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CONF-840932--1

UCRL-91512
PREPRINT

EVOLUTION OF THE MIRROR APPROACH TO FUSION: SOME CONJECTURES

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This paper was prepared for submittal to
ICTP Trieste Commemorative Meeting
on the Next Twenty Years in Plasma Physics
Trieste, Italy
September 24-28, 1984

September 18, 1984

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EVOLUTION OF THE MIRROR APPROACH TO FUSION: SOME CONJECTURES

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ABSTRACT

Some possible directions for the future evolution of the mirror approach to fusion are outlined, in the context of economically-motivated criteria. Speculations are given as to the potential advantages, economic and otherwise, of the use of axially-symmetric systems, operated in "semi-collisional" regimes of lower Q (fusion power balance ratio) than that projected for present-day tandem mirror designs. These regimes include "barely tandem" modes, and ion-heated modes, in association with higher efficiency direct conversion. Another possible economically advantageous approach mentioned is the use of a tandem mirror plasma to stabilize a FRM (field-reversed mirror) plasma, with potential synergistic advantages.

I. INTRODUCTION

In this article we will present speculations as to some possible future directions for mirror research, viewed in the context of its past history and present status.

Over the now more than three decades that the magnetic mirror approach to fusion has been under investigation, the mirror concept has undergone an evolutionary process involving many modifications and changes. These changes were prompted by perceived deficiencies in the original mirror concept. This concept was that of a hot plasma trapped in an axially-symmetric solenoid

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DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED between two axially-symmetric mirror coils. The first major modification was prompted by the theoretical insight that the simple mirror, owing to the net unfavorable curvature of its field lines, was subject to gross MHD-type instabilities. Although early mirror experiments (with hot electrons) [1] had been able to avoid these instabilities (for reasons that were not then understood) the classic experiments of Ioffe and his co-workers [2] both demonstrated the presence of the instabilities and a means for their control: The multipole magnetic well, with its favorable field curvature, strongly suppressed all MHD instabilities--but at the price of abandoning the simplicity of axially-symmetric fields.

Further evolution of mirror ideas came with the recognition that the inherently non-Maxwellian character of the ion distribution functions in mirror-confined plasmas made these plasmas subject to a variety of high frequency instabilities. Building on earlier work of Harris [3], Rosenbluth [4], Mikhailovskii and Timofeev [5] and others, a theory of these "loss cone" instabilities was developed [6], and, following this development, the idea of "warm plasma stabilization" of loss cone modes was suggested [7] as a possible cure for certain ones of these modes. This technique was first investigated in the Soviet PR-6 and PR-7 mirror experiments [8]. When it was applied to the Livermore 2XII-B experiment [9], the dominant high frequency mode was suppressed and 2XII-B went on to demonstrate near-classical confinement of a high temperature (T_i 10 keV), high beta plasma [10]. This demonstration represented another major milestone in mirror research. Its achievement sparked the next major thrust in mirror research--the reduction of end losses.

From the earliest days of mirror research, a major concern had been the question of the predicted low value of the fusion power balance factor, "Q" (ratio of fusion power output to plasma loss power). This low Q stemmed from the high rates of collision-induced particle losses through the mirrors.

Early Fokker-Planck end loss calculations, based on a simplified plasma model, had indicated Q values of order 5 [11], low but possibly tolerable. Later calculations [12], based on more realistic plasma models, namely ones including the cooling effect of the electron drag and the effect of the positive ambipolar potential in degrading ion confinement, indicated even lower values of Q.

An early response to the problem of low Q in the conventional mirror was due to Kelley [13], who suggested the use of a central mirror cell with small-volume "guard" mirror cells at each end. The function of these guard cells was to flatten the ambipolar potential in the central cell, thus eliminating the parallel electric fields in that cell that lead to enhanced ion loss ion rates. Years later Kelley's idea was used as the starting point for the novel "tandem mirror" idea of Dimov [14] and, independently, of Fowler and Logan [15].

Earlier, in the late nineteen sixties, another approach to dealing with the problem of low Q, direct conversion, had been examined by Post [16]. The idea was to reduce the impact of energy loss via particle losses through the mirrors by efficient recovery of this energy, in electrical form, using electrostatic deceleration of the escaping ions (which carry the lion's share of the escaping energy). Follow-up experiments [17] showed that it is possible to achieve very high conversion efficiencies (86 percent in the experiments) by the electrostatic direct conversion technique.

Though viewed as a helpful adjunct, being incorporated in present mirror fusion system designs, direct conversion was not and still is not considered a sufficient answer to the problem of low Q in mirrors. The answers that have by now emerged directly attack the basic problem: mirror end loss.*

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*For a review of many of the concepts proposed for reducing mirror losses, see an article by Gormezano [22].

The front-runner and most actively pursued approach to reducing mirror losses, mentioned earlier, is the tandem mirror idea, with its evolutionary development, the thermal barrier [18], and other improvements. By now the basic concept behind the tandem mirror, electrostatic plugging of ion losses, has been successfully demonstrated in several experiments [19] and work is now in progress to prove out the newer idea of the thermal barrier and other associated improvements.

Another idea for plugging mirror end losses, one also with earlier historical roots (the ASTRON of Christofilos [20]), is the so-called Field-Reversed Mirror [21]. In the FRM, the ion diamagnetic currents are to be made so large as to internally reverse the direction of the magnetic field lines, closing the loss cones for internally trapped particles. The advantages of the FRM would be twofold: Not only would the reversed-field configuration close the loss cones and reduce leakage (assuming adequately low fluctuation levels), but also the self-augmentation of confinement associated with the diamagnetic currents would a priori lead to a very high plasma beta. In fact, the "magnetic efficiency" (confined pressure vs applied magnetic field intensity) of the FRM is probably the highest of any known magnetic configuration, potentially leading to very compact fusion power systems.

Appealing through the FRM idea is, its implementation obviously places severe requirements on plasma stability, raising basic plasma physics questions that are not yet resolved. Recent experiments, in the Soviet Union [23], the US [24] and in Japan [25], with field-reversed configurations similar to those envisaged for the FRM (i.e., ones with purely poloidal field components) have been encouraging, but not yet conclusive. These experiments have been carried out in so-called "reversed-field theta pinch" devices, using high plasma densities (of order 10^{15} cm^{-3}) and, thus far, fairly short confinement times (of order 100 microseconds).

Returning to the subject of the tandem mirror concept and its evolutionary development, it is important to note that its recent development has been stimulated mainly by what are essentially economic considerations. Paper design studies of the original tandem mirror idea showed that the plug cells were technologically demanding and would probably be costly. In the simple tandem mirror, the plugging potentials are obtained by driving up the plasma density and ion mean energy in the plugs well above their value in the central cell. This scheme thus relies solely on the Boltzmann-like response of the plasma electrons to produce the confining potentials, which are here given by the expression

$$e\Delta\phi = e(\phi_p - \phi_c) = kT_e \ln(n_p/n_c) \ , \qquad (1)$$

where ϕ_p, n_p and ϕ_c, n_c are the plug and central cell potential and electron density, respectively, and T_e is the electron temperature, the same in both regions.

The introduction of the thermal barrier idea, a negative-going depression between central cell and plugs, allows the electron temperature in the plugs to be made higher than that of the central cell. It thus permits high plugging potentials to be achieved without the requirement for high plasma density in the plugs or needing excessive power to maintain the enhanced electron temperature in the plugs.

Paper studies of tandem mirror fusion systems employing the thermal barrier concept indicate substantial cost reductions relative to the simple tandem mirror, but at the price of increased complexity of the plugs. There seems little doubt, however, that this latest version of the tandem mirror idea provides a feasible avenue to the control of end losses in mirror systems and thus presumably to the development of successful fusion power systems based on the mirror concept.

The historical sequence we have just described is shown on Figs. 1 and 2. Figure 1 depicts the sequence: simple mirrors, magnetic wells, building to the simple tandem mirror (TM), and the field-reversed mirror (FRM). Figure 2 shows the evolution of TM to the TM with thermal barriers.

II. THE QUESTION

The question that we will explore in this paper is the following one: Are there new directions for mirror research, ones building on its past and present accomplishments, that might offer simpler, less expensive, or smaller mirror fusion systems than those now projected following the scenario of the tandem mirror with thermal barriers?

In what follows some conjectures are made as to future initiatives which mirror researchers might undertake in order to seek still further improvements in either, size, cost or technical simplicity relative to the present mainline approach. The treatments given will be limited to simple analyses and examples; no attempt at rigor will be made. The purpose is to stimulate further exploration of the potentialities of the mirror concept; it is not to present conclusive arguments or results. No doubt some (or perhaps all?) of the ideas discussed will not prove technically viable in the form suggested. They may, however, serve the purpose of suggesting approaches that will be found to be fruitful.

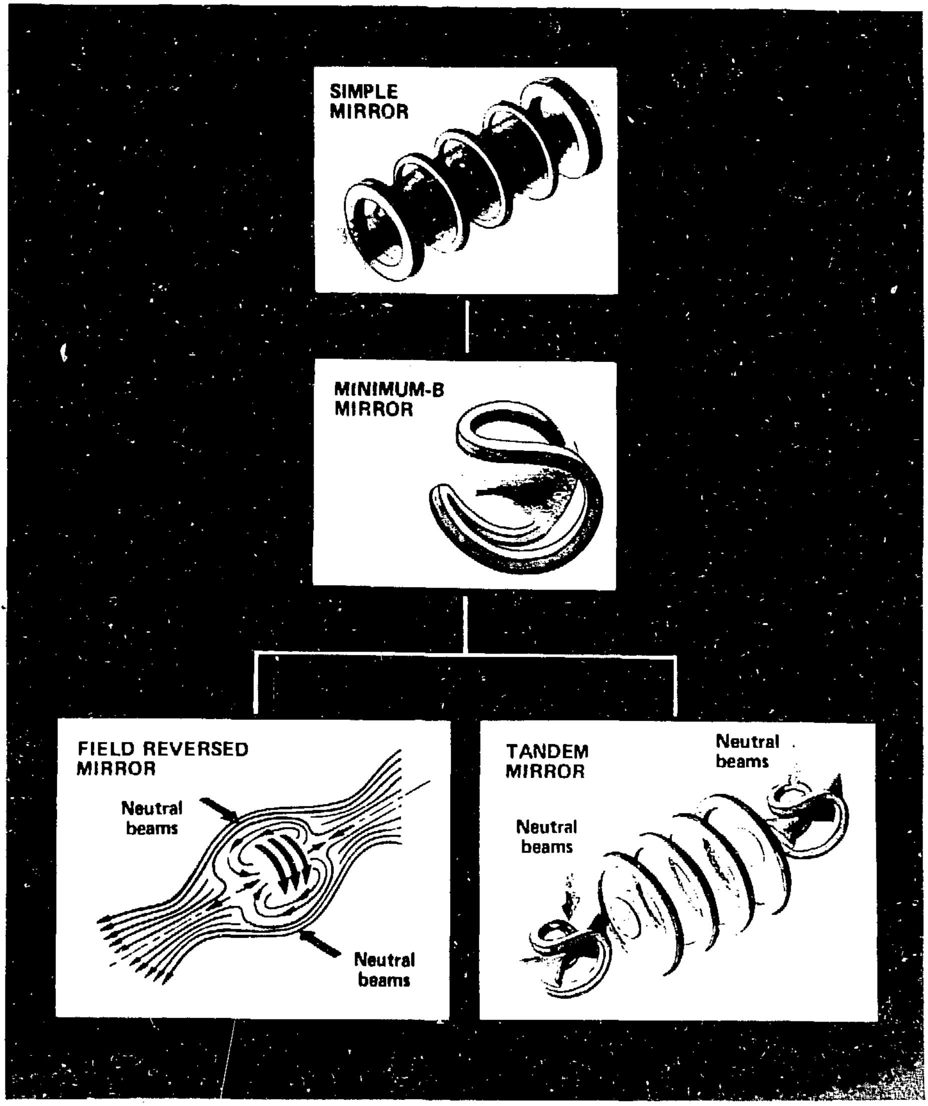

Figure 1. Evolution of the mirror concept from the simple mirror to the tandem mirror and the field reversed mirror.

