} - p_{\alpha_2} \delta_{\alpha_1 \nu}) + 2 p^2 \delta_{\mu\nu} \right\} .$$

We introduce the following change of variables :

$$\begin{aligned} \bar{\tau}_3 &= \tau_1 (1 - \bar{\alpha}_3) & \quad 0 \le \bar{\alpha}_3 \le 1 \\ \tau_3 &= \tau_1 (1 - \bar{\alpha}_3 \alpha_3) & \quad 0 \le \alpha_3 \le 1 \\ \tau_2 &= \tau_1 (1 - \bar{\alpha}_3 \alpha_3 \alpha_2) & \quad 0 \le \alpha_2 \le 1 . \end{aligned} \tag{4.17}$$

We find that

$$PP(b) = \frac{e^2}{2\pi} \delta_{\mu\nu} \frac{e^{-s\tau_1}}{\beta} [N \delta_{ad} \delta_{bc} - \delta_{ab} \delta_{dc}] . \tag{4.18}$$

Therefore

$$PP(a) + PP(b) = 0 . \tag{4.19}$$

5. CONCLUSIONS

In the present work we have exhibited certain limitations of the method of Stochastic regularization. Our analysis shows that S.R. is still appropriate to regularize renormalizable theories, which are in any case the ones for which a consistent stochastic (Langevin) description exists.

In addition to this, we have introduced a form of stochastic regularization for which a minimal substraction prescription is defined. We believe it to be an important step towards practical utilization of the S.R. idea.

Further work is necessary to explicitly exhibit the nature of the singularities of arbitrary stochastic graphs in the new parameter β. Progress made on it will be reported elsewhere [7].

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FIGURE CAPTIONS

Fig.1 (a) One-loop contribution to $S^{(2)}$ in $\lambda\phi^3$ theory

(b) One-loop contribution to $S^{(2)}$ in $\lambda\phi^3$ theory

Fig.2 (a)-(b) One-loop contributions to $\Gamma^{(2)}$ in two-dimensional gauge theory

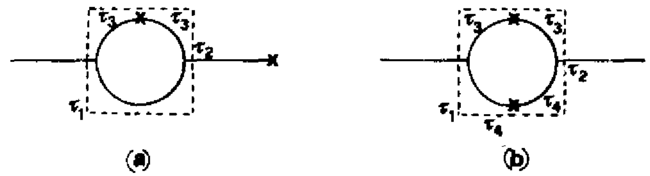

Fig.1

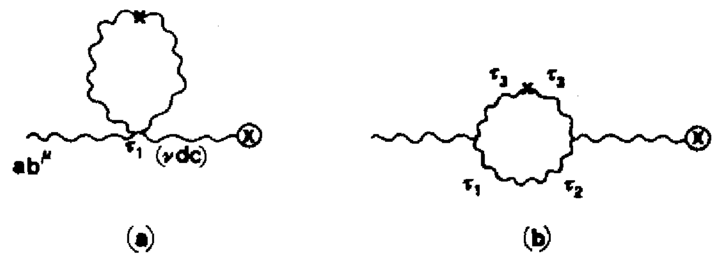

Fig.2