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THEORY OF FRACTIONAL QUANTUM HALL EFFECT *

I.Z. Kostadinov **
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

A theory of the fractional quantum Hall effect is constructed by introducing 3-particle interactions breaking the symmetry for $\nu = 1/3$ according to a degeneracy theorem proved here. An order parameter is introduced and a gap in the single particle spectrum is found. The critical temperature, critical filling number and critical behaviour are determined as well as the Ginzburg-Landau equation coefficients. A first principle calculation of the Hall current is given. 3, 5, 7 electron tunneling and Josephson interference effects are predicted.

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** Permanent address: Department of Physics, University of Sofia, 1126 Sofia, Bulgaria.

1. INTRODUCTION

The fractional quantum Hall effect (FQHE) was discovered by Tsui, Stormer and Gossard [1], who observed two-dimensional (2-D) Hall conductivity σ_{xy} in Ga As-Al$_x$ Ga$_{1-x}$ As heterostructures equal to fractions of the fundamental constant, $\sigma_{xy}=\frac{e^2}{h}\nu$ with $\nu=1/3$ and 2/3. The filling factor is defined as ratio of the total number of electrons N_b to the total magnetic degeneracy of the Landau level N_B. To date the fractions $\nu=1/3,2/3,4/3,5/3,1/5,2/5,3/5,2/7,4/7$ have been observed and identified with fractional fillings of the Landau levels of the 2-D electronic liquid in strong magnetic field B. In a sense this is the first observation of fractional quantum numbers in physics. Extensive experimental work demonstrated peculiar temperature and ν-dependences [2,3] of σ_{xx} and σ_{xy}. A strange behaviour with increasing frequency of an a.c. field was observed by Pepper et al [4,5]. The disorder influence was studied in samples with high and low mobility [6,7].

There is no complete theoretical understanding of this exciting phenomenon. The most promising explanation was suggested by Laughlin [8,9], who discovered a set of new macroscopic quantum states of the 2-D Fermi liquid. In a very intriguing way he related the fractional conductivity to the fractional charge transport suggested by Su and Schrieffer [10] (see also rfs [11,12]). Apart from the exact equality of the Hall conductivity to fractions of the fundamental constant with high precision, much less theoretical work has been devoted to the other experimental data already mentioned.

The present communication describes the framework of a theory, which accounts for a large part of the observations and predicts new phenomena, such as multiparticle tunneling and Josephson-type interference effects.

2. DEGENERACY THEOREM. SYMMETRY BREAKING INTERACTION

The theory is based on a degeneracy theorem, which is briefly described here. The sample of area $L_o \times L$ is rolled onto a cylinder of radius $R = L_o/2\pi$ and is periodically continuated in z-direction. The radial magnetic field B is realized by a magnetic monopole wire producing a vector potential $A_z = q.\varphi$ and the monopole charge per unit length is $q = B.R$. The result for a flat surface is obtained in the limit $R \to \infty$. The following "complete" orthogonal set spans the interior of the lowest Landau level n=0 (complete is the set of all $n \geq 0$, is complete; magnetic units $r_H = \hbar\omega_c = 1$ are assumed and the periodical boundary condition with respect to φ is applied):

$$|p\rangle = const \cdot e^{ipz - \frac{R^2}{2}(\varphi - \varphi_p)^2}, \quad p = \frac{2\pi m}{L}, \quad m = 1, 2, 3, \dots LR, \quad \varphi_p = p/R \qquad (1)$$

The relevant quantum number here is φ_p covering the circle only once. For the case when the total number of states $N_B = L.R$ is of the type $N_B = 3NR$ (or $N_B = (2k+1)NR$ in general) the set is specified by the symbol

$$|p\rangle = |m, \ell, \alpha\rangle, \quad \varphi_p = \frac{2\pi m}{NR} + \frac{2\pi \ell}{R} + \frac{2\pi}{3}(\alpha - 1), \quad m = 1, 2, \dots, N; \ \ell = 1, 2, \dots R; \ \alpha = 1, 2, 3 \qquad (2)$$

The degeneracy theorem now states that for arbitrary potential U the intersector matrix elements $U_{\alpha,\alpha'}$ vanish for $\alpha \neq \alpha'$ and $R \to \infty$. Conversely this theorem states that, whatever the potential, the huge subsets $\alpha = 1, 2, 3$ remain degenerate. By direct calculation it is seen that

$$U_{\alpha,\alpha'} \simeq e^{-\frac{R^2}{2}(\varphi_{p} - \varphi_{p'})^2} \sim e^{-\frac{R^2}{2}\left(\frac{2\pi}{3}\right)^2} \to 0 \qquad (3)$$

This is a property of the von Neuman lattice in p,q space of unit cell = $\hbar$.