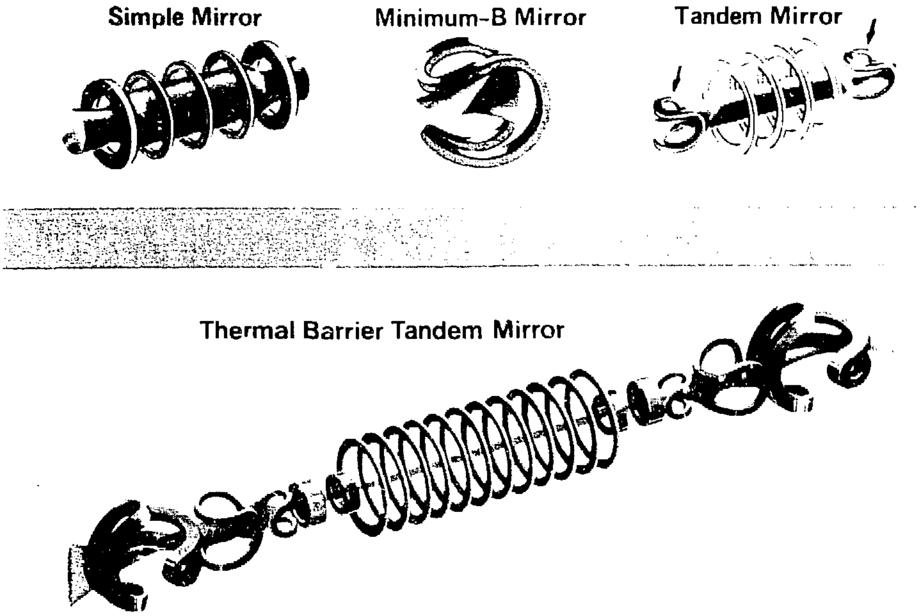

Figure 2. Evolution of the tandem mirror from the simple mirror to the TM with thermal barriers.

III. COMMENTS ON ENERGY RESEARCH AND DEVELOPMENT STRATEGIES

In a certain sense the distinction between "low Q" vs "high Q" systems that has been made here permeates the development of all energy systems, not merely fusion systems. Correspondingly, the strategy for the development and the evolution of such systems depends on in which of the two regimes they exist.

An example of a high Q system is a fission reactor. Here, when the chain reaction is initiated, "Q" becomes infinite. Once this objective is achieved, net energy production is assured, and the development consists of finding economical ways of converting the energy to useful form. Issues of efficiency of conversion of the energy are secondary to those of control and safety.

An example of a low Q system is a gas turbine. Here the issues are primarily those of efficiency and simplicity. In order to compete economically (indeed in order to generate any net power at all!) the main elements--turbine and compressor--must both achieve high efficiency. At the same time the substantial recirculation of energy attendant to a low intrinsic Q must be efficiently and cheaply accomplished--as it obviously is in the gas turbine by the simple shaft that couples turbine and compressor.

We can discuss the same two strategies in the case of the development of mirror fusion systems. The tandem mirror with thermal barriers is a high-Q device by virtue of its efficient end plugging. Its development must then proceed by finding economical ways of accomplishing the complexities of end plugging. Here high efficiency is not the central problem. By contrast, the earlier design approaches to mirror fusion systems sought to maximize the efficiencies of all the components--heating, energy conversion, etc., so as to compensate for the intrinsically low Q of the simple mirror.

In most of our discussions to follow, we will follow a course that is nearer to the latter strategy than it is to the former. Specifically, we will be mainly examining regimes that lie intermediate between the very low Q of the simple mirror and the high Q of present-day tandem mirror designs.

Clearly, the two approaches, i.e., "high-Q" and "low-Q", have complementary advantages and disadvantages. Taking the High-Q route largely relieves concerns over efficiencies and minor degradation of confinement but may introduce complexity accompanied by issues of capital cost. Settling for lower Q may result in cost-saving simplifications of the fusion confinement components, while at the same time requiring greater care with respect to the efficiency and cost of auxiliary systems. Only time will tell which approach, or which compromise between the extremes, will be the most efficacious one.

IV. SOURCE AREAS FOR INNOVATION

A starting point for coming up with suggestions for future directions for mirror research is to delineate general areas from which innovative approaches might come. Some of these areas are:

A1) Re-examination of older ideas in the light of new understanding or new approaches.

A2) Consideration of the insights which might be gained from an "information theory" approach to the problem.

A3) Combination of existing approaches to produce a new form, one with synergistic advantages.

A4) Investigation of "tradeoffs" having economic advantages, i.e., substitution of "more" of conventional, lower-cost, elements in return for needing "less" of unconventional higher-cost ones, with a net gain in economics and/or simplicity.

A5) Exploitation of new or specially-developed technology to simplify or to lower the cost of the overall system.

The meaning and content of these several categories will be made more clear by citing examples in subsequent sections.

In addition to the above rather general categories to examine in the search for innovative improvements, there are features inherent to the mirror concept that substantially enhance the possibilities for innovative change relative to, for example, "closed" or toroidal systems such as the tokamak. Among these inherent characteristics, associated in many cases with the open topology of the field lines of the applied fields in mirror systems, are the following:

M1) Open field line topology has some unique characteristics: for example it permits the use of absolute magnetic wells, including ones of axial symmetry (Andrioletti-Furth configurations" [26]).

M2) The axially-directed forces on particles associated with the magnetic mirror effect permit the creation of gradients in plasma density and temperature parallel to field lines, together with associated ambipolar electric fields (for example as employed in the tandem mirror).

M3) Plasma equilibria in mirror systems do not require the presence of currents flowing in the plasma parallel to field lines. This fact has favorable consequences not only for minimizing anomalous cross-field transport but also for permitting the existence of stable equilibria at high beta values (approaching unity).

M4) Particle losses can be controlled and selectively directed through the mirrors (natural divertor action), facilitating the use of direct conversion techniques and relieving the problem of handling plasma-wall interactions.

M5) The linear geometry of mirror systems allows the controlled translation of plasmas from one region to another, with accompanying new possibilities.

The five general areas listed (labeled A1-A5) earlier and the five mirror-specific attributes (labeled M1-M5) just listed provide a framework within which to initiate the conceptual phase of the innovative process in a search for improved mirror systems. In the sections to follow, we will illustrate the use of these lists by outlining some specific examples, together with "back-of-the-envelope" analyses and calculations, where appropriate.

V) DEJA VU: OLD MIRROR IDEAS REVISITED

More than once in fusion research has an old idea or old approach, once rejected, been re-examined in the light of new understanding or innovative new twists. A classic example in the reversed field pinch, dating back at least as far as the British ZETA experiment of the nineteen fifties. In the mirror context we will here re-examine the original axially-symmetric "mirror machine", with an eye to exploiting its simplicity.

Our starting model is that of a long solenoid capped at each end with a single mirror coil (or a short mirror cell) of circular cross-section, such as is shown in Fig. 1. The virtue of this configuration is that it is probably one of the the simplest versions of a mirror system that can be visualized, having a low cost, easy-to-fabricate, magnet system. It, however, is also a configuration with two obvious and well-known deficiencies: poor MHD stability properties and low Q (as mentioned previously). To exploit any potential benefits, both of these limitations must be addressed, in the context of the chosen configuration, without unduly compromising either the simplicity or the economic attractiveness of a fusion power system based on this geometry. In what follows we will sketch some possible ways of addressing these issues, utilizing the guidelines A1 to A5 and the mirror-specific qualities M1 to M5 where appropriate.

A. MHD Stability of Axisymmetric Mirrors

Simple mirror fields of the axisymmetric variety have long been known to be subject in theory to MHD interchange instabilities, as illustrated by their failure to satisfy the usual MHD stability criterion [27]

$$\oint \frac{p_\perp + p_\parallel}{R_c r B^2}\, dl > 0 \text{ , stable .} \tag{2}$$

For typical pressure distributions, the negative field curvature ($R_c < 0$), in the low field region between the mirrors always dominates over the positive curvature in the high field region near the mirrors.

Not understood in earlier days was the experimental observation that mirror plasmas, in particular ones with a hot electron population, were MHD stable [28]. Today we believe we understand these results as arising from various circumstances not taken into account in the simple MHD picture used in deriving the criterion. Some of these circumstances are:

a) In a mirror with a long central cell with short mirrors the growth rate, γ_{MHD}, predicted by MHD theory is slow, γ_{MHD} scaling roughly as L_c^{-1}, where L_c is the length of the central cell.

b) Finite orbit effects, ignored in MHD theory, can stabilize all higher order ($m > 1$) mirror modes [28].

c) When γ_{MHD} is lower than the drift frequency of the hot electrons, as can occur, the electrons can stabilize the plasma when their beta is sufficient to reverse the negative radial field gradient in the bad curvature regions [29].

d) Communication with good curvature regions (or conducting surfaces) outside the mirrors can stabilize MHD modes [30].

Looking toward the re-examination of axisymmetric mirror systems, items a-d give hints as to means for insuring interchange stability in such systems. As a result of (a) and (b) it can be insured that only a relatively slowly growing m = 1 mode (a "rigid body" transverse displacement) need be stabilized. Among ways to exploit this possibility are: stabilization by the use of rf pondermotive forces, recently experimentally demonstrated [31]; employment of feedback systems of various types [32]; or, as mentioned below, stabilization by hot electron or superthermal ion populations.

The stabilizing effect of hot electrons, observed in early mirror experiments, is exploited in the electron bumpy torus [33]. Recently, Berk and coworkers [34], have shown that energetic ion populations, together with nearby conducting surfaces, can exert strong stabilizing effects.

The general question of stabilizing axisymmetric mirror systems is currently under intense re-examination. For example, Arsenin [35] has examined several techniques theoretically, and has advanced some novel approaches to the problem, including the use of new field configurations and other techniques.

On the basis of the above-cited considerations it seems reasonable to conclude that mirror systems with axisymmetric fields can be employed. Theory, in particular that of Berk et al. previously cited, also indicates that stability in axisymmetric mirror fields can be maintained even at high beta values. We will therefore assume in what is to follow that solenoidal and mirror fields using only circular coils can be employed in a search for improved mirror systems.

