

**SEMILEPTONIC $\Delta S = 1$ DECAYS OF BARYONS
WITH BOOSTED BAGS**

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Abstract: We calculate recoil effects for strangeness-changing semi-leptonic decays of baryons by means of boosted quark mode solutions of the MIT bag model. We consider both the quark and pseudoscalar part of the axial current. We show that the induced scalar form factor f_3 and the "weak electric" form factor g_2 are proportional to the mass difference of the final and initial baryon. Moreover, we show that the quark wave function mismatch, which decreases the conventional vector form factor f_1 , can be compensated by recoil effects. Thus a better agreement with experiment seems to be achieved.

1. Introduction. The knowledge of recoil effects in strangeness-changing ($\Delta S = 1$) semileptonic decays of baryons has been poor. In an analysis of recent experiments [1] it is found a small increase in the conventional vector form factor f_1 compared to the naive SU(3)-value. On the other hand, it has been reported [2] that bag model calculations give a small decrease in f_1 due to mismatch of quark wave functions. One of the purposes of this work is to study how f_1 is influenced by recoil effects. More generally, we want to calculate all six form factors involved using boosted bag model wave functions. Especially, we want to study the induced scalar form factor f_3 and the "weak electric" form factor g_2 which cannot be calculated without including recoil effects. f_3 and g_2 are often assumed to be zero due to CVC and SU(3)-symmetry respectively. With recoil effects properly included, we expect that f_3 and g_2 are proportional to the mass difference ΔM of the initial and final baryon.

A proper treatment of recoil effects have always been regarded as a problem within the bag model because the model does not obey translation invariance. And it has been thought that to boost a bag might lead to even more difficulties. On the other hand, a quark mode wave function in the (MIT) bag model is a solution of the free Dirac equation inside the bag. [3]. Performing a Lorentz boost to this solution yields a solution of the Dirac equation in the new frame. It can therefore be argued that this boosted solution for a quark confined in a moving cavity is as physical as the standard static MIT bag solution ^{*1}. Some fundamental problems occurring when bags are boosted are to some extent discussed in recent papers by Guichon [4], and Betz and Goldflam [5].

Boosted bag model wave functions were first used by Picek and Tadić [6], and later by other authors [4,5,7,8] to study recoil effects on the magnetic moment μ_p of the proton. Axial form factors have also been discussed [5,9]. The ~30% increase in μ_p [6,7] due to the "spin precession effect" was later shown [4,5,8] to be numerically more than compensated by the "retardation effect" due to the Lorentz transformation of coordinates. It should be emphasized that the basic problem of boosting a hadron with three quarks inside is not solved. However, we think we have an acceptable model to deal with recoil effects for momentum transfers relevant to semileptonic decays.

^{*1} However, the boosted solution is not an eigenstate of the Dirac Hamiltonian in the new frame. This is natural because the static MIT bag solution is not an eigenstate of (zero) momentum.

To describe the induced pseudoscalar form factor g_3 and the axial current in general, we use the "hybrid chiral bag model" [10] with non-zero pseudoscalar mass [11,12]. To have a non-zero mass is particularly important for $\Delta S = 1$ processes, where the pseudoscalar field represents a kaon.

In this letter we will give mainly the qualitative results of our work. Further details will be postponed to a later and more complete paper.

2. Currents and form factors. The matrix elements of the vector and axial current operators between baryon states (- hereafter called currents on baryon level -) are given by the well known expressions [13]:

$$\langle B' | j_{\mu}^V | B \rangle \equiv J_{\mu}^V = \bar{u}(B') \{ \gamma_{\mu} f_1 - i \frac{f_2}{M+M'} \sigma_{\mu\nu} k^{\nu} + \frac{f_3}{M+M'} k_{\mu} \} u(B), \quad (1a)$$

$$\langle B' | j_{\mu}^A | B \rangle \equiv J_{\mu}^A = \bar{u}(B') \{ \gamma_{\mu} g_1 - i \frac{g_2}{M+M'} \sigma_{\mu\nu} k^{\nu} + \frac{g_3}{M+M'} k_{\mu} \} \gamma_5 u(B), \quad (1b)$$

where the form factors $f_1 \dots g_3$ (- which are Lorentz scalars) are functions of the squared momentum transfer k^2 . $M(M')$ and $u(B)(u(B'))$ are the mass and the Dirac spinor of baryon $B(B')$. In β -decay, $f_1(0) = 1$, $g_1(0) \equiv g_A \cong 1.25$ and g_3 is dominated by the pion pole:

$$\frac{1}{2M} g_3 \cong \frac{G_{\pi N} f_{\pi}}{k^2 - \mu^2}, \quad (2)$$

where $G_{\pi N}$, f_{π} and μ are the pion-nucleon coupling constant, pion decay constant and pion mass respectively. For $\Delta M \equiv M - M' = 0$ it is consistent to put f_3 and $g_2 = 0$ in (1), but not in general.