In a periodic potential the 2-D character of the Landau subbands is easily restored by applying a unitary transformation to H. The diagonal H has eigenvalues $E(\vec{p})$, where $p_x = 2\pi\, m/N$, $p_y = 2\pi\, l/R$, $m = 1, 2, \dots N$, $l = 1, 2, \dots R$ and the 3-fold degeneracy remains intact. Close to the Fermi level the difference $\xi_{\vec{p}} = E(\vec{p}) - E_F$ can be written as usual $\xi_{\vec{p}} = v_F(p - p_F)$. Thus the single particle potentials fail to split the magnetic degeneracy of the Landau subband and next the electron-electron interaction is to be examined in respect to intersector elements. It appears that the intersector elements are of order e^2/R and vanish in the limit $R \to \infty$. The term really breaking the symmetry is the three electrons bound state wave function of the type $\langle a^+_{\vec{p}_1\alpha_1} a^+_{\vec{p}_2\alpha_2} a^+_{\vec{p}_3\alpha_3} \rangle$, where $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$, $\alpha_i \neq \alpha_j$. This term appears in the second link of the chain of equations for the Green functions and due to the degeneracy is not small. Instead of using second order splitting of the equations for the Green functions it is simpler to consider a model 3-particle interaction of the type

$$H_{int} = \frac{g}{3!} \int d\vec{r}\ \psi^+_\alpha(\vec{r})\, \psi^+_\beta(\vec{r})\, \psi^+_\gamma(\vec{r})\, \psi_\gamma(\vec{r})\, \psi_\beta(\vec{r})\, \psi_\alpha(\vec{r}) \quad ; g \sim \left(\frac{e}{\sqrt{\varepsilon_o}}\right)^3 \qquad (4)$$

The operators $\psi_\alpha(\vec{r}), \psi^+_\alpha(\vec{r})$ here are fermions and in general the interaction strength is $g \sim (e/\sqrt{\varepsilon_o})^{2k+1}$, the interaction being (2k+1) particles type. In the present model the theory is restricted to the rigid three electron wave function, but soft wave functions can easily be constructed. The model leaves out the internal excitations which do not contribute to the a.c. charge transport and are of higher energy.

3. EXCITATION SPECTRUM. CRITICAL BEHAVIOUR.

Constructing the Gorkov type equations for the single particle Green function $G_{\alpha\beta}(x, x')$; $x = (r, t)$ and the anomalous three-particles wave function* $\Upsilon^+_{\alpha\beta\gamma}(x, x')$ defined as

$$G_{\alpha\beta}(x, x') = -i \langle T\, \psi_\alpha(x)\, \psi^+_\beta(x') \rangle \quad ; \Upsilon_{\alpha\beta\gamma}(x, x') = \langle T\, \psi^+_\beta(x)\, \psi^+_\gamma(x)\, \psi^+_\alpha(x') \rangle, \qquad (5)$$

the following excitation spectrum is found:

$$\omega^\pm(p) = \frac{1}{2}\left[-\xi_p \pm \sqrt{9\xi_p^2 + 4\Delta^2}\right] \quad ; \quad \Delta^2 = \frac{2}{3}|g|^2 |\Upsilon_o|^2\, n_o \ , \qquad (6)$$

where n_o is the electronic concentration. This spectrum has a gap illustrated in Fig.1a. The gap in the single particle excitations spectrum is of multiparticle origin – in this case it arises from 3-electron correlations. It is maximal, when the filling number is $\nu = 1/3$.

The density of states has remarkable 1-D singularities as demonstrated in Fig.1b. The experimentally observed $\sigma_{xx}(\nu)$ practically reproduce the density of states and especially the gaps in the spectrum. In this respect the observed form of $\sigma_{xx}(\nu)$ [1-7] is in good agreement with this result.