B. Strategies for Dealing With the Low Q of Simple Mirrors

The entire thrust of the Kelley scheme, of high efficiency direct conversion, and now, of the tandem mirror, is of course to reduce (or to ameliorate the effect of) end losses from simple mirror cells with the associated low Q values. The TM is eminently successful in this regard, in theory being able to achieve ignition ($Q \to \infty$) in the central cell. In our search for a possibly simpler approach, and in the spirit of the guidelines, it is worthwhile to consider different approaches which might either:

- Operate at lower Q than the ignited tandem mirror in return for simplification and/or possibly lowered total system cost (guideline A4),
- Employ some elements of both the simple mirror and the tandem mirror to achieve higher Q than that of the simple mirror but with minimal increase in complexity (guideline A3),
- Introduce new or specially developed concepts and technology (guidelines A2 and A5).

We will sketch in what follows some alternative approaches to the Q problem that will illustrate the use of these guidelines.

VI. Q: HOW MUCH IS ENOUGH?

From the definition of Q as the ratio of fusion power generated to heating power required, it can be seen immediately that the question of how high Q must be for a mirror fusion power system (beyond the bare breakeven point) is essentially an economic question. That is to say, accepting lower Q values, though likely resulting in savings in the cost of the fusion core, will be offset by the requirement for either or both of the following:

- A higher recirculating power fraction (therefore less power available for distribution),
- Higher efficiency in heating and energy recovery (therefore higher cost, or at least additional complexity in the auxiliary equipment).

The role of the various competing factors can be illustrated (Fig. 3) through a simple block diagram showing the main channels of power flow in a mirror fusion power system.

From the diagram we see that one unit of heating power introduced into the plasma results in an outflow of Q+1 units of heat energy (charged reaction products, neutrons and thir capture energy, plus the original heating energy). This heat energy is converted to electricity with an average efficiency $\bar{\eta}$ (as averaged over the thermal conversion and the direct conversion efficiencies, as later discussed). To maintain the plasma $(1/\eta_H)$ units of electrical power are fed back to the heating system (using neutral beams, ICRH, etc.).

The minimum "breakeven" value of Q (zero net electrical output), call it Q_M, is given by the easily-derived expression

$$Q_M = \left[\frac{1}{\bar{\eta}\eta_H} - 1\right] . \tag{3}$$

In terms of Q and Q_M then, the fraction of generated electric power that must be recirculated to maintain heating is simply

$$\frac{P_H}{P_{EG}} = \frac{Q_M + 1}{Q + 1} , \tag{4}$$

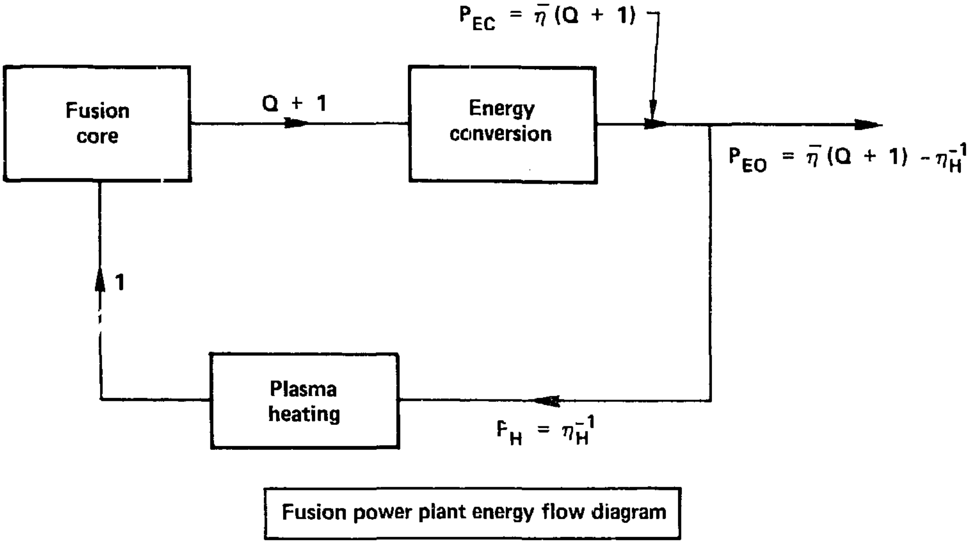

Figure 3. Simplified power-flow diagram for a mirror fusion power plant.

and the ratio of recirculated electrical power to <u>delivered</u> electrical power is

$$\frac{P_H}{P_{ED}} = \frac{Q_M + 1}{Q - Q_M} \quad . \tag{5}$$

To relate these power-related quantities to economic issues, we must make some assumptions about the cost of the various elements involved. The answers we will obtain will clearly depend on the assumptions made, and can therefore only be considered as indicative of areas of potential savings.

As an over-simplified economic model we will divide the direct capital cost per kW of the fusion power plant into four major categories, as follows:

- Buildings and structures and miscellaneous plant equipment, C_B,
- Electrical plant (turbogenerators, substations, etc), C_E,
- Plasma heating equipment, C_H,
- Fusion core, C_F.

The formula for the direct capital cost of the plant per kilowatt of generated electricity is then simply

$$\begin{aligned} \text{cap.cost/kW}_e &= [(\text{fusion core}) + (\text{elec. plant}) + (\text{plasma heating}) \\ &\quad + (\text{bldgs. \& misc.})] \div [\text{kW}_{\text{del.}}] \\ &= \left[\frac{\text{F.C.}}{\text{kW}_e}\right] C_F + \left[\frac{\text{E.P.}}{\text{kW}_e}\right] C_E + \left[\frac{\text{P.H.}}{\text{kW}_e}\right] C_H + \left[\frac{\text{B.M.}}{\text{kW}_e}\right] C_B \quad . \end{aligned} \tag{6}$$

Each term has associated with it a "cost index factor". We will assume that these factors are proportional to the "power handled". That is, the cost of the fusion core is assumed to scale with the fusion output power, that of the electrical plant with its electrical output, and that of the plasma heating system with its electrical input. With this assumption we may then derive values of the coefficients from existing paper studies, such as the

MARS (Mirror Advanced Reactor Study) [36]. Given these coefficients the scaling of the several terms with Q and Q_M may then be determined by use of the previously-derived power-flow quantities. Thus we find:

$$\left[\frac{\text{F.C.}}{\text{kW}_e}\right] = \left[(Q_M + 1)^{-1} - (Q + 1)^{-1}\right]^{-1} \tag{7}$$

$$\left[\frac{\text{E.P.}}{\text{kW}_e}\right] = (Q_M + 1)^{-1} \cdot \left[(Q_M + 1)^{-1} - (Q + 1)^{-1}\right]^{-1} \tag{8}$$

$$\left[\frac{\text{P.H.}}{\text{kW}_e}\right] = (Q + 1)^{-1} \cdot \left[(Q_M + 1)^{-1} - (Q + 1)^{-1}\right]^{-1} . \tag{9}$$

It follows that the total direct capital cost per kW_e of the fusion power plant is given by the relation

$$[\text{cap.cost/kW}_e] = [(Q_M + 1)^{-1} - (Q + 1)^{-1}]^{-1} \left\{C_F + C_E(Q_M + 1)^{-1} + C_H(Q + 1)^{-1}\right\} + C_B . \tag{10}$$

We now insert rounded-off values from the MARS study in order to determine the coefficients.

For MARS:

$Q = 26$, $\overline{\eta} = 0.45$, $\eta_H = .67$, so that $Q_M = 2.32$

$\left[\frac{\text{cost of bldgs. \& misc.}}{\text{kW}_e}\right] \approx \$400/\text{kW}_e$ plant output

$\left[\frac{\text{cost of fusion core}}{\text{kW}_e}\right] \approx \$1400/\text{kW}_e$ plant output

$\left[\frac{\text{cost of elect. plant}}{\text{kW}_e}\right] \approx \$400/\text{kW}_e$ plant output

$\left[\frac{\text{cost of heating}}{\text{kW}_e}\right] \approx \$150/\text{kW}_e$ plant output .

From these values and Q and Q_M we find:

$$C_F \approx 370;\ C_E \approx 350,\ C_H \approx 1000,\ C_B = 400\ .$$

Inserting these values into Eq. (10) we find the (reference) MARS direct capital cost as \$2350/kW_e, in approximate agreement with the figures given in the MARS report.

We may now employ Eq. (10) to investigate trade-offs between lower Q (and Q_M) values and the reduced fusion core and/or heating costs that could result from the use of simpler fusion cores or of less expensive plasma heating methods or from improved efficiencies (lower Q_M).

In the MARS design the fusion core, including the central cell and tandem plugs, is a major capital cost item, and in the fusion core itself the plugs are of order 50 percent of the cost. We therefore first explore the effect of using a simpler, lower cost, lower Q, fusion core, at first assuming the same value of Q_M (i.e., same energy recovery and heating efficiencies) and the same value of C_H as was used in MARS. Figure (4) is a plot of (cap.cost/kW_e) vs Q, with C_F as a parameter. As can be seen from the figure, a reduction of a factor of 2 in C_F would permit Q to drop from 26 to about 8 at approximately the same capital cost.

If now the simpler fusion core permitted the use of less expensive plasma heating methods (ICRH vs ECRH, for example) a still further reduction in Q would be permissible. Figure 5 shows cap.cost/kW_e vs Q with C_F = 180, and C_H as a parameter. At C_H = 500, and the same Q_M as in MARS. Q may now drop to nearly 7.0 before a capital cost penalty is paid.

Finally, if even slightly higher efficiencies are achieved in either energy recovery or heating, still lower Q values can be tolerated. For

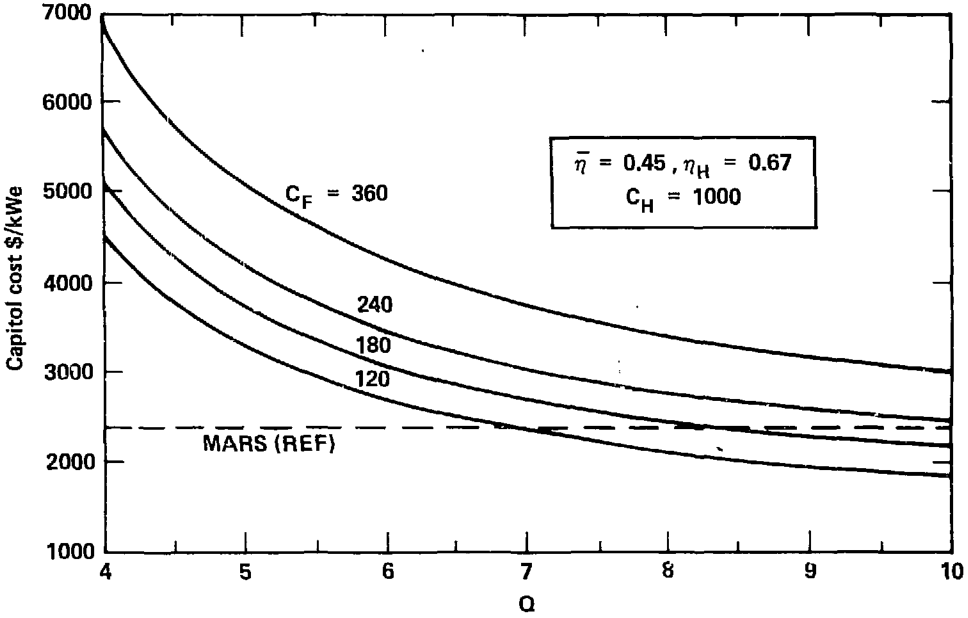

Figure 4. Scaling of mirror fusion power plant capital cost vs Q for various values of the cost parameter, C_F, with C_H = 1000.

