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SOME REMARKABLE SPIN PHYSICS WITH MONOPOLES AND FERMIONS†

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SOME REMARKABLE SPIN PHYSICS WITH MONOPOLES AND FERMIONS†

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Abstract - This review will cover the following topics, which follow the historical evolution of the subject: the Dirac monopole; the Kazama-Yang Goldhaber problem in electron-monopole scattering; the 't Hooft-Polyakov monopole and spin from isospin; the Rubakov analysis; monopole catalysis of proton decay "the Rubakov-Callan effect"; the role of exactly solvable 2-dimensional QFT's and finally observable consequences.

I am going to talk about an intriguing aspect of the gauge theories we believe might describe the fundamental interactions of nature. As most of us have learnt, such gauge theories have magnetic monopole soliton-like states. The latter have remarkable properties as regards spin and angular momentum. In fact, a charge boson interacting with a monopole carrying one Dirac unit of magnetic charge forms a system with half integer total angular momentum, i.e. it behaves like a fermion. The catalogue of the phenomena of this kind associated with monopoles seems limitless. A detailed study of fermions interacting with heavy monopoles leads to some fascinating lessons in non-perturbative spin physics, through exactly solvable two-dimensional quantum field theories. The latter is the first case of such theories being directly relevant to observable elementary particle physics. This is in contrast to condensed matter physics, where there are numerous such examples.

About three years ago, Valerie Rubakov and independently Curt Callan predicted that a remarkable phenomenon would occur if a monopole of a grand unified theory passed through nuclear matter, namely it would cause protons to decay into an electron and pions at rates more characteristic of strong interactions, rather than the "ultra-weak" forces of the grand unified theory, which are known to give rise to baryon number violation. In the last part of this talk I will sketch how these deductions were made and discuss the consequences, but let me begin by describing briefly how Dirac introduced the monopole concept in the 1930's.

THE DIRAC MONOPOLE

In 1931 Dirac /1/ asked the following question. Since Maxwell's equations are almost symmetrical with respect to the exchange of the electric and magnetic fields, is it not possible to have a point magnetic pole analogue of the electron? This would make the Maxwell system completely symmetrical (see Table I). However, he met with a very important technical problem, the solution of which pointed to the monopole having some topological character.

Table I - Equations governing a Dirac magnetic monopole

Modified Maxwell's equations (in units $\hbar = c = 1$)

$$\vec{\nabla} \cdot \vec{E} = 4\pi \, e \, \delta^{(3)}(\vec{r})$$
$$\vec{\nabla} \cdot \vec{B} = 4\pi \, m \, \delta^{(3)}(\vec{r})$$
$$\vec{\nabla} \times E + \frac{\partial \vec{B}}{\partial t} = - 4\pi \vec{J}_m$$
$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = 4\pi \, \vec{J}_e$$

Magnetic field: $B^i = m \, n^i/r^2 - 2\pi \, m \, \delta(x)\,\delta(y)$

Vector potential with string singularity: $A^i = m \, \frac{1 - \cos\theta}{r \cos\theta} \, n^i$

(The string singularity along the z-axis feeds in $4\pi m$ units of magnetic flux.)

Dirac quantization condition:
(In order that the string singularity be invisible in an Aharanov experiment)

$$e \oint_{\substack{\text{around}\\\text{string}}} A \cdot d\ell = 2\pi \, ,$$

i.e. $eg = 1/2$ $(g = m/4\pi)$.

The difficulty I am referring to is that once one tries to write down the corresponding vector potential A_μ (required for a gauge invariant description of QED), then one finds that it must have a string singularity. One can think of this string as a solenoid feeding in a so to speak the monopole's $4\pi m$ units of magnetic flux. In order that this fictitious solenoid be unobservable, for example in an Aharanov electron interference experiment, the line integral of the vector potential around it should be a multiplet of 2π. This has the immediate consequence that the electric and magnetic charges are quantized according to $eg = 1/2$. Notice that the line integral maps the U(1) gauge group around a circle (i.e. in homotopy theory this is denoted by $\pi_1(U(1)) = Z = \{0,1,2,3,...\}$) and this is what I meant above by "the monopole has a topological character".

