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A CLASSICAL PICTURE OF ANOMALOUS EFFECTS IN A TOKAMAK

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RESEARCH REPORT

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Further communication about this report is to be sent to the Research Information Center, Institute of Plasma Physics, Nagoya University, Nagoya 464, Japan.

Abstract

It is demonstrated that the atomic collisions between plasma ions and a very small amount of neutral particles remaining in a hot plasma plays a very important role for plasma transports and may be an origin of anomalous effects observed in a tokamak such as the diffusion coefficient independent of the field strength, a rapid plasma density increase during gas puffing and current penetration with anomalously high speed in the start-up phase. The Ohm's law derived by Cowling is used for the analysis.

It is widely known that in a tokamak there are many anomalous effects which are not understood by Coulomb collisions between ions and electrons. At the present time all the existing theories seek the origin of the anomalies in some turbulent process.[1]

In this paper we examine whether the atomic collisions between the plasma ions and a small amount of residual neutral particles in a hot tokamak plasma can explain some of the observed anomalous effects. As for such examples we consider the following three typical phenomena for which reliable experimental data are available and theoretical treatments are relatively easy; (1) diffusion coefficient D independent of the magnetic field as the one employed for INTOR study[2]; (2) very rapid plasma density increase in the axis during gas puffing, (3) anomalously high penetration speed of the current in the start-up phase.

Among them elastic collision will become very important in a high temperature plasma because the elastic collision cross-section σ_{cs}^{in} between charged and neutral particles keeps an approximately constant value while the Coulomb scattering cross section σ_{cs}^{c} decreases with the square of the temperature. This suggests that the ion-neutral collision may render a dominant friction force to a plasma even in the case that a very small amount of neutral particles are remained in. Therefore it may be inferred that the internal friction force of a high temperature tokamak may be due to this collision process. In such a case the Ohm's law firstly derived by Cowling[3] may be applicable. In this paper we use SI units except temperatures for which eV is employed.

Here we simply assume that the fluid model can be used and the plasma is composed of electrons, hydrogen ions and hydrogen atoms. Then the momentum balance equations of each component may be written as

$$m_e n_e \frac{d\vec{v}_e}{dt} = 0 = -\vec{\nabla} p_e - e n_e(\vec{E} + \vec{v}_e \times \vec{B})$$

$$+ n_e n_i \lambda_{ei}(\vec{v}_i - \vec{v}_e) + n_e n_n \lambda_{en}(\vec{v}_n - \vec{v}_e) \ , \qquad (1)$$

$$m_i n_i \frac{d\vec{v}_i}{dt} = -\vec{\nabla} p_i + e n_i(\vec{E} + \vec{v}_i \times \vec{B})$$

$$- n_e n_i \lambda_{ei}(\vec{v}_i - \vec{v}_e) - n_i n_n \lambda_{in}(\vec{v}_i - \vec{v}_n) \ , \qquad (2)$$

and

$$m_n n_n \frac{d}{dt} \vec{v}_n = -\vec{\nabla} p_n + n_e n_n \lambda_{en}(\vec{v}_e - \vec{v}_n) + n_i n_n \lambda_{in}(\vec{v}_i - \vec{v}_n) \ , \qquad (3)$$