The solution of the gap equation for negative g is

$$\Delta_0 = 6\omega_0 \exp[-1/(\tfrac{2}{3} n_0 |g| \rho_F)] \quad , \tag{7}$$

where ω_0 is the cut-off energy. Both ω_0 and g are parameters in the model Hamiltonian (4). This solution is valid for $\nu=1/3$. The ground state is a coherent wave function $|\Psi_0\rangle$

$$|\Psi_0\rangle = \prod_p{}' \left(u_p + v_p \cdot \frac{\varepsilon_{\alpha\beta\gamma}}{6} a^+_{p_1\alpha} a^+_{p_2\beta} a^+_{p_3\gamma} \right) |0\rangle \quad , \tag{8}$$

where

$$2(u_p^2, v_p^2) = 1 \pm \xi_p / \sqrt{9\xi_p^2 + 4\Delta^2}$$

and $|0\rangle$ is the Fermi liquid ground state. The product is taken over the the 2-D momenta $\vec{p}_1+\vec{p}_2+\vec{p}_3=0$ of electrons of equal energies $\varepsilon_{p_1}=\varepsilon_{p_2}=\varepsilon_{p_3}$. The number of bound electrons is not fixed, leading to the coherent nature of the state $|\Psi_0\rangle$. The gap vanishes at the critical temperature T_c

$$T_c = \Delta_0 \frac{\gamma\sqrt{2}}{6\pi} \quad , \quad \ln\gamma = C = 0.577 \quad . \tag{9}$$

The temperature dependence of the gap in the vicinity of T_c is of the type

$$\Delta = 3\pi T_c \sqrt{\frac{3}{7\zeta(3)}\left(1-\frac{T}{T_c}\right)} \quad . \tag{10}$$

Here $\zeta(x)$ is the zeta function. This prediction is in qualitative agreement with the observations [2] reproduced in Fig.2a. The plateau's of σ_{xx} at various temperatures represent $\Delta_T(\nu)$ for fixed T. For very low temperatures the gap temperature dependence is exponential,

$$\Delta = \Delta_0 - \frac{2}{3}\sqrt{\frac{3\pi T\Delta}{\sqrt{2}}}\cdot\left(1-\frac{3T}{16\sqrt{2}\Delta}\right)\exp\left[-2\frac{\sqrt{2}\Delta_0}{3T}\right] \tag{11}$$

The gap in the single particle excitation spectrum determines a threshold in the microwave absorption coefficient. The calculation of thermodynamic potential, specific heat and absorption coefficient will be given elsewhere.

In order to discuss the critical filling factor ν^* determined by the disappearence of the gap $\Delta(\nu^*)=0$, the density of states at the Fermi level is approximated by a constant $\rho_F = N_B/E_0$, where E_0 is of the order of the Landau level band width. The exponent in the gap equation is different

in the case of V_g or B control. For the B=const case one has

$$\nu^* = \nu_g \ln(\omega_0 \gamma\sqrt{2}/\pi T) \quad ; \quad \nu_g = \frac{E_0}{|g|}\left(\tfrac{3}{2}N_B\right)^2 . \tag{12}$$

Close to the critical filling ν^* the gap depends on ν according to the mean field theory

$$\Delta = 3\pi T\sqrt{\frac{3}{7\zeta(3)}\frac{|\nu-\nu^*|}{\nu_g}} \quad . \tag{13}$$

For the case of magnetic field control (V_g= const.) the results are

$$\nu^* = \nu_B / \ln\left(\frac{\omega_0\gamma\sqrt{2}}{\pi T}\right) \quad ; \quad \nu_B = \left(\tfrac{2}{3}n_0\right)^2 \frac{|g|S}{E_0} \; ; \; \Delta = \frac{3\pi T}{\nu^*}\sqrt{\frac{3\nu_B|\nu^*-\nu|}{7\zeta(3)}} \quad . \tag{14}$$

Here S is the total area of the 2-D liquid. The observed [2] $\Delta(\nu)$-dependence is reproduced in Fig.2b and is seen to be in reasonable agreement with this result. The existence of a maximum situated at $\nu = 2/3$ appears in the theory for the exact symmetry already discussed.

4. GINZBURG-LANDAU EQUATION. FRACTIONAL FLUX QUANTIZATION

Next the Ginzburg-Landau equation is derived for the order parameter $\Psi(\vec{r})$. It has the form

$$\left\{ \frac{1}{2m^*}\left(-i\hbar\nabla + \frac{e^*}{c}\vec{A}\right)^2 - \alpha_0\left[1-\frac{T}{T_c} - |\Psi|^2\right]\right\}\Psi(\vec{r}) = 0 . \tag{15}$$

The effective mass here is 3m and the effective charge is 3e. The calculated coefficient α_0 is related to the correlation length ξ by

$$\xi^2 = \xi_0^2/(1-T/T_c) \quad ; \quad \xi_0^2 = \frac{\hbar^2}{2m^*\alpha_0} = \frac{2\cdot 27}{7}\frac{(\pi T_c)^2}{\zeta(3)|g| n_0 \rho_F v_F^2} \quad . \tag{16}$$