THE KAŽAMA-YANG-GOLDHABER PROBLEM IN ELECTRON-MONOPOLE SCATTERING

Another unforeseen feature of a Dirac monopole emerges when one considers an electron scattering off it. The total angular momentum is made up of three pieces:

$$\text{i.e.} \quad \vec{J} = \vec{L} + \vec{S} + \vec{T} \quad ,$$

where $\vec{L}$ is the usual orbital piece, $\vec{S} = 1/2\,\sigma\hbar$ is the spin of the electron, while the extra piece is due to the interaction of the charge of the electron with the monopole magnetic field. This is given by

$$\vec{T} = eg \, \hbar \, \frac{\vec{r}}{r} \quad .$$

By virtue of the Dirac condition $eg = 1/2$, T also corresponds to half a unit of the angular momentum. Thus the lowest value of the total angular momentum J can take is zero, when the two spin 1/2 pieces exactly compensate one another ($L = 0$ wave) or together compensate an $L = 1$ orbital angular momentum. However, Kazama, Yang and Goldhaber /2/ pointed out a problem. If one inspects $\vec{T}$, one notices that its sign depends on the direction $\vec{r}$, i.e. it changes sign if $\vec{r}$ points in instead of out. This means as an electron passes the core (say in the s-wave), then in order to conserve angular momentum, either

$$e \to -e \qquad \text{(charge exchange, i.e. T flip)}$$

or

$$S_Z \to -S_Z \qquad \text{(helicity flip, i.e. S flip)}$$

i.e. there appears to be an ambiguity.

The problem can be viewed in another way. If one examines the Hamiltonian of the system, namely:

$$H = \vec{\alpha}\cdot\left(\vec{\nabla} - \frac{g(1-\cos\theta)}{r\cos\theta}\,\vec{n}\right) + \beta m$$

then as it stands it does not give rise to a deep self-adjoint Hamiltonian scattering problem. It has to be modified or equivalently one must supplement the problem with a special boundary condition at $r = 0$. However, there is an arbitrariness in doing this, which can be characterized by a phase angle θ.

To be explicit, let us recall some work of Kazama and Yang /3/ on the partial wave analysis of the equation

$$H\psi = E\psi \quad .$$

By virtue of the fact that the total angular momentum, which commutes with H, has two spin 1/2 pieces, the orbital series for a monopole will, in fact, run over integer values

$$\text{i.e.} \qquad L = |\hat{q}| - 1/2, \quad |\hat{q}| + 1/2, \quad |\hat{q}| + 3/2, \quad \ldots \quad ,$$

where $\hat{q} = eg\,\hat{n}$ with $eg = 1/2$ for a Dirac monopole. The relevant harmonics are of the Jacob and Wick type or equivalently those proposed by Yang (see Ref. /3/ and references therein), namely $Y_{\ell,m,q}(\theta,\varphi)$. Let us concentrate on the lowest partial wave, since only this sector has the interesting physics, due to the fact that the corresponding wave function is non-vanishing at $r = 0$. We can write the s-wave function in the form

$$\psi_0 = \frac{1}{r}\begin{bmatrix} u(r)\ \eta \\ v(r)\ \eta \end{bmatrix} \quad ,$$

where η_α is a two-component spinor and is a solution of the equation

$$\vec{\sigma}\cdot\hat{n}\,\eta = \hat{q}\,\eta \quad .$$

The Dirac equation in two component form $\chi_\alpha = \begin{bmatrix} u \\ v \end{bmatrix}$ can be written in the form

$$\hat{H}\chi = E\chi \quad ,$$

where $\hat{H} = i\hat{q}\gamma_5\frac{d}{dr} + \gamma_0 m$.

The solutions are thus classified according to the eigenvalues $v = \pm 1$ of $\hat{q}\gamma_5$ and take the form (for $m = 0$):

$$\chi_v = e^{iE(t+vr)}\begin{pmatrix} 1 \\ v \end{pmatrix} \ ; \quad v = \pm \quad .$$

This means that they decompose into either in-movers or out-movers, in principle with no outgoing wave for a given incoming wave. One has to impose a boundary condition to relate the in and out sectors. In order to figure out what would be the appropriate one, let us consider the self-adjointness of $\hat{H}$, i.e. consider

$$\Delta = (\bar{\chi},\hat{H}\chi) - (\widehat{H\chi},\chi)$$
$$= \bar{\chi}\, \hat{q}\gamma_5\chi\Big|_0^\infty = \bar{\chi}_+(0)\, \chi_+(0) - \bar{\chi}_-(0)\, \chi_-(0)$$

Thus only if we choose $\chi_+(0) = e^{i\theta}\, \chi_-(0)$ do we obtain a self-adjoint Hamiltonian and consequently well defined physics. However, the physics depends in an important way on the value of this arbitrary angle θ as has been demonstrated /4/ by the work of Yang, Wu and others. For example the fermion ground state (i.e. the Fermi sea around the monopole) varies as we change θ. Further the possible bound state spectrum depends on its value.

