

ISTITUTO NAZIONALE DI FISICA NUCLEARE

IT 8500280

Sezione di Catania

INFN/AE-82/7  
17 Giugno 1982

M. Pavšič and E. Recami: CHARGE CONJUGATION AND  
INTERNAL SPACE-TIME SYMMETRIES

CHARGE CONJUGATION AND INTERNAL SPACE-TIME SYMMETRIES<sup>(x)</sup>

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ABSTRACT.

We adopt the relativistic framework in which fundamental particles are regarded as extended objects. Then, we show that the (geometrical) operation which reflects the internal space-time of a particle is equivalent to the operation C which inverts the sign of all its additive charges.

In the present paper we critically comment on the discrete transformations of Minkowski space-time, namely on the effects of space-reflection  $\mathcal{P}$  and time-reversal  $\mathcal{T}$ , by exploiting some results contained in previous papers<sup>(1-3)</sup>. Our aim is to show the connection between the internal discrete transformations<sup>(2)</sup> and the charge-conjugation operator C. We assume fundamental particles to be extended objects, as many theoretical observations suggest to be the case, at least in relativistic theories<sup>(4)</sup>.

First of all, let us recall that an inversion  $\mathcal{I}_{ABC}^{(n)}$  of the axes  $x^A, x^B, x^C, \dots$ , in a n-dimensional space  $M_n$  is equivalent to an appropriate  $180^\circ$ -rotation<sup>(+)</sup>  $\mathcal{R}_{ABC\dots}^{(m)}$  in the hyperplane  $(x^A, x^B, x^C, \dots, x^n)$  of the m-dimensional space  $M_m$  with  $m \geq n$ . If the number

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(x) Work partially supported by CNR and MPI.

(o) On leave of absence from J. Stefan Institute, F. Kardelj University, Ljubljana, Yugoslavia (permanent address).

(+) When M is Minkowskian, "rotation" will mean pseudo-rotation.

k of the inverted axes  $x^A, x^B, x^C, \dots$ , is even, then it may be  $m = n$ ; but if k is odd, then  $m \geq n+1$ . In particular, the total inversion (of all axes) in a n-dimensional space  $E_n$  corresponds to a rotation either in  $E_n$  (if n is even) or in a (n+1)-dimensional space  $E_{n+1}$  (if n+1 is even).

For instance, in the 2-dimensional plane the effect of the inversion  $\mathcal{I}_x(2)$  (i. e.,  $x \rightarrow -x$ , whilst  $y \rightarrow y$ ) is equivalent to the effect of the  $180^\circ$ -rotation  $\mathcal{R}_{xz}(3)$  in three dimensions around the y-axis:

$$\mathcal{I}_x(2) = \mathcal{R}_{xz}(3) \Big|_{E_2} \quad (1)$$

where the subscript  $E_2$  means that eq. (1) is true as far as we confine ourselves to the effect of its r. h. s. into the initial 2-dimensional space.

In Minkowski space there are the following discrete transformations<sup>(x)</sup> (we adopt the notation  $x^\mu \equiv (x^0, x^1, x^2, x^3) \equiv (t, x, y, z)$ ):

$$\begin{aligned} \text{space-reflection:} \quad \mathcal{I}_1(4) &\equiv \mathcal{P} && \text{(inversion of } x^1 \text{);} \\ \text{time-reversal:} \quad \mathcal{I}_0(4) &\equiv \mathcal{T} && \text{(inversion of } x^0 \text{).} \end{aligned}$$

The product  $\mathcal{P}\mathcal{T} = \mathcal{T}\mathcal{P}$  is equivalent to the  $180^\circ$ -(pseudo)rotation in  $M_4$ :

$$\mathcal{T}\mathcal{P} \equiv \mathcal{I}_0(4) \mathcal{I}_1(4) = \mathcal{I}_{01}(4) = \mathcal{R}_{01}(4). \quad (2)$$