The currents at quark level are

$$j_{\mu}^{V,Q}(x) = \overline{\psi}_q(x) \gamma_{\mu} \psi_q(x), \quad j_{\mu}^{A,Q}(x) = \overline{\psi}_q(x) \gamma_{\mu} \gamma_5 \psi_q(x). \quad (3)$$

In our "boosted bag model" we represent in- and out-going quark fields $\psi_q(x)$ and $\overline{\psi}_q(x)$ by boosted bag model wave functions (see section 3). The total axial current is

$$j_{\mu}^A = j_{\mu}^{A,Q} + j_{\mu}^{A,\phi}; \quad j_{\mu}^{A,\phi} = - f_{\phi} \partial_{\mu} \phi, \quad (4)$$

where $J_{\mu}^{A,Q}$ is given by (3) and $f_{\phi}(f_{\pi}, f_k)$ is the decay constant of $\phi(=\pi, k)$. ϕ satisfies the free Klein-Gordon equation outside the bag and is otherwise determined by the requirement of continuous axial current through the bag surface. ϕ will in general also contribute to the vector current. This term give a non-negligible pionic contribution to magnetic moments [14]. But for $\Delta S = 1$ processes when $\phi = K$, this term gives a negligible contribution.

As pointed out by Guichon [4], we have to identify the baryon currents in (1) with the Fourier transform at time $t = 0$ of the corresponding current operator calculated between bag states representing the baryons B and B':

$$J_{\mu}^V = \langle B' | \int d^3 \underline{x} e^{-i \underline{k} \cdot \underline{x}} j_{\mu}^V(0, \underline{x}) | B \rangle, \quad (5)$$

and similarly for the axial vector case. Choosing some definite frame (- the Breit-frame, say), we reduce (1a) and the right hand side of (5) (- with j_{μ}^V given by (3)), to expressions involving two-component rest spinors [5,6]. Comparing (1b) and (5), we will then obtain three linear equations which can be solved to obtain f_1 , f_2 and f_3 in terms of bag integrals (- see section 5). The same procedure is used for the axial current, which also involves integrals over the pseudoscalar field ϕ outside the bag.

3. Boosted bag solutions. The bag model wave function for a quark q in a baryon at rest is [3]:

$$\psi_q^{(o)}(x^{(o)}) = \psi_q^{(o)}(\underline{x}^{(o)}) e^{-it^{(o)} E_q} ; \quad \psi_q^{(o)}(\underline{x}^{(o)}) = \begin{pmatrix} i u_q^{(o)} \chi_q \\ -v_q^{(o)} \underline{\sigma} \cdot \hat{\underline{x}}^{(o)} \chi_q \end{pmatrix}, \quad (6)$$

where χ_q is the Pauli spinor of the quark q and $x^{(o)} = (t^{(o)}, \underline{x}^{(o)})$ are the coordinates in the rest frame of B. $E_q \equiv \omega_q/R$ is the energy of the quark mode and R is the bag radius. The upper ($u_q^{(o)} \equiv u_q(r^{(o)})$) and lower ($v_q^{(o)}$) components of $\psi_q^{(o)}$ are functions of the radius $r^{(o)} \equiv |\underline{x}^{(o)}|$ only. $\underline{\sigma}$ are the Pauli matrices, and $\hat{\underline{x}}^{(o)} \equiv \underline{x}^{(o)}/r^{(o)}$. The boosted quark wave functions for q inside B and q' inside B' are:

$$\psi_q(x) = \Lambda_B \psi_q^{(o)}(x^{(o)}) , \quad \psi_{q'}(x) = \Lambda_{B'} \psi_{q'}^{(o)}(x^{(1)}) , \quad (7)$$

where $x^{(1)} = (t^{(1)}, \underline{x}^{(1)})$ are the coordinates in the rest frame of B' . $\Lambda_B(\Lambda_{B'})$ is the Dirac boost matrix for a Lorentz transformation from the rest frame of $B(B')$ to the frame where we perform our calculations. For this we choose a Breit frame such that the velocities of the baryons are equal and opposite. Then $\Lambda_B^{-1} = \Lambda_{B'} \equiv \Lambda$, which simplifies our calculations. The coordinate transformation from the rest frame of B to the Breit frame can be written