where $\vec{v}$, $\vec{E}$, $\vec{B}$, p, n and m have the usual meanings and the subscripts e, i, and n denote electron, hydrogen ion and hydrogen atom, respectively, and λ abbreviates average collision rate $\langle\sigma_{cs} v\rangle$ multiplied by the reduced mass of colliding particles. We assume that each particle species has its own temperature T_e, T_i and T_n. For simplicity, however, we put $T = T_i = T_n$. Using the σ_{cs}^{in} given by McDowell;[4] σ_{cs}^{en} by Dücks and Griem;[5] and σ_{cs}^{ei} by Braginskii;[6] we have $\lambda_{in} = 2.5 \times 10^{-41}\ T^{0.3}$, $\lambda_{en} = 1.7 \times 10^{-44}\ T_e^{1/2} \cdot [1 + 22.1/(T_e + 1.3)]$, and $\lambda_{ei} = 2.6 \times 10^{-42} L T_e^{-3/2}$, where L is the Coulomb logarithm. Here, we introduce the average velocity $\vec{v}$ defined by $\vec{v} = (m_i n_i \vec{v}_i + m_n n_n \vec{v}_n)/\rho$, where $\rho = m_i n_i + m_n n_n$, and require charge neutrality (i.e., $n = n_e = n_i$). If $d(\vec{v}_i - \vec{v}_n)/dt$ can be neglected compared with $(\vec{v}_i - \vec{v}_n)/\tau$ which are contained in the friction force, it is possible to obtain the Ohm's law derived by Cowling:

$$\vec{E} + \vec{v} \times \vec{B} = \frac{\vec{j}}{\sigma} - \frac{\xi_n^2}{n n_n \lambda_{in}} (\vec{j} \times \vec{B}) \times \vec{B} + \frac{\xi_n}{n n_n \lambda_{in}} \vec{G} \times \vec{B} + \frac{1}{en} (\vec{j} \times \vec{B} - \vec{\nabla} p_e), \qquad (4)$$

where $1/\sigma = (n\lambda_{ei} + n_n\lambda_{en})/(ne^2)$, $\xi_n = m_n n_n/\rho$ and $\vec{G} = \xi_n\vec{\nabla}(P_i + P_e) - (1 - \xi_n)\vec{\nabla}p_n$. Using the pressure balance relation $\vec{j} \times \vec{B} = \vec{\nabla}p$ where $p = p_e + p_i + p_n$, we find Eq.4 turns into

$$\vec{E} + \vec{v} \times \vec{B} = \frac{\vec{j}}{\sigma} - \frac{\xi_n}{nn_n\lambda_{in}}\vec{\nabla}p_n \times \vec{B} + \frac{1}{en}\vec{\nabla}(p_i + p_n) \ . \qquad (5)$$

As may be seen Eq.5 adds the 2nd term in the RHS to usual 2-fluid Ohm's law,[6] so that if neutral particles exist in a plasma their pressure gradient induces additional plasma flow in the direction opposite to the gradient.

In the case that a plasma is in a quasi steady state phase, we assume each pressure component is so distributed as to satisfy

$$\frac{1}{p_e}\vec{\nabla}p_e = \frac{1}{p_i}\vec{\nabla}p_i = \frac{1}{p_n}\vec{\nabla}p_n \ , \qquad (6)$$

so that in such a case we easily have $\vec{\nabla}p_n = f_n/(1 + T_e/T)\cdot\vec{\nabla}p$ where f_n is defined by $f_n = n_n/n$ and is assumed to be $f_n << 1$. Then Eq.5 yields the folloiwng two forms similar to Polovin and Cherkasova[7]:

$$\vec{E} + \vec{v} \times \vec{B} = \frac{\vec{j}}{\sigma} - \frac{\vec{\nabla}p \times \vec{B}}{\sigma_i B^2} + \frac{\vec{\nabla}p}{2en} \qquad (7\text{-a})$$

$$= \frac{\vec{j}}{\sigma} - \frac{(\vec{j} \times \vec{B}) \times \vec{B}}{\sigma_i B^2} + \frac{\vec{j} \times \vec{B}}{2en} \ , \qquad (7\text{-b})$$

where $\sigma_i = (1 + T_e/T)n^2\lambda_{in}/(f_n B^2)$.