The current density has the familiar form

$$\vec{j} = \frac{e^*}{2m^*}\left\{\Psi^*(\vec{r})\left(-i\hbar\nabla + \frac{e^*}{c}\vec{A}\right)\Psi + c.c.\right\} = e^*|\Psi|^2 \vec{V}_c \quad , \tag{17}$$

where the velocity $\vec{V}_c$ is related to the phase variation according to

$$\vec{V}_c = \frac{\hbar}{m^*}\left[\nabla\phi - \frac{e^*}{\hbar c}\vec{A}\right] \quad . \tag{18}$$

Integrating this equation along a closed contour C one reaches exact fractional flux quantization:

$$\Phi = n\,\Phi^* \quad ; \quad \Phi^* = \frac{hc}{3e} \quad \left(\frac{hc}{5e}, \text{etc}\right) \quad . \tag{19}$$

It should be emphasized that the appearence of plateau's in the Hall conductivity is a result of phase transitions into new macroscopic coherent quantum states (Laughlin states). In these states the fermionic liquid has gaps in the single particle spectrum. At T=0 the dissipative conductivity σ_{xx} vanishes in the region of the gap and the phase transition is of metal-insulator type.

5. FIRST PRINCIPLES FQHE DERIVATION

The Hall current (17) can be calculated from first principles. In an infinite system in the absence of external electric field the wave function can be chosen real and the current is zero. In the presence of external electric field at T=0 the gap in the spectrum ensures $\sigma_{xx}=0$ and the only existing current is the non-dissipative Hall current. Since the electric and magnetic field are perpendicular, a unique reference frame exists, where the electric field is zero. In this frame the Hall current also vanishes. Hence the velocity $\vec{V}_c=0$ and in this way the phase of the rigid wave function is fixed in the moving frame. Returning to the reference frame with actual electric field, a Galilean transformation of the wave function gives

$$\Psi_{\mathcal{E}} = \Psi_{\mathcal{E}=0} \cdot e^{i\frac{m^*\vec{V}}{\hbar}\cdot(\vec{r}+\vec{V}t)} \quad , \quad |\vec{V}| = c\frac{\mathcal{E}}{B}$$

The current has the form

$$\vec{j} = e^* |\Psi_{\mathcal{E}=0}|^2 \cdot \frac{c\,\mathcal{E}}{B} = \frac{e^2}{h} \nu_c\, \mathcal{E} \qquad (20)$$

This result is highly unexpected. For T=0 because of the energy gap the concentration of the coherent electrons coincides with the total density of electrons n_o. For $T\neq 0$ it is different from n_o, being temperature dependent according to (compare with (11))

$$n_c = n_o - \tilde{n}(T) = n_o \left[1 - \frac{4}{3}\sqrt{\frac{3\pi T}{\Delta_o\sqrt{2}}}\left(1 - \frac{3T}{16\sqrt{2}\Delta_o}\right) e^{-\frac{2\sqrt{2}\Delta_o}{3T}}\right] \qquad (21)$$

where a fraction of the electrons $\tilde{n}(T)$ are normal, activated in states with higher energy than the gap E_g. Therefore (20) contains the observed [2] thermally activated behaviour as well. The data of Chang, Paalanen, Tsui, Störmer and Hwang [2] demonstrate the <u>same</u> activation energy for σ_{xx} and σ_{xy}. Note that the activated hopping Hall effect would have different activation energies for σ_{xx} and σ_{xy} [13]. The result (20) relates the plateaus only to the coherent electrons. It gives an remarkable explanation of the observed deviations from the exact quantization. The two fluids model with activated normal electrons contributing to the dissipative transport can also account for the effect of an

a.c. field applied to the 2-D inversion layer [4,5].

6. MULTIELECTRON TUNNELING

Here we give only the prediction of simultaneous tunneling of 3,5,7 etc. electrons through barriers and Josephson interference effects. Following Josephson [14], the Gor'kov integral equations are solved assuming constant values of the order parameter on both sides of the barrier. The current through the barrier has the form

$$J = J_1 \sin\phi \quad , \qquad (22)$$

where ϕ is the phase difference of the two fluids. The time dependence of ϕ is determined by the Gor'kov-Josephson relation

$$\hbar\frac{\partial\phi}{\partial t} = (2k+1)\Delta\mu \quad , \ k=1,2,3,\ldots \qquad (23)$$

Here $\Delta\mu$ is the chemical potential difference on both sides of the barrier. In the presence of an d.c. and a.c. potential $V=V_o+V_1\cos(\omega_d t+\varphi_o)$ the phase depends on t according to

$$\phi(t) = \frac{(2k+1)eV_o t}{\hbar} + \frac{(2k+1)eV_1}{\hbar\omega_d}\sin(\omega_d t+\varphi_o) \qquad (24)$$

giving rise to a.c. and d.c. Josephson effects.