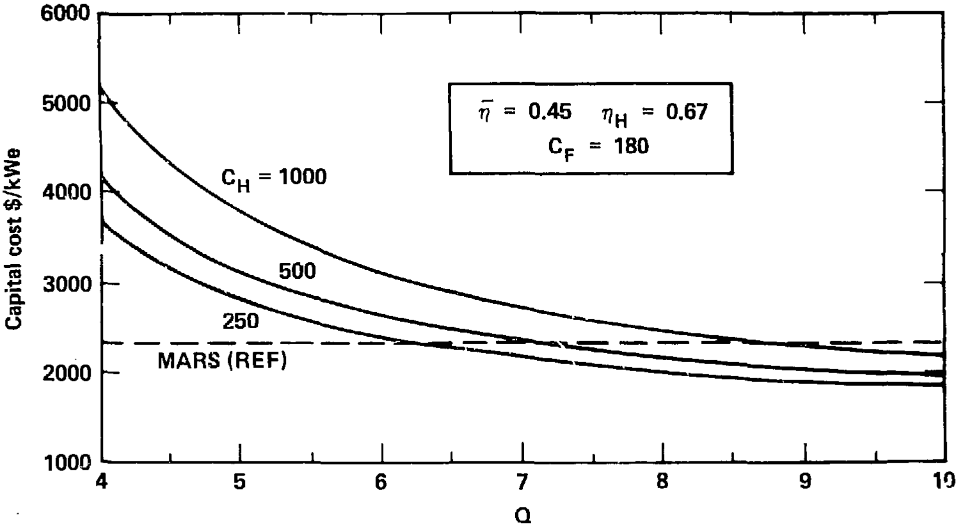

Figure 5. Scaling of mirror fusion power plant capital cost vs Q for various values of the heating cost parameter, C_H, with C_F = 180.

example, if $\bar{\eta} = 0.50$ and $\eta_H = 0.70$, so that $Q_m = 1.86$, then with $C_F = 180$ and $C_H = 500$ one finds that $Q \approx 5.5$ would yield the same direct capital cost as the reference MARS value.

The purpose of the preceding discussion has been to attempt to quantify the intuitively obvious point stated earlier, namely: A fusion core (i.e., the confinement chamber and its peripherals) that is simpler and cheaper than that of a full-blown tandem mirror system need not have as high a Q value in order to be economically competitive with that system.

In the section immediately following, we will attempt also to quantify the degree of improvement (in intrinsic $n\tau$ values) that is needed over that of a simple mirror cell in order to achieve the modest Q values found in the examples given in this section. Although the discussion is aimed at mirror systems fueled with DT, some of the remarks may also apply to D-D-He_3 or D-He_3 fueled systems, where the issue of direct conversion efficiency is of major importance.

A. Mirror Q: $n\tau$ Values and the Role of Alpha Particles

Understandably, one of the most thoroughly investigated topics in mirror research is that of collision-induced mirror losses. The extensive Fokker-Planck studies of this topic have been alluded to earlier. By now there exist large Fokker-Planck computer codes [37] that incorporate in their capabilities a whole spectrum of classical processes, Coulomb collisions, fusion neutrons, charge exchange, and even adiabatic compression and rf heating effects.

The simplest result of such codes is to yield the intrinsic collisional confinement particle $n\tau$ value for a neutral-beam injected plasma (including the effects of electron drag and ambipolar losses). In simplified form, this result is given by the expression

$$n\tau_p = k \times 10^{10}\, E_i^{3/2} \log_{10}(R_m) \simeq \text{cm}^{-3} \text{ sec. ,} \tag{11}$$

with E_i being the injection energy in kilovolts and R_m the mirror ratio. For a DT plasma, $k \simeq 2.2$. Using this result and the definition of Q one obtains the following simple expression for Q in terms of the reaction parameter $<\sigma v>_{DT}$, the injection energy and the fusion energy release (22.4 MeV for DT and Li capture)

$$Q = \frac{1}{4} <\sigma v>_{DT}(n\tau)\left(\frac{E_{fus}}{E_i}\right) \tag{12}$$

i.e.,

$$Q = 5.6 \times 10^{13} \left\{k <\sigma v>_{DT} E_i^{1/2} \log_{10}(R_m)\right\} \tag{13}$$

putting in $k = 2.2$ and $R_m = 10$ yields the value $Q \simeq 1.2$ at $E_i = 100$ keV. From Eq. (11) the corresponding value of $n\tau_p$ is seen to be 2.2×10^{13} cm^{-3} sec.

What classical or other effects might be exploited leading to higher Q values? First, as mentioned previously, any reduction in either electron drag or ambipolar losses could enhance Q (increase $n\tau$) by factors of 2 to 4. The Kelley idea aimed at the latter objective. Electron drag could be reduced if the electrons become heated to higher temperatures than those typically predicted by the Fokker-Planck calculations ($kT_e \simeq 0.1\, E_i$). This possibility was examined by Lazar and Haste [38], [39], who extended previous Fokker-Planck studies to include external electron heating means. They found Q increases of order a factor of 2 to 3 (in the Kelley mode) over that given by Eq. (12). However, when they included in the overall energy balance the external power required to heat the electrons, the effective Q value dropped to about the same level as that without heating. Nevertheless, the point remains that if the intrinsic $n\tau$ value could somehow be increased, then electron heating might be useful, since with an increased $n\tau$ the equilibrium value of T_e would rise so that less external heating power would be required.

Another positive consequence of an increased intrinsic $n\tau$, true even for modest increases over the classical values, is the effect of alpha particles in heating the plasma. This is the same effect of the alphas that results in "ignition" ($Q \rightarrow \infty$) in better-confined plasmas. The heating by alphas in the context of mirror confinement was studied by Kuo-Petravik, Petravic, and Watson [40]. They showed that to a good approximation the increase in Q caused by plasma heating by the alpha particles of the DT reaction could be represented by a simple expression, derived from energy balance considerations, namely:

$$Q_\alpha = \frac{Q_0}{1 - Q_0/6.4} , \qquad (14)$$

where Q_0 is the Q value derived from the intrinsic $n\tau$ confinement value, Eq. (13). Figure 6, a plot of Eq. (14), shows the substantial enhancement in Q that occurs, which would eventually lead to "ignition" if $Q_0 > 6.4$. For our purposes here the significance of heating by alphas is that an increase in Q_0 by only about a factor 2.5 over that of the simple mirror can lead to Q_α values approaching 6.0. Such Q values are comparable to those we earlier indicated could be of economic interest if coupled with a less expensive fusion core, and with adequately efficient plasma heating and energy recovery systems. Implied also would be operation at or near the optimum temperature for the DT reaction (i.e., $T_i \approx 60$ keV).

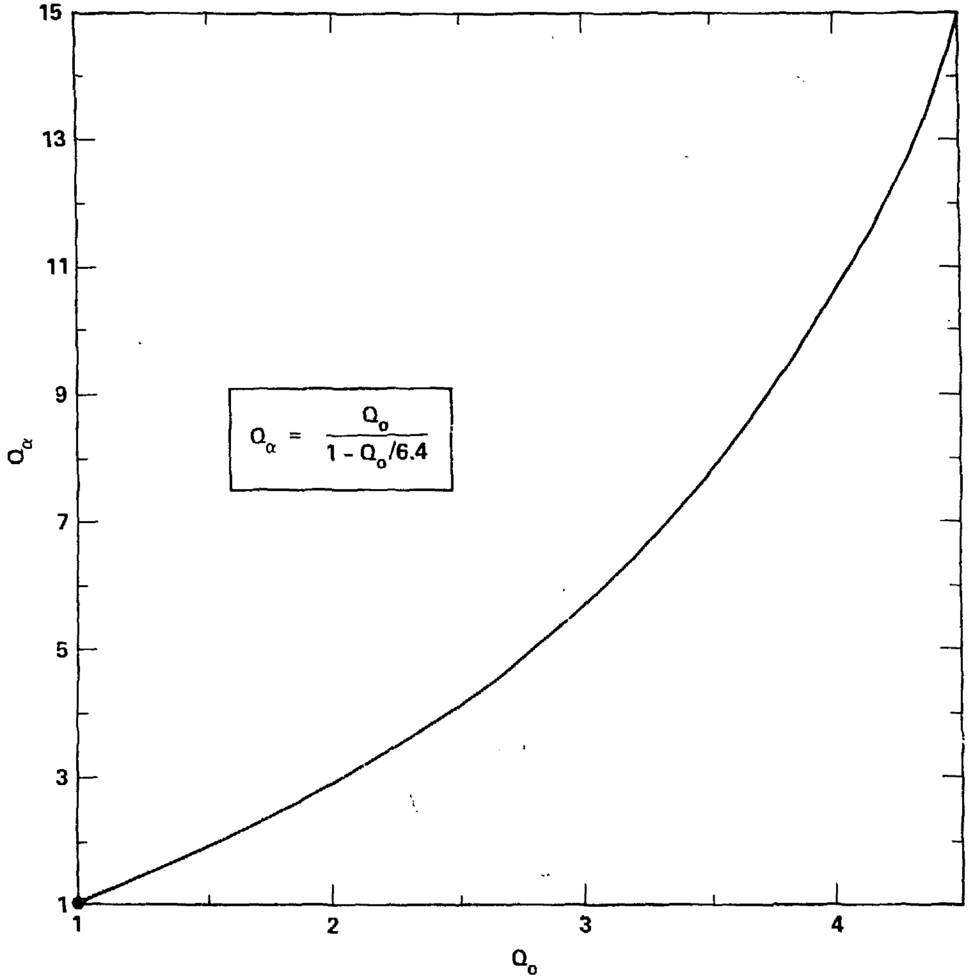

Figure 6. Enhancement of mirror Q values by plasma heating by DT alpha particles.

In summary, in the last two subsections of this report we have made the point that mirror-based systems with DT-alpha-enhanced Q values of order 5 could be of interest if at the same time the fusion cores of these systems, by virtue of the reduced confinement requirements, could be made to be substantially simpler and less expensive than that of present-day tandem mirror designs. To achieve this objective was shown to require only a modest increase (of order a factor of 2 to 3) in the intrinsic confinement $n\tau$ values over that of simple mirrors. In sections to follow we will give some examples of approaches that might lead to $n\tau$ improvements of that order.

VII. COLLISIONAL EQUILIBRIUM: LIVING NEAR THE BORDERLINE

As can readily be seen by comparing the $n\tau$ expression for a simple mirror, Eq. (11), with the usual Spitzer formula [41] for collisional relaxation times, the containment time in a mirror cell with ambipolar and electron drag-induced particle losses is about one ion-ion collisional relaxation time at its ion temperature for maximum Q. By contrast, a tokamak, operating at lower ion temperatures, would typically have an ion containment time hundreds of times longer than the ion-ion relaxation time at that lower temperature. It follows that in the case of mirror systems, if one is only looking for $n\tau$ increases of order of a factor of 2 or 3 over that of an unaugmented mirror cell, it makes sense (where it would not for the tokamak) to discuss plasma states that are still fairly close to the borderline between collisional equilibrium (i.e., several ion-ion collisions in one mean ion containment time) and collisional non-equilibrium (less than one collision in one ion confinement time). Obviously there exists a continuum of confinement states between the two extremes. In what follows we will, however, be discussing ways in which the circumstance, "living near the borderline", might be exploited to advantage.