When there are more than one flavour of charged fermion in the problem, then more complex boundary conditions emerge and consequently quite a different physics.

't HOOFT-POLYAKOV MONOPOLE AND SPIN FROM ISOSPIN

Certain non-Abelian gauge theories in their Higgs can have monopole configurations of their gauge fields /5/. These are topologically excited stable soliton solutions of the equations of motion, which carry one or two Dirac units of magnetic charge, depending on the gauge group. The origin of this phenomenon lies in the self-interactions of the gauge fields. The underlying non-linear system of equations admit soliton solutions, which are characterized by a topological charge. This discovery in 1974 caused considerable excitement, because of the success non-Abelian gauge theories were having, by providing a description of all elementary interactions in nature.

To describe such a monopole, let us consider an SU(2) gauge theory, couple to a charged scalar field, chosen to be in a Higgs phase, in which only a long range U(1) gauge field remains manifest at long wave lengths. The set of equations governing this system is given in Table II, together with 't Hooft-Polyakov monopole solution.

If the vacuum expectation value of the scalar field in the Higgs phase $<\phi^a> = \phi_0 n^a$ takes on the hedgehog configuration shown in Fig.2, in which the isospin points along the radial direction everywhere, then a stable monopole configuration of the gauge fields sits at the centre. For those familiar with homotopy theory in mathematics, this situation is characterized by $\Pi_2(SU(2)/U(1))$ which is equal to $\Pi_1(U(1)) = Z$, i.e. this is a mapping of a surface of a sphere in real space into the SU(2) group, with winding number $Z = 1,2,3,\ldots$ The latter defines the stability class of the monopole, because it cannot be unwound by quantum fluctuations of the system. The U(1) gauge group that is left manifest a long way from the centre, corresponds to rotations around the radial directions as indicated in Fig.2. Referring to the set of equations governing this system in Table II, we notice unlike a Dirac monopole the vector potential does not have a string singularity. This is due to the presence of the scalar field, which in a sense replaces the Dirac string.

By virtue of the fact that these non-Abelian monopoles are non-singular at $r = 0$ and have finite core radius, there should be a unique solutions to the Kazama-Yang-Goldhaber problem, even as we let the core radius (i.e. 1/mass) go to zero.

Let us consider N_D SU(2) doublets of left-handed two-component fermion fields, coupled to the SU(2) monopole system we have just described. The f ion part of the action is given by:

$$S_{fermion} = \int d^4x \sum_{k=1}^{N_D} \bar{\psi}_L^{(k)}(x)\, i\sigma^\mu[\partial_\mu + A_\mu^a \tau^a + a_\mu^a \tau^a]\psi_L^{(k)}(x) \quad ,$$

where $\sigma^\mu = (\mathbb{1},\sigma^i)$; $A_\mu^a(x)$ is the monopole static potential given in Table II and $a_\mu(x)$ is the quantum fluctuation of this configuration. If we do a partial wave analysis of fermions scattering off the monopole centred at $r = 0$, then we see that the total angular momentum is made up of the three pieces:

$$\vec{J} = \vec{L} + \vec{S} + \vec{T} \quad ,$$

where L is the usual orbital piece, $S = 1/2\ \sigma\hat{n}$ is the fermion spin and $T = 1/2\ \tau\hat{n}$ is half a unit of angular momentum coming from the interaction of the fermion charge with the monopoles static magnetic field. Thus the monopole promotes isospin to spin. This means, just like to the Dirac case and unlike the usual central potential problem, in which the lowest partial wave is $J = 1/2$, for a monopole the lowest partial wave is a $J = 0$, i.e. s-wave. The latter is built up either by $L = 0$, $S + T = 0$ or $L = 1$, $S + T = 1$. This is reflected in the two independent functions $g(r)$ and $h(r)$ in the following decomposition of the s-wave field /6/