For evaluating the diffusion coefficient D of a tokamak we assume that a tokamak is approximated by a straight cylinder and all the parameters are only functions of a radial position r. In such a system, D can be defined by $nv_r = - Ddn/dr$ if a plasma is in a steady state. As a result, we are able to have D from Eq.7-a and find that D is composed of the sum of the two terms D_e and D_i written as $D_e = kT_e n(1 + T/T_e)g/(B^2\sigma)$ and $D_i = kTf_n g/(n\lambda_{in})$, where $g = 1 + [d\ln(T_e + T)/dr]/[d\ln n/dr]$. We can see that D_e is the usual classical diffusion coefficient resulting from electron's collision with ions and neutrals while D_i arises by the contamination of neutral particles. In the case of a tokamak we believe that the neoclassical effect may be taken into account if D_e is multiplied by the factor $(1 + q^2)$ where q is the safety factor. Therefore, using λ given before we can estimate D (= $D_e + D_i$) for a tokamak by

$$D_e = (2.4 \times 10^{-22} T_e^{-1/2} + 1.0 \times 10^{-25} T_e^{3/2} f_n)(1 + \frac{T}{T_e})\frac{ng(1 + q^2)}{B^2} \quad (8)$$

and

$$D_i = 6.4 \times 10^{21} \frac{f_n T^{0.7} g}{n} . \quad (9)$$

The data of PLT by Strachan et al.[8] may be utilized for D_e and D_i. Here we duplicate the necessary data for D_e and D_i: a (minor radius) = 0.4 m, R_0 (major radius) = 1.35 m, B_t (toroidal field) = 2.5 T, I_p (plasma current) = 400 kA, q(a) = 3.7, q(0) ≅ 1, $n_n(0) \cong 1 \times 10^{14}$ m^{-3}, $n_e(0) \cong 2.6 \times 10^{19}$ m^{-3}, $f_n(0) \cong 4 \times 10^{-6}$, $T_e(v) \cong 1800$ eV, T(0) ≅ 750 eV, $n_n(a) \cong 2 \times 10^{15}$ m^{-3}, $n_e(a) \cong 3.5 \times 10^{18}$ m^{-3}, $f_n(a) \cong 5.4 \times 10^{-4}$, $T_e(a) \cong 62$ eV, T(a) ≅ 45 eV. In addition to these we use g(0) ≅ 7.8 and g(a) ≅ 1.8 deduced from profiles in Fig.3 of Ref.8. Plugging all the necessary data into Eqs.8 and 9, we find $D_e(0) \cong 6.2 \times 10^{-4}$ m^2/s, $D_e(a) \cong 1.1 \times 10^{-3}$ m^{-2}/s, $D_i(0) \cong 0.79$ m^2/s and $D_i(a) \cong 27$ m^2/s. Therefore, it can be said that the diffusion due to neoclassical effect is smaller by 3 to 4 orders of magnitude than the one caused by ion-neutral collisions, which eventually shows that even a ppm range contamination of neutral particles may be enough to spoil the neoclassical particle transport. In order to compare Eq.9 with the empirical law[2] $D = 1.25 \times 10^{19}/n_e$, we inserted the observed values of f_n, T and g to have $D(0) = 2.1 \times 10^{19}/n_e$ and $D(a) = 9.4 \times 10^{19}/n_e$. As is seen the agreement between the two is satisfactory.

The smallness of D_e compared with D_i in a tokamak discharge suggests that the 1st term in the RHS of Eqs.5 and 7 can be neglected. In such a case we know from Eq.5 that the radial component of $\vec{v}$ is given by

$$v_r = -\frac{\xi_n}{nn_n\lambda_{in}}\frac{dp_n}{dr} , \qquad (10)$$

which suggests that if $dp_n/dr > 0$ the plasma does not diffuse out but pinches in. We consider that during gas puffing $dp_n/dr > 0$ may be realized because neutral particle density becomes higher on the plasma boundary. Actually, we can read in Ref.8 the data just after gas puffing having the values $n_n(0) \cong 5 \times 10^{13}\ m^{-3}$, $T(0) \cong 600$ eV, $n_n(a) \cong 7 \times 10^{15}\ m^{-3}$, and $T(a) \cong 20$ eV, by which $p_n(a) > p_n(0)$ may be well expected. Provided that p_n is assumed to have a profile given by $p_n(x)/p_n(0) = 1 + [p_n(a)/p_n(0) - 1]x^c$ where $x = r/a$ and c is a constant, Eq.10 can be reduced to

$$v_r = - 6.4 \times 10^{21}[n_n(a)T(a) - n_n(0)T(0)]\ \frac{cx^{c-1}}{n^2T^{0.3}a} . \qquad (11)$$