$$\begin{aligned} \underline{x}^{(0)} &= \underline{x} + (\gamma - 1)(\underline{x} \cdot \hat{\underline{v}})\hat{\underline{v}} + \gamma \underline{v}t, \\ t^{(0)} &= \gamma(t + \underline{v} \cdot \underline{x}) \end{aligned} \quad (8)$$

with $\gamma \equiv (1 - v^2)^{-1/2}$ and $\hat{\underline{v}} = \underline{v}/v = -\hat{\underline{k}}$. The relations between x and $x^{(1)}$ are given by (8) with $\underline{v} \rightarrow -\underline{v}$. We observe that for $t = 0$ in the Breit frame $\underline{x}^{(0)} = \underline{x} + O(v^2)$ and $\underline{x}^{(1)} = \underline{x} + O(v^2)$ which means that the deformation of the bag because of Lorentz contraction is an effect of order v^2 . From (6), (7) and (8) we find

$$j_{\mu}^V(0, \underline{x}) = \{\overline{\psi}_{q'}^{(0)}(\underline{x}^{(1)}) \Lambda_{Y_{\mu}} \Lambda_{\psi_q^{(0)}}(\underline{x}^{(0)})\}_{t=0} \exp\{i \frac{E_q + E_{q'}}{M+M'} \underline{k} \cdot \underline{x}\}, \quad (9)$$

and a similar expression for the axial current. It is important to note that the exponential factor in (9), which is $\exp\{-it^{(0)}E_q + it^{(1)}E_{q'}\}$ at $t = 0$ in the Breit frame, is responsible for the "retardation effect" [4,8]. Combining this exponential factor with the one occurring in the Fourier transform in (5), we obtain $\exp\{-i\underline{k} \cdot \underline{x}\} \rightarrow \exp\{-i\beta \underline{k} \cdot \underline{x}\}$, where

$$\beta = 1 - \left(\frac{E_q + E_{q'}}{M+M'} \right) \quad (10)$$

is the "retardation" factor.

Looking at (9) for the quark transition $q \rightarrow q'W$ in the Breit frame, one may wonder what happens to the "spectator quarks" which does not take part in the weak interaction. For physical reasons they have to be boosted in the same way. Betz and Goldflam [5] have shown that one obtains an overall factor $\sim 1 + O(v^2)$ for each spectator quark.

4. The pseudoscalar field ϕ satisfies

$$(\partial^\nu \partial_\nu + \mu^2)\phi(x) = 0 \quad (11)$$

outside the bag, and

$$-i \oint n^\mu \delta_\mu \phi(x) = n^\mu j_\mu^{A,Q}(x) \quad (12)$$

at the bag surface. To find the time component $j_0^{A,\phi}$ of the pseudoscalar part of the axial current, we need $\partial\phi/\partial t$ at $t = 0$ in the Breit frame. We find $(\partial\phi/\partial t)_{t=0}$ by expanding ϕ and $j_\mu^{A,Q}$ around $t = 0$, using (7), (8), (11) and (12). To determine ϕ we must know the bag normal n_μ in (12) for the boosted case. Intuitively it seems to be an advantage to work in the Breit frame where the deformed bags have the same shape. In the rest frame of B only the bag corresponding to B' is deformed. The bag normal n cannot be chosen freely with respect to the boundary condition for the MIT-bag solutions (6). In the Breit frame these can be written

$$in_B^\mu \gamma_\mu \psi_q(x) = \psi_q(x) \quad , \quad in_{B'}^\mu \gamma_\mu \psi_q(x) = \psi_q(x) \quad , \quad (13)$$

where ψ_q and $\psi_{q'}$ are the boosted solutions in (7), and where $n_B(n_{B'})$ is by definition the four vector which is a Lorentz transformation of $n_B^{(0)} = (0, \hat{x}^{(0)})$ ($n_{B'}^{(1)} = (0, \hat{x}^{(1)})$) in the rest frame of B(B'). From (13) we deduce that:

$$\frac{1}{2} i(n_B + n_{B'})^\mu \overline{\psi_{q'}}(x) \gamma_\mu \psi_q(x) = 0 \quad , \quad (14a)$$

$$\frac{1}{2} i(n_B - n_{B'})^\mu \overline{\psi_{q'}}(x) \gamma_\mu \psi_q(x) = \overline{\psi_{q'}}(x) \psi_q(x) \quad (14b)$$

at the bag surface. (14a) tells us there is no vector current out of the bag if the bag normal for the boosted case is chosen as $n = \frac{1}{2}(n_B + n_{B'})$. Working in the Breit frame, we have $n = (0, \hat{x}) + O(v^2)$. In the rest frame of B, however, the time component of n will be of first order in v .