A. The Continuum: $n\tau$ vs $e\Delta\phi/E$ Scaling

One informative (and useful) way to define the continuum of confinement states lying between the simple mirror, the "Kelley Mode, and the tandem mirror, is via the scaling of $n\tau$ vs the parameter $e\Delta\phi/E$, plasma ambipolar confining potential divided by ion energy. In the simple mirror $e\Delta\phi/E$ is negative; in the Kelley mode it is zero; and in the TM it is positive. Fokker-Planck and/or analytical forms (Pastukhov scaling [42]) can be used to define a curve of $n\tau$ vs $e\Delta\phi/E$. We will then locate ourselves on this curve in terms of the previously-developed economic criteria.

Properly the scaling of $n\tau$ vs $e\Delta\phi/E$ should be developed theoretically from a full sequence of Fokker-Planck calculations. In the absence of these we will deduce the relationships by interpolation between some existing Fokker-Planck calculations and an analytical form. The F-P points are ones calculated by Kuo-Petrovik, Petrovik and Watson (KPW) who give the following: (for $R_m = 10$)

$e\Delta\phi/E$	$(n\tau)_{kw}$
-0.5	$2.3 \times 10^{10} E^{3/2}$
0.0	$2.6 \times 10^{10} E^{3/2}$

The Pastukhov form that was used is a generalized one given in a paper by Najmabadi, Conn, and Cohen [43].

Figure 7 was prepared by using the two KPW data points, at $e\Delta\phi/E = 0.5$ and 0.0 with graphical interpolation between those points and the analytical

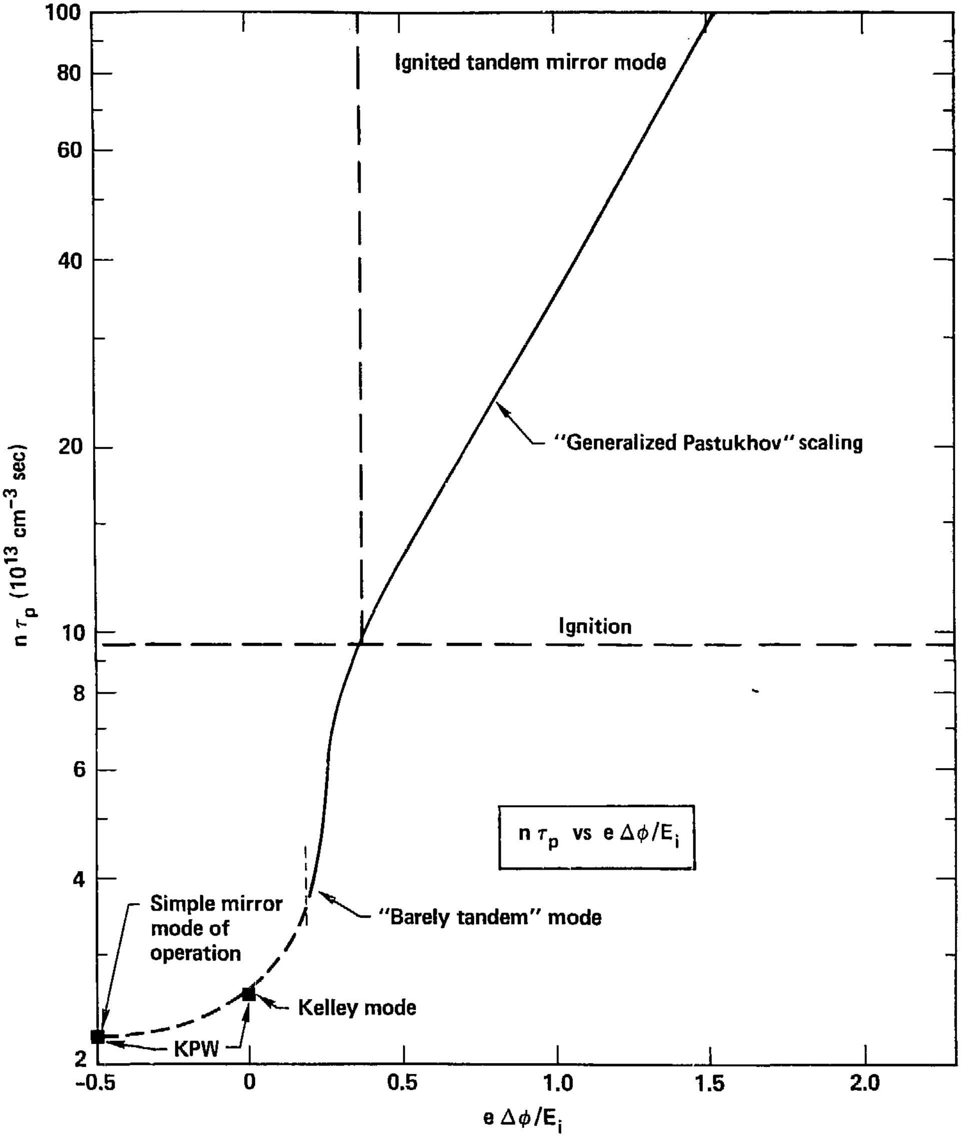

Figure 7. The scaling of $n\tau_p$ with the confining potential parameter $e\Delta\phi/E_i$.

Pastukhov form (*used between* $e\Delta\phi/E$ = 0.2 *and* 1.5, *as shown*). The ion energy, $E = 3/2\ T$, was chosen to be 100 keV, near the optimum value for the simple mirror case. Labeled on the curve are the regimes represented, i.e., "simple mirror", "Kelley mode", and "TM mode". Also shown is the "ignition" value of $n\tau$, corresponding to the value determined by KPW from the effect of alpha particle heating (see Eq. (14)).

If the goal is to achieve $n\tau$ values about 2 times that of a simple mirror, then Fig. 7 shows one possible route, namely to operate in what might be called a "barely tandem" mode, with $e\Delta\phi/E$ in the range of 0.2 to 0.3. Such a mode might be achieved by simpler and less expensive means than the use of thermal barriers and their auxiliary equipment as employed in present-day tandem mirror designs. In the next section we will explore some of the implications relative to electron temperatures that operation in a "barely tandem" mode may imply.

B. Scaling of Electron Temperature and Ambipolar Potential

Implicit in the idea of operating in modes that are not far removed from the ion-ion collision time boundary is the question of the scaling of kT_e and $e\phi$ in the vicinity of this boundary. These scalings can be deduced approximately from some simple collision-time arguments, as follows:

In all mirror systems, tandem or otherwise, the confinement of the general electron population is by means of the ambipolar potential, operating to bring the electron and ion losses into balance. Taking the collision rate and population density in the tail of the Maxwellian (beyond $e\phi$) approximately into account, and measuring ion losses in units of τ_{ii} ($n\tau_i = Kn\tau_{ii}$) the balance condition gives the equation, with $\eta \equiv e\phi/kT_e$:

$$\eta \exp(\eta) = K[(n\tau)_{ii}/(n\tau)_{ee}] \ , \tag{15}$$

with

$$(n\tau)_{ii} = 2.5 \times 10^{10}(E_i)^{3/2} \tag{16}$$

$$(n\tau)_{ee} = 5.3 \times 10^{8}(kT_e)^{3/2} \ , \tag{17}$$

energies in keV.

A second equation is obtained by balancing the energy carried out by the escaping electrons against the heat input from the plasma ions and from any external source of heat. This equation is

$$\frac{\eta \left[\frac{3}{2}\, kT_e\right]}{K\tau_{ii}} = \left[\left(\frac{E_i}{\tau_{ie}}\right) + p_{eh}\right] \ , \tag{18}$$

where $(n\tau_{ie}) = 1.1 \times 10^{12}(kT_e)^{3/2}$ is the electron-ion drag time coefficient and p_{eh} is the external heating rate in keV/sec. Rewriting the heating in terms of a normalized heating rate coefficient $\lambda = (\tau_{ie}\, p_{eh}/E_i)$ there results an expression for kT_e/E_i:

$$\left(\frac{kT_e}{E_i}\right) = \frac{2}{3}\left(\frac{K}{\eta}\right)\left(\frac{\tau_{ii}}{\tau_{ie}}\right)(1 + \lambda) \ . \tag{19}$$

Solving the two equations simultaneously for η and kT_e/E_i gives the result

$$\eta^{9/10}\exp(\eta) = 5.83 \times 10^{2}\left[\frac{K^{2/5}}{(1 + \lambda)^{3/5}}\right] \tag{20}$$

$$\left(\frac{kT_e}{E_i}\right) = 0.187\left[\frac{K}{\eta}(1 + \lambda)\right]^{2/5} . \tag{21}$$

These equations give the scaling vs ion confinement time (proportional to K) and heating (proportional to λ; $\lambda = 1$ corresponds to an external heating rate equal to the plasma ion-electron transfer rate). Some results calculated from these equations are given in Table I.

Table I

K	λ	η	kT_e/E_i	$e\phi/E_i$
1.0	0.0	4.93	0.099	0.488
	1.0	4.70	0.133	0.625
	2.0	4.56	0.158	0.720
2.0	0.0	5.17	0.128	0.662
	1.0	4.93	0.172	0.848
	2.0	4.79	0.205	0.982
3.0	0.0	5.31	0.149	0.791
	1.0	5.07	0.200	1.014
	2.0	4.93	0.238	1.173

Thus, for example, if the ion confinement time is 3 times τ_{ii} (K = 3), and an amount of external heating power equal to the plasma ion-electron heat transfer rate is added ($\lambda = 1$), then $(kT_e/E_i) = 0.200$ and $(e\phi/E_i) \approx 1.0$. Note that these results indicate a degree of "regenerative feedback" between $n\tau_p$ and kT_e for plasma regimes near the collisionality borderline. That is, an increase in $n\tau_p$ (larger K) results in an increase in kT_e, which then reduces ion-electron drag and thus tends to increase $n\tau_p$, etc. (and similarly for the effects of external heating of the electrons).

VIII. INCREASING MIRROR $n\tau$ VALUES OR FUSION REACTIVITY: SOME POSSIBLE APPROACHES

Having quantified the level of $n\tau$ gains (over the simple mirror) or improvements in efficiency that may be of interest, and having estimated the scaling laws for plasma parameters such as $e\phi$, $e\Delta\phi$, and kT_e, we will now examine some possibilities for accomplishing these ends.

Three topics will be discussed, in order to illustrate these different points. The first topic concerns mirror operation with small positive confining potentials, close to the Kelley mode, but with some augmentation in $n\tau$ in order to achieve the modest Q values previously discussed. The second topic deals with the subject of ion heating, specifically adiabatic compression/decompression processes, illustrating how these can compete with collisional losses when one is "operating on the borderline". The third topic concerns ICRF and how it can compete with collisions to modify the ion distribution function and the spectrum of the energy losses. Such modifications may have advantages in terms of increased fusion reactivity or increased energy recovery efficiency.

A. The "Barely Tandem" Model

Our model here is of an axially-symmetric central cell with high mirror ratio, bounded on each end by short, axially symmetric plug cells of modest mirror ratio (i.e., higher midplane magnetic field than the central cell). We presume neutral beam injection in both the central cell and the plugs. The central cell is to be injected with deuterium and tritium beams at energies of order 80-100 keV; but the plugs are maintained with higher Z, higher energy, beams. We will aim at an $n\tau$ enhancement (over simple mirror values) of order 2, leading to alpha-enhanced Q values for the central cell of order 6.