In order to compare v_r from Eq.11 with experimental results, we also refer Strachan et al.[8] because they found that the experimental inward velocity v_r^{exp} may be estimated by the relation $v_r^{exp} = v^A - (D/n)\cdot dn/dr$ where v^A is the artificial inward velocity to satisfy the particle conservation law. Careful reading of the curves in Figs.3 and 13 of Ref.8 gives $v_r^{exp}(a) = -10$ m/s, and $v_r^{exp}(0.1\ m) = -0.45$ m/s. We find c = 0.3 in Eq.11 gives the best fit to v_r^{exp} and yields $v_r(a) = -9.9$ m/s and $v_r(0.1\ m) = -0.16$ m/s. As may be seen v_r given by Eq.11 predicts v_r^{exp} fairly well. Therefore we would say that the present model may well explain the rapid density increase during gas puffing.

Lastly we discuss the anomalously high speed current penetration observed in the start-up phase of a tokamak where the effective ion charge Z_{eff}^{fd} deduced from the field diffusion is always greater than the one Z_{eff}^{sp} due to spectroscopic measurements. A typical such data may be the one presented by Hawryluk et al.[9] using PLT. They showed that, in the case of the deuterium discharge, the start-up phase at 10-20 ms records $Z_{eff}^{fd} = 32$ in spite of $Z_{eff}^{sp} < 2$. For understanding this paradoxical effect we need the field diffusion equation under the present model. We employ the slab model for simplicity and introduce the (x, η, ξ) coordinate system fixed on the field line as shown in Fig.1. Since we choose the direction of the field line for ξ axis, $\vec{B}$ is characterized by B and the angle α between ξ and y axis. For deriving the field diffusion equation we utilize Eq.7-b. In the case of Ref.9 the deuterium discharge at 10-20 ms has the following parameters: $T_e \simeq 250$ eV, $n_e \simeq 1 \times 10^{19} m^{-3}$, $B_t = 3.2$ T and $I_p = 280$ kA. If we assume $T_e \simeq T$ and $f_n \simeq 10^{-5}$, these data yield $\sigma/\sigma_i \simeq 2 \times 10^4$, which means η-component of Eq.7-b can be approximated by $E_\eta - v_r B = j_\eta/\sigma_i$ and ξ component is written as $E_\xi = j_\xi/\sigma$. Here we write $v_r = v_{ro}(1 + \varepsilon)$ where $\varepsilon = v_{r1}/v_{ro}$, v_{ro} is the diffusion velocity in a steady state plasma, and v_{r1} is a small pinching velocity appearing due to small imbalance between the field and the plasma pressure. This implies that for ε the solution of the equation of motion $\rho d\vec{v}_1/dt = -\vec{\nabla}p + \vec{j} \times \vec{B}$ is necessary. However, we proceed the study considering ε to be a small parameter. Then, Eq.7-b, the Ampere's law, and the ξ-component of the Faraday's law yield the diffusion equation,

$$\frac{\partial}{\partial t} B = -\frac{B}{(\mu_o a\sigma)}(\alpha')^2 - (\frac{\varepsilon}{\mu_o a^2 \sigma_i} B')' , \qquad (12)$$