$$\psi(r)_{ai} = 1/\sqrt{8\pi}[g(r)\epsilon_{ai} + h(r)\ i(\sigma\cdot n)_{\alpha\beta}\,\epsilon_{\beta\alpha}]/r \quad ,$$

where a refers to spin and i to isospin. If we collect the functions g and h in the form of a two component spinor $f = \begin{bmatrix} g \\ h \end{bmatrix}$, then $f(t,r)$ satisfies a free Dirac equation on the half space (t,r) $(0 < r < \infty)$, namely

$$[\sigma_3 \frac{\partial}{\partial t} + i\ \sigma_1 \frac{\partial}{\partial r}]\ f(t,r) = 0 \quad ,$$

where so far we have only taken into account the monopoles static field. Thus we appear to have a free two-dimensional fermions, except for a special boundary condition at the monopole core $r = 0$. The latter is given by the solution to the classical scattering problem and if we express each fermion doublet in terms of an upper component of charge $Q = +1$ and a lower component of charge $Q = -1$, the boundary condition corresponds to pure charge exchange $Q \to -Q$. If we stay within the one particle scattering system, then this charge $2e$ must go somewhere. It has been known for some time that a 't Hooft-Polyakov monopole has also a dyon degree of freedom, which when excited corresponds to a monopole with both electric and magnetic charge. However, the energy required to excite this degree of freedom is of order $10^{14}\ m_p$. On the other hand, in order to conserve probability in the s-wave sector some process must take place. Clearly the answer is particle creation occurs and one needs a full quantum treatment of the problem.

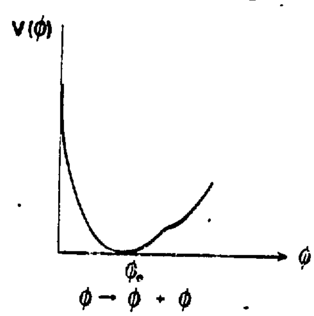

Fig.1 - Potential in Higgs phase

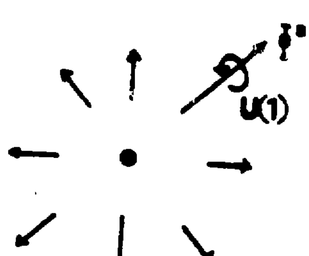

Fig.2 - Hedgehog configuration of Higgs field around a monopole

Table II - The 't Hooft-Polyakov monopole system

Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^{a}_{\mu\nu} F^{a\mu\nu} + \frac{1}{2} \left| D^{\mu}_{ab} \phi^{b} \right|^{2} - V(\phi^{a}\phi^{a}) \quad ,$$

where

$$F^{a}_{\mu\nu} = \frac{\partial}{\partial X^{\mu}} A^{a}{}_{\nu} - \frac{\partial}{\partial X^{\nu}} A^{a}{}_{\mu} + 2e \, \epsilon_{abc} A^{b}{}_{\mu} A^{c}{}_{\nu}$$

$$D^{\mu}_{ab} = \delta_{ab} \frac{\partial}{\partial X_{\mu}} + 2e \, \epsilon_{abc} A^{c,\mu}$$

$$V(\phi^{2}) = \frac{\lambda}{4}(\phi^{2} - \phi^{2}{}_{0})^{2}$$

Classical equation of motion

$$B^{a}{}_{i} = D^{ab}_{i} \phi^{b}$$

't Hooft-Polyakov monopole solution

$$\phi^{a}(x) = \frac{n^{a}}{r} \phi_{0}$$

$$A^{a}_{i}(x) = \frac{1}{2e} \epsilon_{aij} \frac{n^{j}}{r} \qquad (A^{a}_{0} = 0)$$

The monopole's magnetic field

$$B^{em}_{i} = \frac{1}{2e} \frac{n_{i}}{r^{2}}$$

Magnetic charge

$$g = \frac{1}{4\pi} \int d^{3}r \; \nabla \cdot B^{em}$$

i.e. e.g. = 1/2 (the Dirac condition)