From the $n\tau$ vs $e\phi/E_i$ curve, Fig. 6, we estimate that $e\Delta\phi/E_i$ in the range of 0.2 to 0.3 is required to achieve the desired $n\tau$ value. From Table I, taking $K = 2$ and $\lambda = 2$, for example, we find $(kT_e/E_i) = 0.20$ and $(e\phi/E_i)$ 1.0. Since there are no thermal barriers kT_e is constant throughout at 20 keV. We may therefore use the usual Boltzmann relationship to find the required electron density ratio between the plugs and the central cell. We have:

$$e\Delta\phi = kT_e \ln\left(\frac{n_p}{n_c}\right), \tag{22}$$

so that

$$\frac{n_p}{n_c} = \exp\left(\frac{e\Delta\phi}{E_i} \bigg/ \frac{kT_e}{E_i}\right) . \tag{23}$$

Putting in the numbers for the table we find

$$\frac{n_p}{n_c} \approx 2.7 \text{ to } 4.5 \quad .$$

One possible way to achieve both the increased electron density in the plugs and at the same time introduce an external heating source at high efficiency could be to feed the plugs with a high energy neutral beam composed of lithium atoms. In this case the electron density in the plugs is 3 times the plug lithium ion density. The latter then becomes roughly equal to the central cell ion density. Injecting these ions at energies of order 1 MeV would lengthen the plug ion confinement time while at the same time it would provide a heating source to the electrons. Normalized to plug electron density, and relative to deuterons at lower energy, the plug ion lifetimes would be of order $(M_{Li}/M_D)^{1/2}(Z_{Li})^{-3}\ (W_{Li}/W_D)^{3/2}$ times that of a deuteron, i.e., $.069\ (W_{Li}/W_D)^{3/2}$ times. Thus at $W_{ci} \approx 600$ keV and $W_D = 100$ keV they would be equal. The actual Li ion energy chosen would depend on considerations of efficiency, on the ratio of plug-to-central cell volume, and on electron heating questions.

Recalling the earlier discussion of the stabilization of axially-symmetric mirror systems with energetic particles, it can be seen that the plug ions may play an important role in this respect.

The numbers given suggest that there may be viable "barely tandem" systems where the simplification and relaxation of parameters allowed by operation at lowered Q values and with axially-symmetric coils becomes economically attractive.

B. Ion Heating in Competition with Collisional Effects

If we try to move even closer to the simple mirror, not relying on the gains from operation in the "barely tandem" mode, the path to increasing $n\tau$ becomes both more difficult and more conjectural. One possible approach is to employ ion heating in such a way as to increase the ion reactivity. Though the heating process costs extra energy that must be factored into the balance, it does have some potential advantages: (1) the role of alpha-heating in increasing Q may be enhanced, and (2) it may be possible to use the additional heating process so as to increase either the efficiency of direct energy recovery, or overall efficiency of the ion heating, thereby decreasing Q_M (the second avenue to decreasing costs).

Ion heating to increase reactivity is an example of "living on the borderline" by using an "ordering" process (selective heating) to compete with a disordering process, Coulomb collisions. A calculation based on a simple model will illustrate the kinds of effects that are sought. However, to obtain reliable quantitative estimates requires both a detailed model and Fokker-Planck and/or Monte Carlo calculations. First, the simple case:

We will model the losses from a simple mirror by defining "loss" as having occurred when the integrated value of the probability of a large angle scatter (by Coulomb collisions) becomes unity, i.e., when

$$P_L = \int_0^T n\langle\sigma v\rangle_s \, dt = 1 \ . \tag{24}$$

For Coulomb collisions we then take

$$n\sigma_s = A/v^4 \ . \tag{25}$$

Using a stochastic heating model (heating proportional to t), and ignoring energy dispersion about the mean, one has

$$v^2(t) = (1 + ht)\, v_0^2 \ , \tag{26}$$

where the parameter h determines the rate of heating. Inserting this assumption in Eq. (24) integrating, and normalizing to the case of no heating (h = 0), there results for the factor of increase of the confinement time,

$$\frac{T}{T_0} = \frac{\left[1 - \frac{hT_0}{4}\right]}{\left[1 - \frac{hT_0}{2}\right]^2} \ , \tag{27}$$

and for the ratio of energy at t = T compared to that at t = o:

$$\frac{E(T)}{E_0} = \frac{v^2(T)}{v_0^2} = (1 + hT) = \frac{1}{\left[1 - \frac{hT_0}{2}\right]^2} \ . \tag{28}$$

If the upper energy is limited to the value given by the ratio

$$\frac{v^2(T)}{v_0^2} = \frac{E_{max}}{E_0} \ , \tag{29}$$

then the gain in confinement time will be correspondingly limited to

$$\frac{T_{max}}{T_0} = \frac{1}{2}\left[1 + \left(\frac{E_0}{E_{max}}\right)^{1/2}\right]\left[\frac{E_{max}}{E_0}\right] . \qquad (30)$$

For example, if $E_{max}/E_0 = 3.0$, then $T_{max}/T_0 = 2.37$.

In this example we see that the increase in confinement time is less than the increase in energy. There could be a net gain, therefore, only if the mean reactivity of the plasma during the heating process was sufficiently greater to make up the difference, or if a major improvement in energy recovery efficiency resulted. We will give below examples (not necessarily practical ones) of the effect of ion heating based on two specific models, namely adiabatic compression and ICRF heating.

1. Increasing $n\tau$ by Adiabatic Compression

Though it obviously would introduce some difficult design issues for a fusion power system, the concept of heating mirror plasmas by magnetic compression is by now both well documented experimentally and accurately predictable theoretically through Fokker-Planck calculations.

The time scale on which compression needs to proceed in order to compete with collisions in the mirror fusion context is relatively slow, being of order tenths of seconds to seconds. Thus, compression might either be implemented electrically, or even possibly mechanically (by plasma translation). A related concept has been discussed previously [44], in which Monte-Carlo code calculations were used to obtain rough estimates. In those earlier calculations, the magnetic "compression" used was actually a longitudinal decompression, occasioned by moving the mirrors farther apart. Since this action preferentially decreases $v_{\parallel}^2$, it delays the time for diffusion of a typical particle into the loss cone, thus increases the confinement time.

Similar calculations, but now involving both radial magnetic compression (increases $v_{\perp}^2$) and axial decompression (decreases $v_{\parallel}^2$) were performed for the author by A. Mirin, using the Livermore "Hybrid II" Fokker-Planck code [45]. This code allows the introduction of a time-varying magnetic field ($\dot{B}$) and a time-varying separation between the mirrors ($\dot{L}$). From elementary considerations one sees that the instantaneous effect of the variation of these parameters on the perpendicular and parallel energies of the ions is given by the relationships

$$\dot{v}_{\parallel}^2 = -2\left(\frac{\dot{L}}{L}\right) v_{\parallel}^2 \tag{31}$$

$$\dot{v}_{\perp}^2 = +\left(\frac{\dot{B}}{B}\right) v_{\perp}^2 \quad , \tag{32}$$

i.e., the total energy of a given ion varies instantaneously as

$$\dot{E} = M\left(\frac{\dot{B}}{B}\right) v^2 - \frac{\dot{L}}{L}\, v_{\parallel}^2 \quad . \tag{33}$$

The specific case that was run had the following parameters:

Injection energy (deuterons): - - - $E_i = 80$ keV

Injection current density: - - - $j = 1.25 \times 10^{15}$ cm^{-3} sec^{-1}

Injection angle (narrow gaussian): - - - 90^0

Mirror ratio: - - - $R_m = 4.0$.

The case was first run with $\dot{B} = \dot{L} = 0$, when it was found that n reached the steady-state value 1.23×10^{14} cm^{-3}, corresponding to $n\tau_p = n^2/j = 1.21 \times 10^{13}$ cm^{-3}, or $\tau_p \approx 0.1$ sec. The electron temperature was 7.0 keV, the plasma potential was 37 kilovolts, and the mean ion energy was 87 keV. After the above steady-state conditions were reached, radial compression and axial expansion was turned on, with $\dot{B}/B = \dot{L}/L = 3$ sec^{-1}, i.e., with a time constant about 3 times longer than τ_p for the steady state. Under these conditions the density rose to 1.74×10^{14} cm^{-3}, corresponding to a $n\tau_p$ of 2.42×10^{13} cm^{-3} sec., an increase of a factor of 2.0 over the static field case. At the same time the electron temperature rose to 11.8 keV, the potential to 60 kilovolts, and the mean ion energy to 149 keV. Note that since the rate of charge of plasma volume, being proportional to $(\dot{L}/L - \dot{B}/B)$, is zero in this case, the increase in plasma density found directly reflects the increased $n\tau$ values resulting from the adiabatic compression/expansion process.

Since adiabatic compression/decompression was shown to favorably influence $n\tau$, it follows that it should also enhance the effect of alpha heating. The Hybrid II code was again used to demonstrate this effect, by altering the source function to include both deuterium and tritium while retaining the same values of $\dot{B}/B$ and $\dot{L}/L$ (3 sec^{-1}) as in the previous runs. Despite the relatively low mirror ratio, the effect of alpha heating was found to be very strong, essentially corresponding to ignition, with rapidly climbing plasma temperatures and densities occurring during the course of the compression/decompression.

Figure 8 shows plots of the tritium density and particle energy with and without compression/decompression, and Table II gives a comparison between the plasma parameters reached at the end of 2 seconds, with and without compression/decompression. Although it would obviously be impractical to

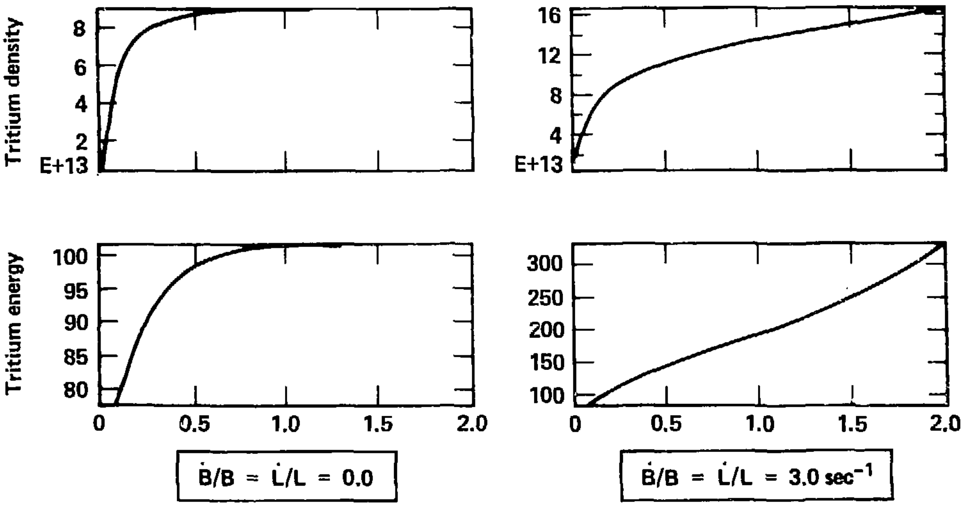

Figure 8. Time history of tritium density and mean energy without (left) and with (right) adiabatic compression/decompression when the heating effects of alpha particles are included.

Table II. Comparison between alpha heating effects with and without adiabatic compression/decompression.