7. CONCLUSIONS

We here emphasize the main results of the present work. A theorem was proved stating that any potential field leaves the 3-fold degeneracy intact. A symmetry breaking 3-particle interaction was considered and the order parameter was found to be the three particle rigid anomalous propagator. A mean field critical behaviour was described by (10),(13),(14),(16). The data of Chang et al. [2] give a strong indication of such a behaviour. The Ginzburg-Landau equation coefficients were calculated, the effective charge being 3e. Fractional flux quantization was found in accordance with this value. A first-principles FQHE derivation was given accounting for the activated temperature dependence. The 3,5,7,etc. electron tunneling and Josephson interference effects were predicted. The fluctuations and the disorder can be taken into account in the framework of the Wigner [15,16] Q-matrix approach. The discussion of the filling fractions with even denominators will be given elsewhere.

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*New symbol Ⰰ is introduced for the new quantum state.It is the first letter of the Glagolic alphabet of Cyril (863) pronounced like "az".

FIGURE CAPTIONS

Fig.1 a) Two branches of the excitation spectrum $\omega^{\pm}$ as vs energy ξ measured from the Fermi level. b)Density of states without impurity scattering

Fig.2 a) The data of Chang et al.[2] for ρ_{xx} and ρ_{xy} for B=104.5 kG and Vg sweep. The flat region in ρ_{xx} and ρ_{xy} around $\nu=2/3$ decrease with increasing temperature.

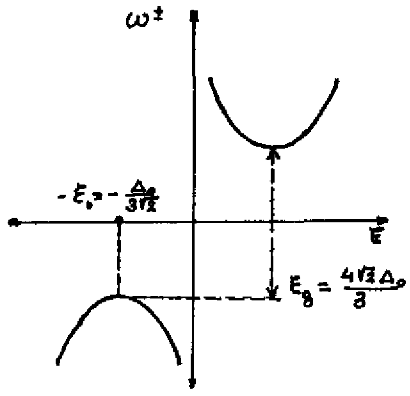

Fig. 1 a

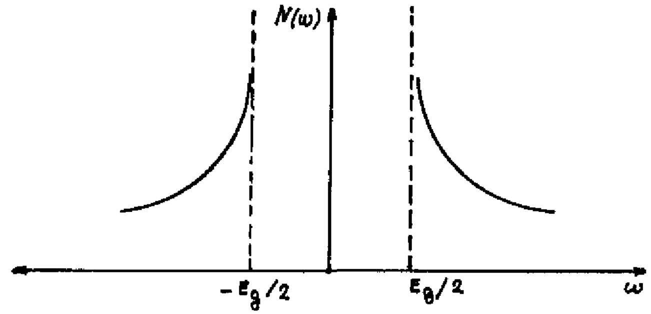

Fig. 1 b

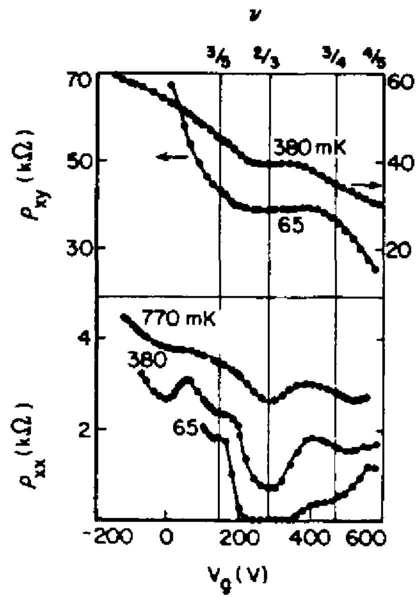

Fig. 2 a

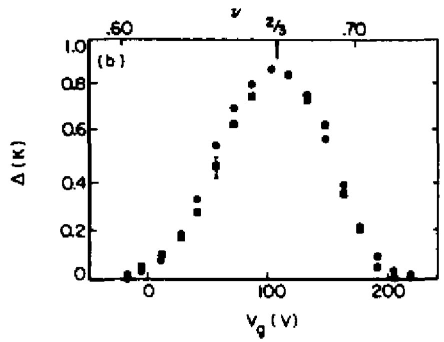

Fig. 2 b