THE RUBAKOV ANALYSIS

In order to study this Rubakov noted /7/ that the quantum electro-dynamics of s-wave fermions is governed by the following action

$$S = \int_{-\infty}^{\infty} dt \int_{0}^{\infty} dr \left[2\pi r^{2}/e \; E \cdot E + \sum_{k=1}^{N_D} \bar{f}^{(k)} i\gamma \cdot D \, f^{(k)} \right] \quad ,$$

where $x_{\mu} = (t,r)$ and $\gamma_{\mu} = (\sigma_{3}, i\sigma_{1})$. The b.c. at $r = 0$ is $(1+\hat{\gamma}_{5})f(0) = 0$; $\hat{\gamma}_{5} = i\sigma_{2}$. The Rubakov system is in fact exactly integrable and is very similar to the 2-dimensional QED model of Schwinger. They differ in two essential respects. Firstly for the monopole the fermions live on a half space with a special boundary condition at $r = 0$ and secondly the dimensional coupling constant of the Schwinger model is replaced by e/r^2, which becomes indefinitely large as we approach $r = 0$. These differences are the ones that are responsible for the unexpected new physics.

The system is solved by simply making a rotation of the fermion field so it becomes a free field, i.e.

$$f(x) = \exp[i\alpha(x) + \sigma_2\beta(x)]f_0(x) \quad ,$$

where the functions $\alpha(x)$ and $\beta(x)$ are chosen to cancel the gauge interaction. The appropriate choice is $a_\mu(x) = \epsilon_{\mu\nu}\partial^\nu\alpha(x) + \partial_\mu\beta(x)$, where the second term is simply a gauge choice. Thus the only non-trivial dynamical variable is $\alpha(x)$. The action splits into two parts $S = S(\alpha) + S(f_0)$, where $S(f_0)$ is the action of free fermions and $S(\alpha)$ is given by

$$S[\alpha] = \int_{\infty}^{\infty} dt \int_0^{\infty} dr \,\{ 4\pi r^2/e(\Box\, \alpha)^2 + \frac{N_D}{2\pi}\, \alpha \Box \alpha) \quad ,$$

where $\Box = \frac{\partial}{\partial t^2} - \frac{\partial^2}{\partial r^2}$.

The fact that this action is Gaussian means that the system is integrable and can be studied non-perturbatively. In fact, the underlying differential equations can be solved exactly and consequently all fermion Green's functions explicitly computed. By these means Rubakov was able to demonstrate that the pairing or condensate parameter $< f_1(r)f_2(r)> \sim r^{-1}$ for the case of two fermion doublets. This corresponds to an anomaly in the fermion number current $J_\mu^F = \sum_{k=1}^{N_D} \bar{f}^{(k)}\gamma_\mu f^{(k)}$, namely as

$$\partial^\mu J_\mu^F = \frac{N_D}{2\pi} E(x)$$
$$= \frac{N_D}{2\pi} \Box\, \alpha(x) \quad .$$

Integrating this equation gives the change in fermion number, namely

$$\Delta n_F = \frac{N_D}{2\pi}\int dt\; dr \Box\, \alpha \quad .$$

On can show that the above condensates correspond to instanton-like configurations in the monopole's U(1) radial quantum gauge field. This in turn corresponds to the monopole being in a superposition of states with different fermion number characterized by multiples of N_D.

The condensate

$$< f_\pm^{(1)}(r)\; f_\pm^{(2)}(r) \ldots f_\pm^{(N_D)}(r) > \sim t^{-N_D/2}$$

correlates fermions with the same helicities and is associated with virtual fermion number violating processes occurring in the vacuum around the monopole. These have probabilities which fall off very slowly as we move away from the core.

To see the implication of the above phenomenon, let us consider the case of a monopole in the SU(5) grand unified theory of Georgi and Glashow, which is the minimal one that contains the SU(3) x SU(2) x U(1) gauge theories of strong, weak and electro-magnetic interactions. The lightest generation of fermions i.e. e, ν, u_1, u_2, u_3, d_1, d_2, d_3 form the following SU(5) multiplets

$$5_L = \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ e^- \\ \nu \end{pmatrix}_L \begin{matrix} \} SU(3) \\ \\ \} SU(2) \end{matrix} \qquad 10_L = \begin{pmatrix} 0 & \bar{u}_3 & \bar{u}_2 & u_1 & d_1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & u_2 & d_2 \\ -\bar{u}_2 & -\bar{u}_1 & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & e^+ & 0 \end{pmatrix}_L ,$$