Though the product  $\mathcal{P}\mathcal{T}$  can be considered as a rotation in  $M_4$ , of course neither  $\mathcal{P}$  nor  $\mathcal{T}$  alone can be replaced by any rotation in  $M_4$ . However, if instead of the 4-dimensional space  $M_4$  we consider the 5-dimensional space  $M_5$ , so that an event e is described by the five coordinates

$$e: \quad x^A \equiv (x^0, x^1, x^2, x^3, x^4), \quad e \in M_5,$$

then the effect in  $M_4$  of the reflection  $\mathcal{P}$  is equal to the effect in  $M_5$  of the  $180^\circ$ -rotation around the space  $(x^0, x^2, x^3)$ , i. e. of the  $180^\circ$ -rotation of  $M_5$  "in the plane"  $(x^1, x^4)$ :

$$\mathcal{P}x^\mu \equiv \mathcal{I}_1(4)x^\mu = \mathcal{R}_{14}(5)x^A \Big|_{M_4}; \quad \begin{matrix} \mu = 0, 1, 2, 3 \\ A = 0, 1, 2, 3, 4 \end{matrix} \quad (3)$$

and the effect of the time-reversal  $\mathcal{T}$  is equal to the  $180^\circ$ -rotation of  $M_5$  in the plane  $(x^0, x^4)$ :

(x) The inversion in  $M_4$  of one of the space-axes  $x^1, x^2$  or  $x^3$ , e. g.  $\mathcal{I}_1(4)$ , is called space-reflection. Applying, after  $\mathcal{I}_1(4)$ , also the  $180^\circ$ -rotation in the plane  $(x^2, x^3)$  is equivalent to the inversion  $\mathcal{I}_{123}(4)$  of all the three space-axes  $x^1, x^2$  and  $x^3$ :  $\mathcal{I}_{123}(4) = \mathcal{I}_1(4)\mathcal{R}_{23}(4)$ .

$$\mathcal{F}_x^\mu \equiv \mathcal{F}_0(4)x^\mu = \mathcal{R}_{04}(5)x^A \Big|_{M_4}. \quad (4)$$

The subscript  $M_4$  means that, after having performed the rotation, we take account only of the events in  $M_4 \subset M_5$ .

At this point let us stress that, if the considered space-time  $M_4$  contains a particle  $a$ , we are going to assume that: (i) particle  $a$  is - as we already mentioned - an extended object<sup>(4)</sup>, so that the interior of its world-tube is a finite portion of space-time; (ii) our operations  $\mathcal{P}$ ,  $\mathcal{F}$  are to be regarded as acting both on the external space-time and on the internal one ("internal" and "external" with respect to the particle world-tube). Since the ordinary parity and time-reversal act on the contrary only on the external space-time, to avoid possible confusion we shall call  $P \equiv \mathcal{P}_E$  the ordinary space-reflection and  $T \equiv \mathcal{F}_E$  the ordinary time-reversal (E = external)<sup>(2)</sup>.

Then, we shall show - among the others - that the charge-conjugation C is equal to the product  $\mathcal{P}_I \mathcal{F}_I$ , where  $\mathcal{P}_I$  is the internal space-reflection and  $\mathcal{F}_I$  is the internal time-reversal (I = internal)<sup>(2)</sup>. So that  $\mathcal{P}\mathcal{F} = \text{CPT}$ .

Let us explicitly write:

$$\mathcal{P} = \mathcal{P}_E \mathcal{P}_I = \mathcal{P}_I \mathcal{P}_E; \quad (5)$$

$$\mathcal{F} = \mathcal{F}_E \mathcal{F}_I = \mathcal{F}_I \mathcal{F}_E, \quad (6)$$

where  $\mathcal{P}_I$  ( $\mathcal{F}_I$ ) is the internal,  $\mathcal{P}_E$  ( $\mathcal{F}_E$ ) the external, and  $\mathcal{P}$  ( $\mathcal{F}$ ) the total space-reflection (time-reversal).

More precisely, the transformations  $\mathcal{P}$ ,  $\mathcal{P}_E$ ,  $\mathcal{P}_I$ ,  $\mathcal{F}$ ,  $\mathcal{F}_E$  and  $\mathcal{F}_I$  can be defined with the aid of the suitable rotations in  $M_5$ . The total space-reflection  $\mathcal{P}$  is defined by eq. (3) and the total time-reversal by eq. (4). See Figs. 1, 2, where quantity  $s^A$  is chosen to be a space-like vector lying inside the particle world-tube<sup>(x)</sup> and orthogonal to the world-tube axis (specified by its unit-vector  $\tau^A$ ). The world-tube lies in the ordinary  $M_4$ .

The internal space reflection  $\mathcal{P}_I$  can be defined as the  $180^\circ$ -rotation in  $M_5$  of the particle world-tube around the space  $\Sigma_p \equiv (x^0, x^2, x^3)$  orthogonal to the plane  $(x^1, x^4)$ : See Figs. 1a. Notice that the space  $\Sigma_p$  around which one has to perform the rotation in  $M_5$  contains the time-axis  $x^0$ . When the particle  $a$  is considered at rest, then the tube axis coincides of course with the time-axis; in such a particular case, therefore,  $\Sigma_p$  contains  $\tau^A$ : See Figs. 1b.