	$\dot{B}/B = \dot{L}/L = 0$	$\dot{B}/B = \dot{L}/L = 3\ \mathrm{sec}^{-1}$	
n_D	$4.7 \times 10^{13}\ \mathrm{cm}^{-3}$	$7.3 \times 10^{13}\ \mathrm{cm}^{-3}$	(at 2 sec.)
n_T	$8.9 \times 10^{13}\ \mathrm{cm}^{-3}$	$16.8 \times 10^{13}\ \mathrm{cm}^{-3}$	"
W_D	90 keV	246 keV	"
W_T	101 keV	333 keV	"
kT_e	7.7 keV	19 keV	"
$e\phi$	41 keV	101 keV	"

continue the compression process for this long a time (6 e-folds), the "runaway" character of the plasma parameters during the course of the compression demonstrates closely the effectiveness of the process in competing with plasma losses.

Although the introduction of adiabatic compression into mirror design would likely be both difficult and expensive, the Fokker-Planck calculations confirm the concept that ion heating processes carried out on a relatively slow time scale can nevertheless favorably influence confinement. In the next section we will discuss another ion heating technique--second harmonic ion cyclotron resonance heating--that could be more readily implemented than adiabatic compression, and which might have some special uses.

2. ICRH and Mirror Losses

Ion cyclotron resonance heating bears a special relationship to mirror confinement. Since particle confinement in a conventional mirror system depends on the magnetic moment $\mu = W_{\perp}/B$, and since ICRH performed at the midplane between the mirrors preferentially increases $W_{\perp}$, under the proper conditions heating the ions by rf should result in modifying their confinement conditions.

The quasilinear theory of stochastic heating of ions in rf fields, such as that put forth by Davidson [46] can be used to predict the nature of the heating process. Since stochastic rf heating results in a dispersion in $W_\perp$ in addition to the net heating that occurs, it is advantageous to choose conditions that will optimize $W_\perp$ heating (which improves confinement) relative to downward energy dispersion (which degrades confinement).

The 2-D diffusion equation governing the evolution of the perpendicular velocity distribut*ion function in the presence of the rf fields has the* general form

$$\frac{\partial F}{\partial t} = \frac{1}{v_\perp} \frac{\partial}{\partial v_\perp} \left[v_\perp D(v_\perp) \frac{\partial F}{\partial v_\perp} \right] . \qquad (34)$$

The diffusion coefficient $D(v_\perp)$ is given by the expression [47]:

$$D(v_\perp) = \frac{e^2}{2Mv_\perp^2} \sum_n n^2 \omega_{ci}^2 \int k_\perp dk_\perp \; |\phi_k|^2 \, J_n^2\!\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right) \frac{\Delta\omega_k}{(\omega_k - n\omega_{ci})^2 + (\Delta\omega_k)^2} \qquad (35)$$

where ϕ_k is the rf potential, n is the harmonic number, and $\Delta\omega_k$ is the correlation frequency (inverse of the correlation time) for the particle moving in the rf field. If the rf field is concentrated in the vicinity of the cyclotron frequency or one of its harmonics, then only one term in the series need be considered.

From the form of the equation it can be seen that for the fundamental (n=1), D is independent of $v_\perp$, whereas for the second harmonic $D(v_\perp) \sim v_\perp^2$, *implying diffusion that is weighted toward higher perpendicular ion energies.**

**Note that the Bessel-function dependence of* $D(v_\perp)$ implies that at a sufficiently large value of $k_\perp v_\perp/\omega_{ci}$ the diffusion would decrease. Advantage might be taken of this effect so as to limit the upward diffusion in energy.

Whether it is advantageous to use rf fields at the fundamental or at higher harmonics depends on the model under consideration. As will be later discussed, we will be considering a model for mirror confinement in which particles are preferentially lost after they have reached a well-defined upper energy cutoff (for example as determined by non-adiabatic or resonance processes). In such a case it should be advantageous to employ, for example, second harmonic ICRH in order to emphasize upward energy diffusion as compared to downward energy dispersion.

In the presence of an upper cutoff in $v_\perp$, and when the effect of Coulomb collisions is relatively small (say near the high energy end of the distribution function), a steady state solution to the diffusion equation (Eq. (34), can be given in the form [40]

$$F = A \int_{v_\perp}^{v_{max}} \frac{dv}{v_\perp D(v_\perp)} \quad . \tag{36}$$

If we now take $D(v_\perp) \sim v_\perp^2$, then

$$F(v_\perp) \sim \left[\frac{1}{v_\perp^2} - \frac{1}{v_{max}^2}\right] \quad , \tag{37}$$

i.e.,

$$F(E_\perp) \sim \left[\frac{1}{E_\perp} - \frac{1}{E_{max}}\right] \quad .$$

As might be expected, as compared to a Maxwellian distribution this distribution function is distorted toward the higher energy end, with the gradient in perpendicular energy becoming very steep near the cutoff energy. Under the right conditions the fusion reactivity of this distribution function could be higher than that of a Maxwellian.

a. Monte Carlo Code Calculations

In order to assess the level of gains in reactivity or in other factors that might be achieved by ICRH heating of the ions at rates competitive with Coulomb scattering, Monte Carlo calculations were performed for the author by T. Rognlien, using a computer code developed by him [49]. In this code both the effect of Coulomb collisions and of rf diffusion can be modeled, together with losses through a loss cone or at a high energy cutoff. It is also possible to introduce ambipolar potentials of either sign, so as to simulate either conventional mirror cells or tandem mirrors.

The cases run simulate a mirror plasma created by perpendicular neutral beam injection. The rf fields concentrated at the midplane and in frequency at the second harmonic further heat the ions until they are lost, either at an upper cutoff energy, or out the loss cone as a result of scattering of the ions against a "background" plasma.

To determine the effect on confinement time and on the energy loss spectrum resulting from rf heating, a comparison is made with runs in which the rf field is turned off, so that only Coulomb scattering is present.

The parameters used were as follows:

Injection energy	80 keV
Confining potential	40 keV
Upper loss boundary	200 keV
Mirror ratio	4.0
rf frequency (rad./sec.)	$2\,\omega_{ci}$.

Runs were made in which the ratio of rf to beam power was varied, and the effect on $n\tau$ and on the fraction of ions lost at the upper loss boundary was listed. Some results (obtained by graphical interpolation) are given in Table III.

As can be seen from the table, the effect of the rf was to reduce $n\tau$ somewhat, while at the same time greatly increasing the fraction of particles lost at the upper cutoff. As will be discussed in a later section, this kind of a shift in the loss spectrum could be employed to increase the direct conversion efficiency. Together with the potentially increased fusion reactivity of the plasma a net gain might be achieved under optimized conditions.

Table III

P_{rf}/P_{beam}	$(n\tau)/(n\tau)_0$	Frac. lost at upper cutoff
0.0	1.00	0.40
0.2	0.85	0.46
0.4	0.73	0.54
0.6	0.63	0.62
0.8	0.56	0.70

IX. THE OTHER HANDLE: DECREASING Q_M

In Section V, in the discussion of the tradeoffs between Q and simplicity/lowered cost, two approaches were mentioned. The first one, discussed in the previous section, was to find sufficiently inexpensive ways to increase the $n\tau_p$ values of simple mirror cells. The second, augmenting, approach mentioned was to find methods, again sufficiently inexpensive, to raise either the heating efficiency, η_H, or the mean energy recovery efficiency, $\bar{\eta}$, so as to decrease $Q_M \equiv [(\bar{\eta}\eta_H)^{-1}-1]$. In this section we will explore the possibility of increasing η_{DC}, the direct conversion component of $\bar{\eta}$. Our guideline will be item A2: " ---- insights which might be gained from an "information theory" approach ----".

In the present context the reference to "information theory" has to do with the particle loss channels from mirror systems and the "information content" of these losses that can possibly be used to advantage in increasing the efficiency of the direct recovery of this energy. The situation is as follows:

In an axially-symmetric mirror cell operating in the "semi-collisional" limit, the particle loss channel will be essentially only through the mirrors; radial transport will be negligible by comparison. Furthermore, apart from a very small fraction of order $[R_m \ln(\Lambda)]^{-1} \approx 1$ percent, arising from large-angle scattering events, the ion losses will be entirely made up of particles that gradually diffuse up the mirror gradients until they reach the peak mirror field, when they are promptly lost. The "information" carried by these lost particles is that we knew "where" they are in both configuration space and in pitch angle space at the moment they are about to be lost. That is, they are at the peak of the mirror and at a pitch angle equal to 90^0. Information content, that is, order--can be related to the efficiency with which energy can be recovered. Therefore, we should look carefully for ways to take advantage of the full information content of the particle losses.

In the past, this information content has been used in an elementary way in the direct converters incorporated into all mirror fusion systems, for example in the MARS study previously alluded to. These systems only take advantage of the fact that particle losses occur through the mirrors. Additional "information" carried by the lost particles, as outlined above, is not used.

In the previous section, it was suggested that some advantages might be gained by introducing rf or other heating methods. The result of ion heating was of course to drive up the mean energy at which the ion was lost to a higher level than that which it was injected. If it could now be arranged that such ions were preferentially lost within a narrowly-defined energy range, the energy conversion efficiency of even a simple direct converter would be correspondingly increased. This result can be seen from the expression for the theoretical efficiency of an electrostatic direct converter with N stages which is [50]:

$$\eta_{DC} = \frac{\sum_{j=1}^{N} W_j \int_{W_j}^{W_{j+1}} I(W)\, dW}{\int_0^{\infty} W I(W) dW} \; . \tag{38}$$

For a single stage converter (N = 1) upon which is incident, for example, a "rectangular" ion current distribution with energy width ΔW and mean energy W, the theoretical efficiency is

$$\eta_{DC} = \frac{1}{1 + \frac{1}{2}\left(\frac{\Delta W}{W}\right)\left(1 - \frac{\Delta W}{W}\right)^{-1}} \tag{39}$$

$$\approx 1 - \frac{1}{2}\left(\frac{\Delta W}{W}\right) , \qquad \Delta W << \overline{W} \; .$$

If passive or energy-efficient active ways could be found to define more precisely the energy at which an ion is lost then η_{DC} might be substantially increased, and with it $\overline{\eta}$. An example will illustrate the order of the gains that are possible.

The average energy conversion efficiency is made up of two components, (1) the thermal conversion efficiency, operating on the neutron and neutron capture energies and the rejected heat from the direct converter, and (2) the direct converter efficiency, operating on the charged particle energy of the plasma and the alpha particle energy (bremsstrahlung losses are ignored in this calculation as being a small correction). We therefore have for the energy balance equation defining $\bar{\eta}$

$$\eta_{DC}\left\{Q\ \frac{3.5}{22.4} + 1\right\} + \eta_{T}\left\{Q\left(\frac{18.9}{22.4}\right) + (1 - \eta_{DC})\left[Q\left(\frac{3.5}{22.4}\right) + 1\right]\right\} = \bar{\eta}\ [Q + 1] \tag{40}$$

(DT reaction: alpha energy = 3.5 MeV) .