where we have indicated the SU(3), SU(2) and lepto-quark transitions, respectively. The corresponding gauge bosons form a 5 x 5 matrix which, in addition to the 8 strong interaction massless bosons, the 3 heavy weak interaction W and Z bosons and the photon,contain 12 very heavy lepto-quark bosons.The latter for example can turn a d-quark into an electron. This enlarged gauge symmetry only becomes manifest at mass scales of 10^{15} m_p, at which all the interactions are said to be unified into a single matrix gauge field, which is coupled to a real scalar field ϕ in the same 24 representation. The latter has a very large vacuum expectation value, which loosely speaking breaks the SU(5) symmetry to the observed low energy SU(3) $\otimes$ SU(2) $\otimes$ U(1) symmetry of nature. The only effect of the heavy gauge bosons is to very occasionally cause a nucleon to decay and to give rise to monopoles as relics of the very early universe. The latter arise from a particular lepto-quark subgroup, e.g. in the space $(\bar{d}_3, e^-)$ as indicated above, finding itself in the topologically excited state. In this case the light fermions form the following 4 SU(2) monopole doublets:

$$\begin{pmatrix} \bar{d}_3 \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} e^+ \\ d_3 \end{pmatrix}_L \quad \begin{pmatrix} \bar{u}_1 \\ \bar{u}_2 \end{pmatrix}_L \quad \begin{pmatrix} \bar{u}_2 \\ -\bar{u}_1 \end{pmatrix}_L \quad .$$

A remarkable feature of the monopole, which is not yet fully understood is that it breaks the $SU(3)_c$ strong interaction gauge symmetry to $SU(2)_c \otimes U(1)_{Y_c}$. In fact, the s-wave fermions experience the following $SU(2)_c \otimes U(1)_{Y_c} \otimes U(1)_Q$ gauge interactions, in which the $SU(2)_c$ interactions turn to the two u-quark doublets into one another. The last $U(1)_Q$ factor is the monopoles electromagnetism and it is made up of the ordinary electric charge generator Q_{em} plus the $SU(3)_c$ strong interaction hypercharge Y_c. If we only take into account this $U(1)_Q$ interactions, then one can simply repeat Rubakov's analysis with $N_D = 4$ and discover the baryon number violating condensates $< u_1 u_2 d_3 e^- > \sim r^{-6}$ surrounding the core of the monopole. This corresponds to the virtual process $p \rightarrow e^-$. The first attempt to take into account the other interactions,made by Callan /8/, in which he replaced the above gauge group by $U(1)_{I_{3c}} \times U(1)_{Y_c} \times U(1)_Q$ and used the so called bosonization transformation confirmed that the catalysis process will occur unhindered and that the monopole can be considered to be in a state of indefinite baryon number. Subsequently, Rubakov, Nahm and myself /9/ showed that one can treat the $SU(2)_c$ 2-dimensional field theory and we also confirmed that the above additional interactions do not in principle switch off the effect. However, they do impose some important selection rules, which could in principle have considerable consequences once we include the heavier fermion generations observed in nature /10/.

Very recently Nahm and I realized that the 2-dimensional SU(2) gauge theory above is actually also exactly solvable as far as its relevance to the s-wave fermion-monopole dyamics is concerned. The reason is an $SU(2)_2$ gauge group has a hidden global symmetry under $u \rightarrow a\,u + b\,\epsilon\,u^*$, with $|a|^2 + |b|^2 = 1$, described by currents satisying the conservation conditions

$$\partial^\mu J^a_\mu(x) = \epsilon^{\mu\nu} \partial_\nu J^a_\mu(x) = 0$$

and the Kac-Moody algebra

$$[J^a(x), J^b(y)] = \epsilon_{abc} J^c(x)\delta(x-y) + k\, \delta^{ab}\, \delta'(x-y)/4\pi$$

which has central charge $k = 1$. Such algebraic systems have been extensively studied in recent years by mathematicians and from their representation theory we know that we are dealing with a unique dynamical system, which can be equivalently described by the following 2-dimensional non-linear sigma model

$$S = \int d^2x\, F^2/4\, \{dg(x)/dx\, dg^{-1}(x)/dx\} + \frac{k}{2\pi}\int_0^1 dt \int d^2x\, \varepsilon_{\mu\nu}\{ g^{-1}\dot{g}g^{-1}dg/dx_\mu g^{-1}dg/dx_\nu\} ,$$