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(x) For simplicity, let us assume the particle  $a$  to be spherical (even if with a non-spherically-symmetric structure).

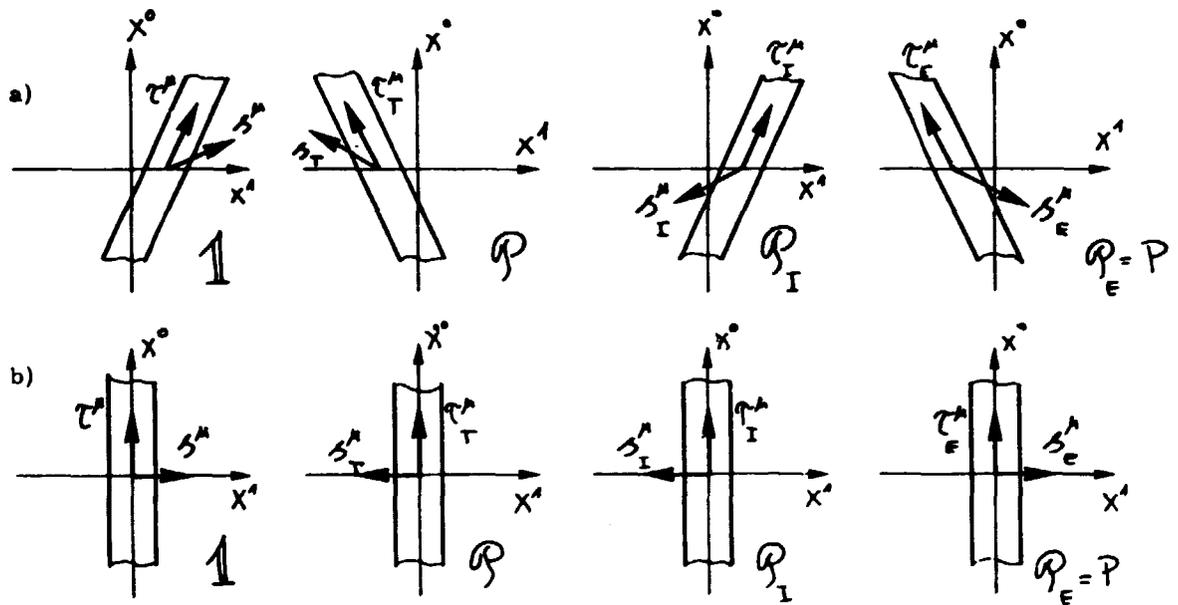


FIG. 1 - The effect of the total space reflection  $\mathcal{P}$ , the internal space reflection  $\mathcal{P}_I$  and the external space reflection  $\mathcal{P}_E$  on the world-tube of a particle. Fig. a refers to a moving particle, and b to the simpler case of a particle at rest. The world-tube is characterized by the time-like 4-vector  $\tau^\mu$  and the space-like 4-vector  $s^\mu$  (see the text). The transformations  $\mathcal{P}$ ,  $\mathcal{P}_I$  and  $\mathcal{P}_E$  change  $\tau^\mu$  into  $\tau_T^\mu$ ,  $\tau_I^\mu$  and  $\tau_E^\mu$ , respectively; and analogously for  $s^\mu$ .