where ' denotes the derivative with respect to x. Since the ratio γ of the 1st and the 2nd term of the RHS of Eq.(12) becomes $\gamma \cong \sigma_i/(\sigma\varepsilon A^2 q^2)$ in order of magnitude where A (= R_o/a) is the aspect ratio, we have $\gamma = 9.8 \times 10^{-8}/\varepsilon$ for the data of Ref.9. Therefore, if ε is not smaller than 10^{-6}, Eq.(12) may be simplified into

$$\frac{\partial}{\partial t} B = (\frac{Z_{eff}^{fd}}{\mu_o a^2 \sigma} B')' , \qquad (13)$$

where we put $Z_{eff}^{fd} = -\varepsilon\sigma/\sigma_i$ in correspondence to the deffinition of Z_{eff}^{fd}. It is noted that Eq.13 does not include σ, so that the field diffusion may proceed without any correlation to the electron conductivity to which Z_{eff}^{sp} is directly connected. Therefore we would say that if Eq.13 holds it solves apparent paradox that Z_{eff}^{fd} is much greater than Z_{eff}^{sp}. Since Z_{eff}^{fd} involves ε, it is not possible to evaluate it unless the equation of motion is solved. However, we can estimate Z_{eff}^{fd} experimentally after the method by Hawryluk et al.[9] where Z_{eff}^{fd} is determined so as the power flow $V_{loop} \cdot I_p$ into a tokamak is equibrated to the ohmic heating power w_{ohm} and the field increasing power w_{mag} through the iterative process using the field diffusion equation. Although $w_{ohm} = j_\xi^2/\sigma + j_\eta^2/\sigma_i$ generally holds, we may use $w_{ohm} = j_\xi^2/\sigma$ in the present case, because the relation $j_\eta/j_\xi \cong -\beta_p/(2Aq)$, where β_p is the poloidal beta value, suggests $(j_\eta/j_\xi)^2 \cdot \sigma/\sigma_i \cong 3 \times 10^{-2}$ for the data in Ref.9. However, the neoclassical picture claims $w_{ohm} = Z_{eff}^{fd} \cdot j_\xi^2/\sigma$, so that it overestimates w_{ohm} by Z_{eff}^{fd} times. In this sense we believe Hawryluk et al. underestimate

Z_{eff}^{fd}. The value of Z_{eff}^{fd} given in Ref.9 is always greater than unity, so that we can infer that Eq.13 may be valid for the start-up phase of any tokamak because the criteria $\epsilon \geq 10^{-6}$ for Eq.14 is equivalent to $Z_{eff}^{fd} \geq 0.02$.

A physical picture of the particle transport across the magnetic field is considered. Using Eqs.3 and 5 we have $n_n v_{nr} = -p_n'/(an\lambda_{in})$, $nv_{ir} = -np'/(a\sigma B^2)$ and $nv_r = nv_{ir} + n_n v_{nr}$. A rough estimation after the PLT data on the axis gives $(n_n v_{nr})/(nv_{ir}) \cong 3 \times 10^3$, which shows that almost all the portion of nv_r is carried by $n_n v_{nr}$. It is noted that, since $nv_r \cong -p_n'/(an\lambda_{in})$ holds, the particle transport becomes the pressure diffusion process of the atoms through the clouds of ions. We infer, however, that the ions are also lost approximately with the same rate to $n_n v_{nr}$ because of frequent charge exchange. McDowell's results[4] show that the mean free path ℓ^{in} and ℓ^{ex} for elastic collision and charge exchange of the atom are related each other by $\ell^{in} \cong 0.6\ \ell^{ex}$ in the energy range below 10^4 eV, so that if the conditions $\ell^{ex} \ll a$ and $\ell^{ex} \ll \ell^{i}$ (mean free path of the atom for ionization by electron and ion impact) are satisfied, charge exchange renews 60 % of the constituent of n_n during an atom passing the distance ℓ^{in} although n_n itself keeps an constant value. Therefore, in such a case we can expect (ion loss rate by charge exchange) $\cong n_n v_{nr}/a$, and $T_n = T_i$ or $p_n = f_n p_i$. Actually the PLT data and the curve by Freeman and Jones[10] give $\ell^{in} \cong 4.7$ cm, $\ell^{ex} \cong 7.8$ cm and $\ell^{i} \cong 31$ cm in the plasma core, and $\ell^{in} \cong 20$ cm, $\ell^{ex} \cong 34$ cm and $\ell^{i} \cong 48$ cm on the edge. Considering a = 40 cm we would say that the conditions are satisfied in the core but somewhat crucial on the edge.