where $F^2 = 1/2\pi$ and the matrix field $g(x)^{ij}$ is an element of a global $SU(2) \otimes SU(2)$ symmetry group. The above fermionic currents are described equivalently by $J = g^{-1}(x)dg(x)/dx$. In the case of free fermions, this non-Abelian generalization of bosonization was recently pointed out by Witten /11/. However, Polyakov and Wiegmann /12/ have given arguments that indicate that the above non-linear sigma model represents a wider class of fermion theories and is exactly solvable. Thus the full SU(5) monopole-s-wave fermion action separates according to

$$S = S[\alpha] + S[e,d] + S_{u\bar{u}}[g] ,$$

where $S[\alpha]$ is the Rubakov action; $S[e,d]$ is essential for the free electron d_3 quark action (i.e. there are no instanton-like effects in their remaining U(1) interactions); $S_{u\bar{u}}[g]$ is a 2-dimensional non-linear sigma model action describing the di-u-quark zero mass $SU(2)_c$ singlet bound states, which interact with the monopoles core.

The basic currents satisfy the conservation condition

$$\partial^+ J_-^{\,a} - 2\,\varepsilon_{ab3}\, a^+ J_-^{\,b} = \frac{1}{\pi}\Box\,\alpha\,\delta^{a3} \quad \text{for the } u\bar{u} \text{ system}$$

and

$$\partial^+ J_-^{\,e,d} = \frac{1}{\pi}\Box\,\alpha \qquad \text{for the } e,d \text{ system} .$$

These anomalous conservation conditions can be solved in a factorized way and the exact solvability of the above actions can be used to compute arbitrary massless s-wave fermion Green functions in the case of an SU(5) monopole [10].

SUMMARY

If one tries to embed the observed $SU(3)_{QCD} \otimes [SU(2) \otimes U(1)]_{electroweak}$ system of forces in some semi-simple grand unified gauge group, e.g. SU(5), SO(10) or SU(16), which has no elementary U(1) to begin with, then inevitably these unified theories have magnetic monopole configurations of the gauge field, which are topologically stable. If these theories have anything to do with the observed elementary particle spectrum, it is clear that this higher gauge symmetry can only become manifest at mass scales $M \sim 10^{16}$ GeV. The corresponding monopole state has a mass of order M/e and can be thought of as a dense coherent state of the very heavy gauge boson quanta. The radius of such a system is very tiny indeed namely less than 10^{-30} cm., in fact one could even contemplate monopoles, which sit inside their Schwarzchild radius, so they would in addition be black holes. Leaving the latter possibility aside, for most practical purposes one could imagine that this tiny non-Abelian monopole would in the laboratory look like a Dirac monopole. This is certainly true as regards its long range electro-magnetic force, however at typical number scales they behave in quite a different way. As Rubakov pointed out in 1981 these monopoles will catalyze proton decays as they pass through nuclear matter, at rates typical of strong interactions, rather than the ultra-weak effects associated with the massive gauge boson, which are responsible for ordinary spontaneous proton decay. The latter causes a proton to decay only once every 10^{32} years or so. Of course, a monopole is a very unlikely configuration to occur in the first place. However, once it exists, it has bottled up in its core a system, which violates baryon and lepton conservation and this profoundly changes the structure of the fermion sea around its core for some distance. We have tried to depict this situation in Fig.3.

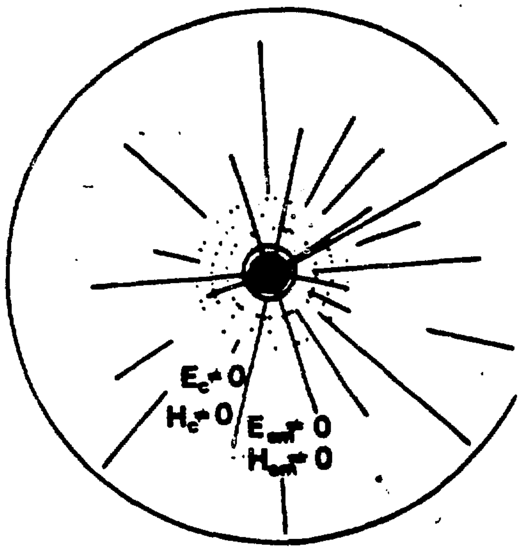

Fig.3 depicts a monopole.