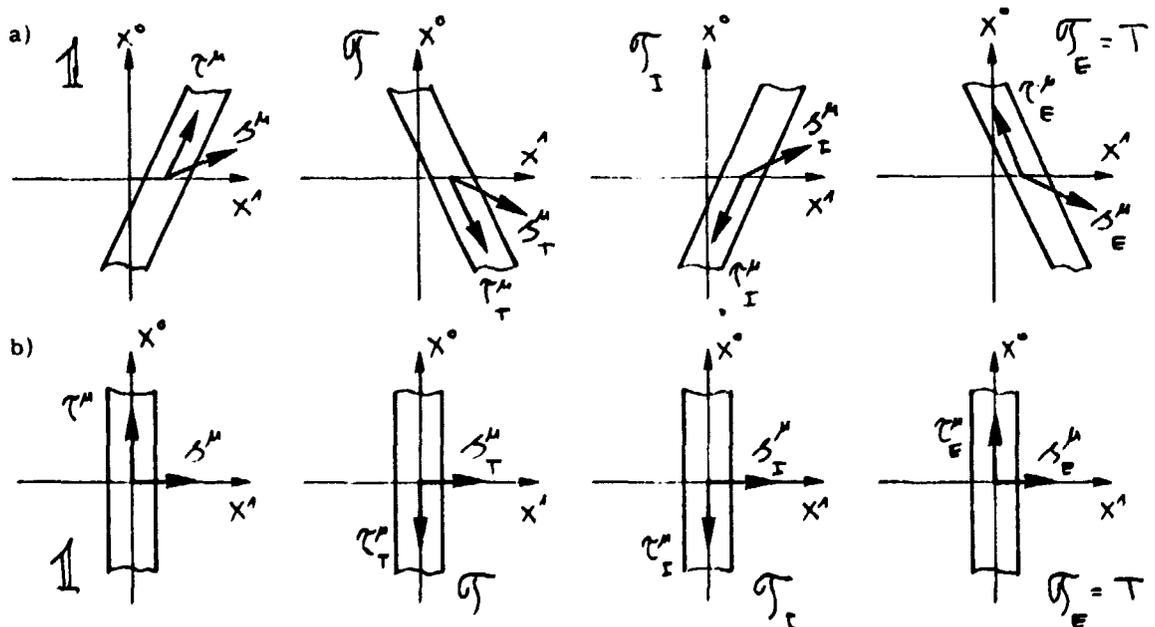


FIG. 2 - The effect of the total time reversal  $\mathcal{T}$ , the internal time reversal  $\mathcal{T}_I$  and the external time reversal  $\mathcal{T}_E$  on the world-tube of a particle. Again, Fig. a refers to a moving particle, and b to the simpler case of a particle at rest. As to  $\tau^\mu$  and  $s^\mu$ , the same notations are used as in Fig. 1.

The internal time reversal  $\mathcal{F}_I$  can be defined as the  $180^\circ$ -rotation of the particle world-tube in  $M_5$  around the space  $\Sigma_T \equiv (x^1, x^2, x^3)$  orthogonal to the plane  $(x^0, x^4)$ : See Figs. 2a. When the particle  $a$  is in particular at rest,  $s^A$  can be chosen so to coincide with the  $x^1$ -axis: See Figs. 2b.

The external space reflection  $\mathcal{P}_E$  in  $M_4$  affects a particle only by reflecting the world-line of its center-of-mass (the position of all other world-lines within the particle world-tube remaining unchanged relatively to the center-of-mass world-line). The external space reflection  $\mathcal{P}_E$  is therefore nothing but the ordinary space-reflection  $P$ :

$$\mathcal{P}_E \equiv P. \quad (7)$$

The external time reversal  $\mathcal{F}_E$  in  $M_4$  is equivalent - with regard to a chosen particle  $a$  - to the operation transforming its velocity  $\vec{v}$  into  $-\vec{v}$  (Figs. 2a), without affecting its internal structure. The external time reversal  $\mathcal{F}_E$  is therefore nothing but the ordinary time reversal  $T$ :

$$\mathcal{F}_E \equiv T. \quad (8)$$

We shall also generalize to the case of extended particles the Stückelberg-Feynman re-interpretation procedure<sup>(5)</sup>.

Let us start by applying (from the active point of view) the total space-time reflection  $\mathcal{P}\mathcal{F}$  to the world-tube  $W$  of a particle  $a$ . We depict  $W$  as consisting in a sheaf of world-lines  $w$  which represent - say - its "constituents" (Fig. 3a); in Fig. 3 - besides the c. m. world-line - we show  $w_1 \equiv A$ ;  $w_2 \equiv B$ . The operation  $\mathcal{P}\mathcal{F} \equiv \mathcal{P}_E \mathcal{F}_E \mathcal{P}_I \mathcal{F}_I$  will transform  $W$  into a new world-tube  $\tilde{W}$  consisting of the transformed world-lines  $\tilde{w}$  (Fig. 3b). The world-tube  $\tilde{W}$  differs from  $W$  in the fact that its world-lines  $\tilde{w}$  point in the opposite time-direction and occupy - with respect to the center-of-mass world-line - the position symmetrical to the corresponding  $w$ .