The discussions above suggest the validity of Eq.3 even if it were applied to an almost fully ionized tokamak plasma because the Maxwellian distribution of neutral particles with $T_n = T_i$ may be established by charge exchange regardless of self collision process. Much detailed elucidations on Eq.3 using kinetic model may be preferable.

In conclusion it is demonstrated that the Ohm's law by Cowling can be the basic principle to explain anomalous effects in a tokamak predicting that even a ppm range of residual neutral particles is enough to destroy the usual classical or neoclassical behavior of a plasma.

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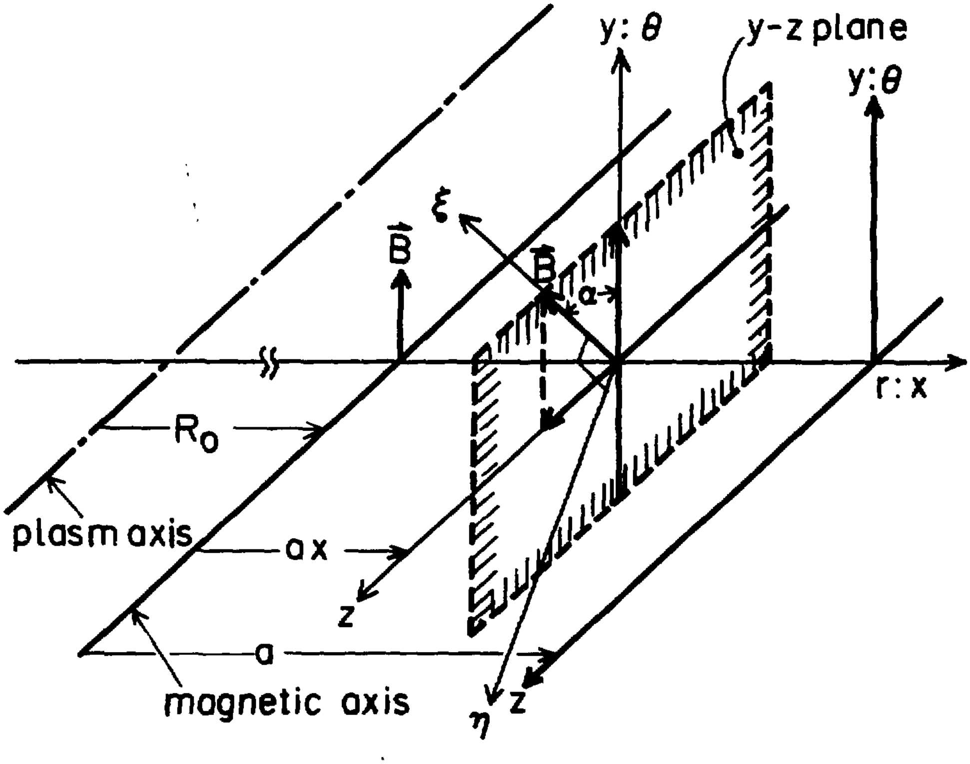
Figure Caption

Fig.1 Two slab coordinate systems (x, y, z) system is fixed on the laboratory from while (x, η, ξ) system on the magnetic field line.



y: θ
y-z plane
y:θ
ξ
$\vec{B}$
$\vec{B}$
α
r : x
R_0
plasm axis
ax
z
a
magnetic axis
η
z