Needless to say due to the Rubakov-Callan effect, an abundant source of these virtually indestructible objects will have some quite astonishing consequences, not to mention a mind boggling new source of nuclear energy. Let me end this talk by mentioning the limits on the flux of monopoles in the universe that have been obtained on the basis of the Rubakov cross-section.

$$\sigma_{\text{Rubakov}} \approx \pi(v/c)^{-2} \text{ millibarns} .$$

The first kind of limit comes from the giant proton decay detectors. If a magnetic monopole of the type we have been considering above passes through such a chamber, then every 10 cm to 10 meters, depending on the above cross-section, there will be an induced proton decay, corresponding to one of the following interactions:

1). $p + M \to M + e^+ + \text{pions}$

2). $P + M \to M + \bar{P} + e^+e^- + \text{pions}$

The signature being suggested here is shown in Fig.4 and all the proton decay experiments have put limits on the monopole flux [13], close to the so-called Parker bound, namely $< 10^{-15}$ cm^{-2} sr^{-1} sec^{-1}. The latter corresponds to the maximum flux of magnetic monopoles, that the observed galactic magnetic fields can tolerate before being quenched. These limits are summarized in Fig.5 taken from the IBM proton decay experiment [14].

Other limits come from astrophysical observations. By virtue of the Rubakov-Callan effect objects like neutron stars and white dwarfs and are very efficient monopole detectors. The essential point is that the monopoles captured by the gravitational fields of these objects, can cause them to considerably heat up and radiate. From the observation of radiation from known neutron stars and white dwarfs, limits of between 10^{-8} and 10^{-10} smaller than the Parker bound have been obtained [15]. If correct, this means that monopoles are very rare indeed and unlikely to be observed on earth.

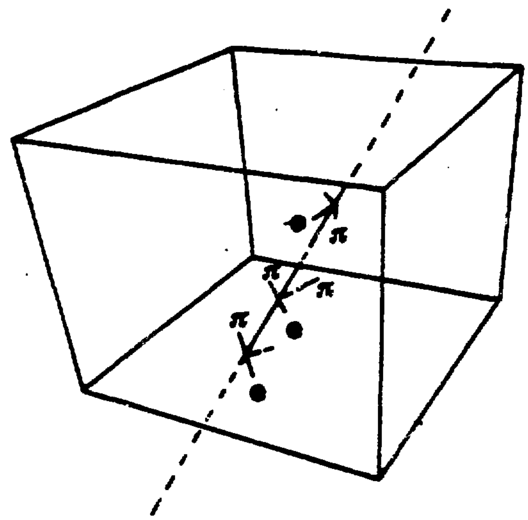

Fig.4 - A schematic drawing of a monopole passing through a giant proton decay detector

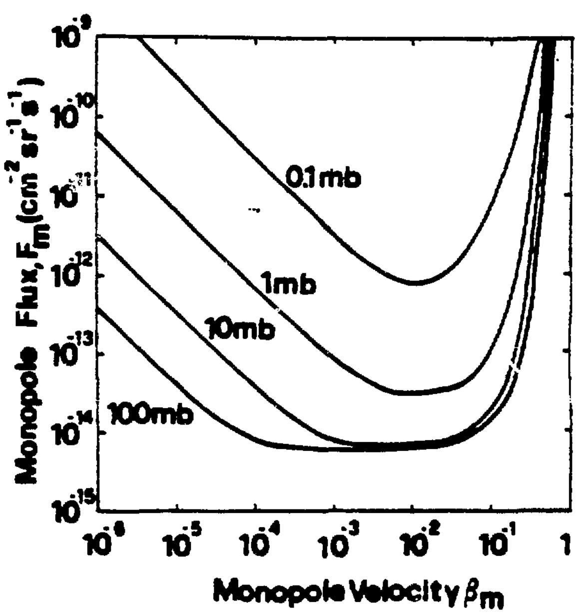

Fig.5 - Upper limits on the monopole flux as function of its velocity $\beta_m = v/c$ obtained by looking for multiple interactions in the IBM.

Let me end by saying that the phenomenon we have just discussed is one of the most fascinating examples of spin physics, in which we saw that a 4-dimensional problem in elementary particle physics can be reduced to an exactly solvable 2-dimensional QFT problem. The latter enables us to predict some really remarkable new phenomena.

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