By applying the Feynman procedure<sup>(5)</sup> each world-line  $\tilde{w}$  transforms into the corresponding world-line  $\bar{w}$  (Fig. 3c). Each world-line  $\bar{w}$  points in the positive time-direction, but represents an anti-"constituent". We now identify the sheaf  $\bar{W}$  of the world-lines  $\bar{w}$  of the "anti-constituents" with the antiparticle  $\bar{a}$ ; and therefore  $\bar{W}$  with the world-tube of  $\bar{a}$ . This identification corresponds to assume that the overall time-direction of a particle  $a$  (or  $\bar{a}$ ) as a whole coincides with the time-direction of its "constituents". Such a procedure is an explicit generalization of Feynman procedure for extended particles.

A preliminary conclusion is that the antiparticle  $\bar{a}$  of  $a$  can be regarded (from the chronological, geometrical point of view) as derived from the reflection of its internal space-time.

Let us repeat what precedes in a more rigorous way, and recall that the Stückelberg-

-Feynman reinterpretation procedure has been recently reformulated into one of the fundamental principles ("Third Postulate") of Special Relativity: See Refs. (3, 1, 2). Let us also recall that Special Relativity can be based<sup>(x)</sup> on the whole proper group  $\mathcal{L}_+$  of both ortho- and anti-chronous Lorentz transformations,  $\mathcal{L}_+ = \mathcal{L}_+^{\uparrow} \cup \mathcal{L}_+^{\downarrow}$ , since a clear physical meaning can be given also to antichronous (i. e. non-orthochronous) Lorentz transformations<sup>(3, 1)</sup>. The central elements of  $\mathcal{L}_+$  are  $(+ \mathbb{1}, - \mathbb{1})$ , where  $\mathbb{1}$  is the identity matrix in four-dimensions. That is to say, in such a formalization of Special Relativity the operation  $- \mathbb{1}$  does represent an actual (even if antichronous) Lorentz transformation, corresponding to the 180° space-time "rotation":

$$\bar{P}\bar{T} = - \mathbb{1} . \quad (9)$$

Notice explicitly that in eq. (9) the operators  $\bar{P}$ ,  $\bar{T}$  have a meaning different from the one of the ordinary space-parity  $P$  and time-reversal  $T$ . Namely, for the very fact that eq. (9) represents a Lorentz transformation, quantities  $\bar{P}$ ,  $\bar{T}$  and  $\bar{P}\bar{T}$  will act not only on the chronotopical space, but also on the "dual" four-momentum space, etc. (This means that  $\bar{T}$ , in particular, when acting on a four-momentum vector, will change also the sign of energy). But let us go back to the mere chronotopical space.

Now, if we apply  $\bar{P}\bar{T} = - \mathbb{1}$  from the active point of view to the world-tube  $W$  in Fig. 3a, we have to rotate it (by 180°, in four dimensions) into  $\tilde{W}$  (Fig. 3b). Such a rotation will effect also a reflection of the internal 3-space of a particle  $a$ , transforming it - among the others - into its mirror image. Analogously, from the passive point of view, if we apply  $\bar{P}\bar{T}$  to the space-time in Fig. 3a, containing also  $W$ , we shall pass to a  $\bar{P}\bar{T}$ -ed frame whose space-time derives from the complete 180°-"rotation" of the initial space-time. Again, this will operate also the reflection of the internal space-time of particle  $a$  (relatively to the new observer).

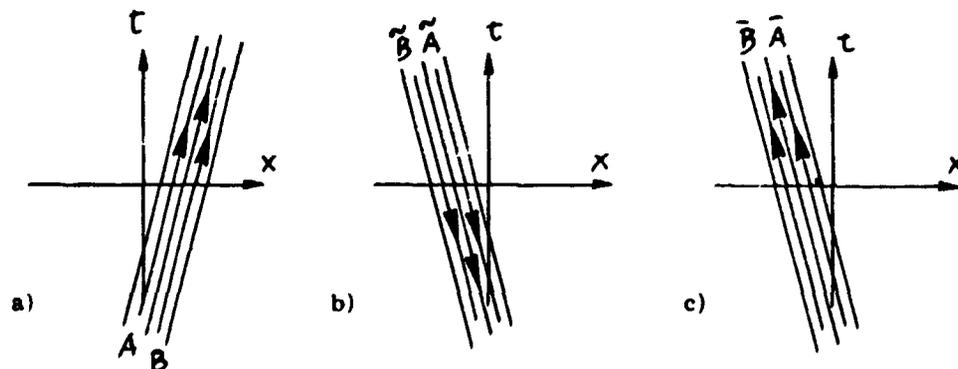


FIG. 3 - Given a world-tube (Fig. a), we show the effect of the (antichronous) Lorentz transformation  $\bar{P}\bar{T} = - \mathbb{1}$  before (Fig. b) and after (Fig. c) the application of the "Reinterpretation Principle"<sup>(5, 1, 2)</sup>. See the text.