Solving for $\bar{\eta}$:

$$\bar{\eta} = \frac{[\eta_{DC}(1 - \eta_{T}) + \eta_{T}][.136\ Q + 1] + \eta_{T}[.844\ Q]}{[Q + 1]} . \tag{41}$$

At $Q \simeq 5$, for example, with $\eta_T = 0.4$ and $\eta_{DC} = 0.5$, $\bar{\eta} = 0.48$, whereas $\bar{\eta} = 0.55$ if $\eta_{DC} = 0.8$. Taking $\eta_H = 0.7$ leads to $Q_M = 1.98$ in the first case and $Q_M = 1.60$ in the second case. With the cost factors $C_F = 180$ and $C_H = 500$, the first case requires $Q > 5.9$ to be cost-effective against the MARS example (see Sec. V) whereas in the second case $Q > 4.6$ is sufficient. The corresponding "intrinsic" Q values (heating by alphas not included) are 3.1 and 2.7 respectively. The latter value is within about a factor 2 of the basic Q value of a simple mirror cell. In a squeeze the indicated reduced Q requirements (between Q = 3.1 and Q = 2.7) could be worth the extra effort.

Though we have not discussed it here, the use of the D-D-He_3 or the D-He_3 fuel cycle in a mirror system would obviously benefit even more from an increase in η_{DC}, owing to the large fraction of energy in charged reaction products.

A. Energy-Selective Losses from Bounce Resonance

A possible way to achieve energy selectivity of ion losses is to drive the bounce motion of particles that have nearly reached the loss cone in an energy-selective resonant manner. If this resonance is sharp, then ions that are being heated by rf and which are close to being lost will be preferentially spilled out at a sharply-defined energy.

To illustrate the idea we will consider the bounce motion in mirror fields with a parabolic axial variation,

$$B = B_0 \left[1 + \frac{z^2}{L_2}\right] . \tag{42}$$

Solving the equation of motion,

$$M\ddot{z} = - \mu \nabla B \ , \tag{43}$$

in this field we are led to the result

$$z_0 = z_0 \sin(\omega_0 \ t) \ , \tag{44}$$

with $\omega_0 = v_{\perp 0}/L$.

For those particles that approach the mirror peak then, at $z = z_{max} = \alpha L$, the resonance occurs at a frequency

$$\omega_0 = \left\{\frac{v_0}{z_{max}} \left[\frac{\alpha}{(1 + \alpha)^{1/2}}\right]\right\} . \tag{45}$$

As can be seen for these particles, the resonance depends only on the field geometry and the scalar velocity of the ion.

We consider now the parametric excitation of bounce oscillations by a weak perturbation of the magnetic field. For simplicity, we will assume that this parametric drive is accomplished by a.c. coils that weakly perturb the parabolic scale length, i.e.,

$$L = L_0 \left[1 - \varepsilon \cos(\omega t)\right] . \tag{46}$$

The equation of motion now takes the form (ignoring terms of order ε^2):

$$\ddot{z} = \omega_0^2 \left[1 + 2\varepsilon \cos(\omega t)\right] z = 0 . \tag{47}$$

Setting $\omega t = 2u$ the equation reduces to Mathieu's equation:

$$\frac{d^2 z}{du^2} + \left[\eta + \gamma \cos(2u)\right] z = 0 , \tag{48}$$

with

$$\eta \equiv \left(\frac{4\omega_0^2}{\omega^2}\right), \quad \gamma = 2\varepsilon\left(\frac{4\omega_0^2}{\omega^2}\right) .$$

The lowest order parametric resonance solution (exponentially growing amplitude) occurs in the vicinity of $\eta = 1$, (i.e., $\omega \approx 2\,\omega_0$. In this region from the theory of Mathieu's equation [51], the width of the unstable region is given approximately by the relationship

$$\Delta\eta \simeq 0.3\,\gamma, \quad \gamma << 1 . \tag{49}$$

Converting this result to an energy interval through the definitions of η and γ we find

$$\frac{\Delta E_0}{E_0} \approx 0.60\ \varepsilon\ . \tag{50}$$

Thus, if the perturbing field is 1 percent of the main field (i.e., $\varepsilon = .01$),

$$\frac{\Delta E_0}{E_0} \approx 0.60 \text{ percent}\ .$$

In actual application, the perturbing fields would probably be more highly localized in the mirror region, but adequate energy selectivity should still be achievable. The amount of added energy needed to cause escape could probably be arranged to be a small fraction of the total particle kinetic energy, so that the non-reactive power input to the field perturbing system could probably be made to be small.

The idea discussed here resembles the "rf pumping" technique proposed by Baldwin [52] for use in the plugs of a tandem mirror with thermal barriers, except that here the pumping is axial rather than radial.

B. Energy-Selective Losses from Non-Adiabatic Effects

An alternative way to introduce energy-selective losses would be to take advantage of the (typically) exponential behavior of non-adiabatic loss processes on ion energy. If, for example, the end mirrors were to be modified so as to have a short parabolic-shaped depression near their peak, then particles that have climbed the mirrors and have sufficient energy would be preferentially lost. This idea is reminiscent of very early work, such as that of Sinelnikov [53], Robson [54], and Moir [55] where non-adiabaticity was to be used as a capture process for injected monoenergetic molecular ions that "resonate" with a spatially-periodic magnetic field.

C. Utility of Energy-Selective Losses

Introducing energy selectivity into the mirror loss process could have the result of increasing the efficiency of direct conversion (or simplifying its use). It could also have other uses. Such processes could be used in principle to selectively remove alpha particles after they had delivered most of their energy to the plasma. These and other possible applications are probably most likely to be realizable for mirror systems that operate in the semi-collisional regimes that we have been discussing.

X. JOINING FORCES: THE FRM-TANDEM

One of the "areas for innovation" mentioned earlier (A3) was the "combination of existing approaches to provide a new form, one with synergistic advantages". The FRM and the tandem idea provide such an opportunity. The FRM offers the possibility of compactness coming from its extremely high magnetic efficiency. This high magnetic efficiency could be a strongly positive economic factor for a DT system, and could be crucially important for the success of "advanced fuel" systems, such as D-D-He_3 or D-He_3.

The TM idea may provide an effective way to create a (plasma) environment for the FRM plasma that could be highly conducive to its stability. The thought is not a new one, having been discussed in a general way early on in TM research [56]. What could be different is the concept of using totally axially-symmetric coils to create the confining field. One possible tandem configuration within which to immerse the FRM plasma would be based on a combination of the "inside out" mirror geometry of Post [57] and the Andrioletti-Furth geometry [26]. As shown schematically in Fig. (9), the TM

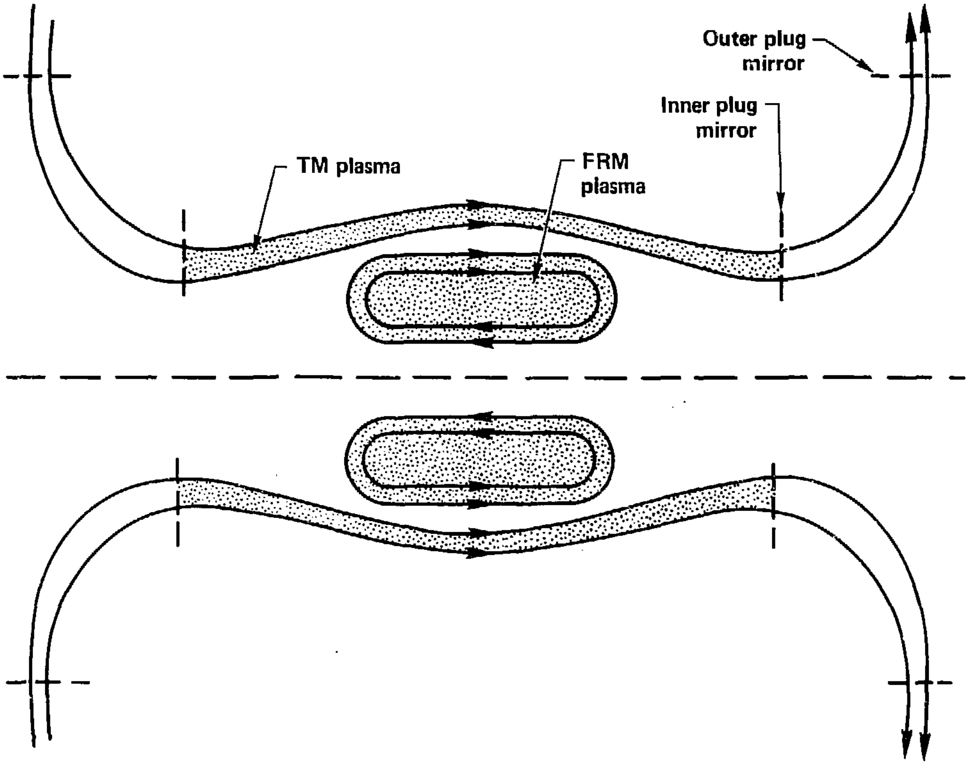

Figure 9. Schematic of the magnetic field configuration of a combined FRM-TM utilizing Furth-Andrioletti axially-symmetric plug mirror cells.

plasma is contained in an annular region surrounding the FRM plasma. The TM plasma would fulfill two functions:

- Provide a stable bounding plasma that could suppress the m = 1 shifting and tilting modes to which the FRM plasma may be prone.
- Provide a finite-pressure bounding plasma that could help to stabilize "surface" modes of the FRM plasma, over at least a portion of its surface.

The open field lines down the middle of the "inside out" geometry could provide an access for the injection of an FRM plasma from an exterior region in which it could be formed. This latter concept, translation of an FRM plasma after formation, has been demonstrated in the Los Alamos FRC experiment [58], and before in field-reversal experiments at Cornell [59].

Since the annular-shaped TM plasma would not need to be large in volume, the power expended in maintaining it could turn out to be minimal, approaching a surface-to-volume scaling in character.

XI. RECAPITULATION

As was stated at its beginning, this article represents some speculations on some of the ways that the mirror concept might evolve on its course toward practical fusion power. The present approach to mirror fusion, the TM with thermal barriers, almost certainly defines an avenue to that goal. However, the intellectual search for ways to simplify or to reduce the cost of fusion systems needs to be a continued one. Nowadays that search is greatly aided by the fact that the experimental and theoretical base of understanding of the physics of mirror plasma has increased spectacularly in the last few years, owing to the resurgence of interest in the mirror approach stimulated by the tandem mirror.

We have here speculated on the following topics:

- The new interest in axially-symmetric mirror systems,
- Ways to exploit regimes of intermediate Q that lie in the continuum between the simple mirror and the tandem mirror,
- The possible utility of plasma heating techniques in modifying the fusion reactivity and/or the ion energy loss spectrum, in semi-collisional regimes,
- Some possible ways to increase the efficiency of and/or simplify direct converters,
- The possibility of using a tandem mirror plasma to improve the stability of an FRM plasma.

In addition to giving the above examples of areas for mirror investigations, we have also tried to list some general "areas for innovation" from which either new concepts or new insights might come, or from which improvements on the suggestions made here might be derived.

The fact that there seem to be many possible avenues for potentially attractive variations on the mirror theme bespeaks the inherent flexibility of this approach to fusion. Though it was once the bete noir of its existence, the linear, open-ended nature of a mirror system may turn out in the long run to be its greatest strength.

ACKNOWLEDGMENTS

I wish to express my sincere appreciation to A. Mirin and T. Rognlein for their assistance in performing Fokker-Planck and Monte Carlo calculations, and to R. Cohen, T. K. Fowler, and L. D. Pearlstein, for many helpful discussions.

This work was performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract number W-7405-ENG-48.

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