For $t = 0$ in the Breit frame we find that the pseudoscalar field has the form

$$\begin{aligned} \phi(0, \underline{x}) &= \chi_q^+ \underline{\sigma} \cdot \hat{x} \chi_q \{ \alpha(r) + \beta(r)(\hat{x} \cdot \underline{k}) \} \\ &+ \chi_q^+ \underline{\sigma} \cdot \underline{k} \chi_q \lambda(r) + O(k^2) \quad , \end{aligned} \quad (15)$$

where α , β and λ are radial functions (involving Hankel functions) of the form

$$\frac{e^{-\mu r}}{r^{(1+\ell)}} \sum_{j=0}^{j_{\max}} c_j \cdot (\mu r)^j, \quad (16)$$

where ℓ and j are integers ≥ 0 . $\partial_t \phi|_{t=0}$, which vanish for equal quark masses, have the same form as (15) and (16).

5. Results The quark contributions to the form factor in eq. (1) are given by linear combinations of standard bag integrals:

$$\begin{aligned} \hat{N} &\equiv \int d^3x [u_q, u_q + v_q, v_q], & \hat{\mu} &\equiv \frac{1}{3} (M + M') \int d^3x [u_q, v_q + v_q, u_q] r \\ \hat{g}_A &\equiv \int d^3x [u_q, u_q - \frac{1}{3} v_q, v_q], & \hat{d} &\equiv \frac{1}{3} (M + M') \int d^3x [u_q, v_q - v_q, u_q] r, \end{aligned} \quad (17)$$

where the upper (u_q, u_q) and lower (v_q, v_q) components of the quark wave functions are functions of $r = |\underline{x}|$ only (see (6)). Taking into account $\sim v^2$ boost effects some more integrals will also occur. For $q = q'$, \hat{N} is the normalization integral; $\hat{N} = 1$. For $q = s$ and $q' = d$, \hat{N} is slightly smaller [2], $\hat{N} \approx 0.97$. $\hat{\mu}$, \hat{g}_A and \hat{d} are standard bag integrals involved in calculation of magnetic moment (transition), axial vector coupling, and electric dipole transition moment respectively. The form factors also involve some well-known [13] SU(6)-factors ξ and η given in table 1 (Symbolically $\xi \equiv \langle \frac{1}{2} \lambda \rangle$ and $\eta = \langle \frac{1}{2} \lambda \sigma_3 \rangle$, where λ is the appropriate combination of Gell-Mann SU(3)-matrices). For the vector current we recover the result for the magnetic (transition) moment [4,5,6,7,8]:

$$\mu = \eta \tilde{\mu}; \quad \tilde{\mu} = \beta \hat{\mu} + \hat{g}_A, \quad (18)$$

where β is given by (10) ($\beta = 1$ if "retardation" is omitted [6,7]). We observe that \hat{d} vanishes for equal quark masses. We define a SU(3) mass splitting parameter δ and a "normalized" electric dipole transition quantity \tilde{d} :

$$\delta \equiv \frac{M-M'}{M+M'}, \quad \beta \hat{d} \equiv \tilde{d} \cdot \delta, \quad (19)$$

in order to express form factors in a transparent way.

To find the full expression for $f_i(k^2)$ to first order in k^2 one needs boost effects of order v^2 - which are doubtful according to ref. 4. To avoid complicated details, we write down the results obtained to first order in the boost velocity $v \sim -\frac{k}{E}$:

$$f_1 = \frac{1}{1-\delta^2} [\xi(\hat{N} - \delta^2 \tilde{d}) - \eta \delta^2 \tilde{\mu}], \quad (20a)$$

$$f_2 = \frac{1}{1-\delta^2} [\eta \tilde{\mu} - \xi(\hat{N} - \delta^2 \tilde{d})], \quad (20b)$$

$$f_3 = \frac{\delta}{1-\delta} [\eta \tilde{\mu} - \xi(\hat{N} - \tilde{d})], \quad (20c)$$