(x) C. f. eq. (11) in the following.

Then, we extend the Reinterpretation Principle<sup>(3,1,2)</sup> to the case of extended objects, i. e. we apply it (e. g. within the active point of view) to the world-tube  $\tilde{W}$  of Fig. 3b. The world-tube  $\tilde{W}$  represents an (internally reflected) particle not only going backwards in time, but also carrying negative energy. Therefore applying the Reinterpretation Principle<sup>(6)</sup> will rigorously transform  $\tilde{W}$  into  $\bar{W}$  (Fig. 3c), the anti-world-tube  $\bar{W}$  representing the anti-particle  $\bar{a}$ .

In conclusion, as far as the chronotopical space is concerned, the (antichronous) Lorentz transformation  $\bar{P}\bar{T} \equiv -\Pi$  can be considered as

$$-\Pi \equiv \bar{P}\bar{T} = \mathcal{P}_E \mathcal{T}_E \mathcal{P}_I \mathcal{T}_I = PT \mathcal{P}_I \mathcal{T}_I, \quad (10)$$

so that in particular:

$$\bar{P}\bar{T} = \mathcal{P}\mathcal{T}. \quad (10')$$

At this point we have to recall that in Refs. (1,3) we showed - by taking account also of the fourmomentum space and by applying the "Reinterpretation Principle" - that

$$\bar{P}\bar{T} = CPT, \quad (11)$$

where C represents the conjugation of all the additive charges<sup>(3,1)</sup>. Let us add, going back to eq. (9), that all known (relativistic) equations and (relativistic) interactions are actually CPT-covariant. From eqs. (10), (11) it is immediate to derive that

$$\mathcal{P}_I \mathcal{T}_I = \mathcal{T}_I \mathcal{P}_I = C. \quad (12)$$

We have thus shown the (geometrical) operation of reflecting the internal space-time of the considered particle to be equivalent to the operation C which inverts the sign of all its additive charges.

We have also seen that the internal transformations  $\mathcal{P}_I, \mathcal{T}_I$  do change the particle intrinsic state. If we convene to write  $\mathcal{P}_I a_{++} = a_{+-}$ ;  $\mathcal{T}_I a_{++} = a_{-+}$ , then:

$$\mathcal{P}_I \mathcal{T}_I a_{++} = a_{--}, \quad (12')$$

where the subscripts denote the internal parameters that transform under the action of  $\mathcal{T}_I$  and  $\mathcal{P}_I$ , respectively; and where  $a_{--}$  represents the intrinsic (= internal) state of the anti-particle  $\bar{a}$ .

All what precedes can be applied also within the realm of quantum theories.

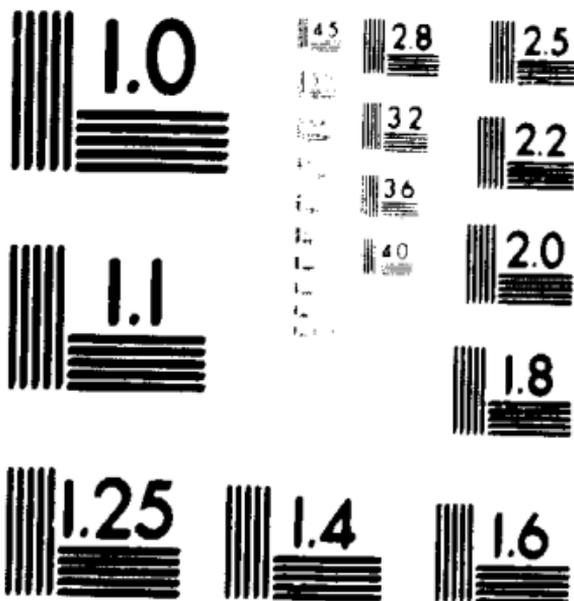
But let us here conclude by emphasizing that - in our opinion, and for the results in this paper and in Refs. (1-3) - we should advantageously substitute in theoretical physics the new operations  $\bar{P} \equiv \mathcal{P}$  and  $\bar{T} \equiv \mathcal{T}$  for the ordinary operations P, T, which are merely external reflections (e. g., only the former do belong to the Full Lorentz Group).

#### ACKNOWLEDGEMENTS

The authors acknowledge useful discussions with R. Mignani and the kind collaboration of L. R. Baldini.

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