where corrections are $\sim O(k^2, \delta^4)$. It is important to note that $f_3 \sim \Delta M$, as expected. Thus we obtain CVC for $\Delta M \rightarrow 0$. The ratios f_3/f_1 and f_1/ξ will vary with the actual process. Using ξ and η from table 1, and reasonable bag parameters [2,3] we find that f_3/f_1 lies between $\cong -0.4$ and $\cong 0$, and f_1/ξ between $\cong 1.02$ and $\cong 0.98$ for different $\Delta S = 1$ processes. (Note that $\tilde{d}/\tilde{\mu} \cong -1.2$). Thus the wavefunction mismatch can be compensated by $\sim \Delta M^2$ recoil effects, and we seem to be in better agreement with experiment [1] than the static bag model calculation [2], which gives $f_1/\xi = \hat{N} \cong 0.97$. A more rigorous discussion of recoil effects in f_1 must be postponed to a more detailed paper.

As expected, the induced axial form factor g_3 is dominated by the pseudoscalar pole. This turns out to be the case also for g_2 . We obtain the axial form factors:

$$g_1 = \eta [\hat{g}_A + \frac{1}{3} \cdot U \cdot \frac{(1+\mu R)}{(1+\mu R + \frac{1}{2}\mu^2 R^2)}] + O(k^2, \delta^2), \quad (21a)$$

$$g_2 = \eta \cdot \delta \cdot \left(\frac{M+M'}{\mu} \right)^2 \cdot U \cdot (\kappa - \varepsilon) + \delta \cdot v_2, \quad (21b)$$

$$g_3 = -\tau \cdot \left(\frac{M+M'}{\mu} \right)^2 \cdot U \cdot \varepsilon + v_3, \quad (21c)$$

where v_2 and v_3 denote non-pole (- i.e. do not behave as $1/\mu^2$) pseudoscalar and quark contributions of same order of magnitude as \hat{g}_A or smaller. U is a quantity which enters integrals over the pseudoscalar field:

$$U \equiv 4\pi R^3 (u_q, u_q)_{r=R} \quad (22)$$

Numerically, $U \cong \frac{3}{2} \xi_A$. For zero quark masses this is exact [10]. ϵ and κ are determined by the volume integrals over $\phi(0, \underline{x})$ and $\partial_t \phi|_{t=0}$ respectively. They are given by:

$$\epsilon = 1 - \frac{2}{3} \frac{(w_q + w_{q'}, -2)}{R(H+H')} \quad (23a)$$

$$R \cdot \Delta M \cdot \kappa = (w_q - w_{q'}) - \frac{2}{3} \frac{1}{R(H+H')} [(w_q - w_{q'})(w_q + w_{q'}) + (\zeta_q - \zeta_{q'})] \quad (23b)$$

where $w_q \equiv R \cdot E_q$ and $\zeta_q \equiv R \cdot m_q$. Numerically $\epsilon \cong 0.7$ and $\kappa \cong 0.3$. We note that $g_2 \sim \Delta M$ as expected. Moreover, we observe that (21c) is compatible with (2). Using different bag model parameters we can estimate the possible variations of form factor ratios, and we find for $\Delta S = 1$ processes ($\mu = m_K$) that $g_2/g_1 \cong -(0.7 \text{ to } 0.9)$ and $g_3/g_1 \cong -(12 \text{ to } 15)$. (Note that these ratios are independent of ξ and η and therefore relatively independent of the particular process).

6. Conclusions: We have obtained two important results: 1) The wave function mismatch which decreases f_1 can be compensated by recoil effects. 2) The form factor f_3 and g_2 are proportional to the mass difference ΔM ; i.e. they are zero in the exact SU(3) limit. Therefore, in spite of the weaknesses of bag model calculations in general, we think our results (18)-(23) obtained in the "boosted bag model" give a reliable description of recoil effects in semileptonic $\Delta S = 1$ decays of baryons.

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Table 1. SU(6) factors for transitions $B \rightarrow B'$.

Transition	$n \rightarrow p$	$\Sigma^+ \rightarrow \Lambda$	$\Lambda \rightarrow p$	$\Sigma^- \rightarrow n$	$\Xi^- \rightarrow \Lambda$	$\Xi^- \rightarrow \Lambda$	$\Xi^0 \rightarrow \Sigma^+$
ξ	1	0	$-\sqrt{3}/2$	-1	$\sqrt{3}/2$	$1/\sqrt{2}$	1
η	5/3	$\sqrt{2/3}$	$-\sqrt{3}/2$	+1/3	$1/\sqrt{6}$	$5/(3\sqrt{2})$	5/3

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