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# THERMODYNAMIC FLUCTUATIONS AND THE MONOPOLE DENSITY OF THE EARLY UNIVERSE

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**BUDAPEST**

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## ABSTRACT

The probability of thermodynamic fluctuations is calculated by explicitely using the Riemannian structure of the thermodynamic state space. By means of this probability distribution a correlation volume can be defined. Identifying this volume with one domain in the GUT continuum at the symmetry breaking phase transition in the early Universe, a prediction can be obtained for the primordial monopole density.



## АННОТАЦИЯ

Вычисляется вероятность термодинамических флуктуаций с использованием Римановской структуры термодинамического пространства состояний. С помощью этого распределения вероятности определяется характеристический объем корреляций. Этим объемом задается характеристический объем структуры доменов в теории большого объединения в ранней Вселенной при фазовом переходе, нарушающем симметрию, и получено предсказание для первичной плотности монополей.



## KIVONAT

Kiszámitjuk a termodinamikai fluktuációk valószinüségét a termodinamikai állapottér Riemann-szerkezetének felhasználásával. E valószinüségi eloszlás definiál egy korrelációs térfogatot. A korai Univerzumban a Nagy Egyesités szimmetriasértő fázisátmenetekor e térfogatot azonositva egy doménnal, jóslat adódik az eredetileg keletkező monopólusok sürüségére.

## 1. INTRODUCTION

GUT type theories are very attractive from the viewpoint of particle physicists, and they give a possibility for following back the past of the Universe almost until Planck time. Nevertheless, the simultaneous application of GUT and General Relativity is not devoid of problems. One of them the so called monopole problem. For detailed formulation of this problem see e.g. Refs. 1 and 2; here we repeat only the most necessary points. The self-interaction of the Higgs sector is quartic, so there is a possibility for a phase transition in which some symmetry spontaneously breaks down. It is unfair to prescribe initial conditions for a Universe, but it is generally assumed that the early (high temperature) state of the continuum was symmetric (i.e. the expectation values of the Higgs fields vanished), and there was a charge symmetry. At some $T_{tr}$ temperature, determined by the scale parameter and dimensionless number parameters of GUT, the asymmetric phase (with nonvanishing expectation values for at least some of the Higgses) becomes energetically preferred, and a phase transition starts. At the end of the transition some bosons (the leptoquarks) possess masses in the order of $T_{tr} \sim M$ (where M is the scale parameter of the theory, cca. $10^{15}$ GeV), so at lower temperatures interactions violating the baryon conservation are practically forbidden, the original /e.g SU(5)/ symmetry is broken down to the SU(3) symmetry of the strong interactions and to the symmetry of the Weinberg-Salam electroweak interactions, so the Universe has arrived at the physics of energies typic?  or today's experiences.

Nevertheless, it see.  :aat the matter does not want to completely forget the original situation. Namely, there may remain monopoles in the Universe. Generally some rotations of the Higgs expectation values are irrelevant energetically. Thus, if the symmetry breaking phase transition starts from separated "condensation nuclei", which is a natural enough assumption, then a domain structure is expected. Now, it is predicted that at some corners of the domain structure topological knots are created, where monopoles are trapped (i.e. there the fields possess such structure which cannot be removed by continuous deformation). If such structures are, in fact, produced in the phase transition, then they are stable, except for pair annihilation, and they carry masses cca. $M/\alpha$, where $\alpha = 1/45$ is the GUT coupling constant.

Now, the total mass of the present Universe possesses an upper observational limit, somewhere at ten times of the observed mass in familiar particles (dominantly in protons) [4]. Then the present monopole/photon ratio (which is roughly the monopole to entropy ratio too) has to be less than

$10^{-23}$. On the other hand, Preskill has shown [5] that the annihilation is ineffective below $n_m/s \simeq 10^{-10}$. (The original calculation is valid only for second order or weakly first order transitions, however, for genuine first order transitions see Ref. 6.) So, the present low abundance indicates that either

1. GUT is incorrect, or
2. something strange happened between the end of the transition and the present, or
3. the initial monopole/entropy ratio was surprisingly low.

The first possibility cannot be excluded, but it is an unconvenient conclusion, which is not necessary until the other possibilities are not exhausted. Possibility 2. has various realizations in the literature, perhaps the most popular is the inflationary scenario [1]. There it is assumed that there is a delay in the phase transition (caused by e.g. potential barriers). Then there is a nonequilibrium phase transition, which leads to extra entropy production [7]. This means that the monopole/entropy ratio is decreasing, if the delay is great enough, the ratio may decrease to the present observation limit. The problem is that there is only one single energy scale parameter in usual GUTs, so if there are potential barriers, their heights and widths are expected to be in the same order of magnitude as $T_{tr}$, therefore they are not too effective against the transition. The so-called new inflation formally avoids this problem, (see e.g. Ref. 8), but this generally needs hand-made self-interaction potentials and one has to believe that quantum and thermal corrections produce such unnatural effective potentials, which is definitely not indicated by e.g. one-loop corrections [9]. If the extra entropy is produced by irreversible processes as e.g. momentum transfer, this problem does not occur, but some estimations yield that such a mechanism would need $M \simeq 7.10^{15}$ GeV [10], and this value is generally regarded as too high for the scale parameter. So one cannot tell that Possibility 2. is definitely the way out of the monopole problem. So it is useful to discuss Possibility 3. too that the monopole/entropy ratio was below $10^{-23}$ at the end of the phase transition.

The usual idea is that the original monopole number is determined by the domain geometry, the monopole/domain ratio is $\tilde{p}$, where $\tilde{p}$ is a number characteristic for the symmetry group of the actual GUT, and for SU(5) it is 1/8 [2]. Then one has to know the average domain size. But that size is more or less the same as the coherence length of the fields, since a domain is produced around a particular "condensation nucleus", it is the volume in which the direction of the fields (specially, of the Higgs fields) are correlated. $L_{cor}$ is obviously less than the horizon distance [5,11]. In a decent field theoretical approximation [2]

$$L_{cor} \sim 1/\lambda T_{tr} \qquad (1.1)$$

in such units when $\hbar = c = 1$, where $\lambda$ is the coefficient of the quartic term in the Higgs potential. Now, the value of $\lambda$ is completely unknown, there are some guesses as $g^2$ [12], or $g^4$ [13,14] where

$$g^2 = 4\pi\alpha \simeq 0.3 \tag{1.2}$$

but it seems that for reasonable values of $\lambda$ one cannot expect an initial monopole/entropy ratio below $10^{-8}$ [2].

Nevertheless, the usual approximation uses some assumptions which are rather mathematically than physically clear, i.e. that the transition of the fields in the domain walls is sufficiently simple and smooth. Furthermore, from the equipartition of the energy one would expect a definitely lower monopole/entropy ratio, and Bais and Rudaz, in fact, has obtained a ratio $\sim e^{-1/\alpha}$, which might solve the whole problem [15]. Therefore the whole problem is unclear in some extent.

In 1979, Ruppeiner introduced a Riemannian structure for the thermodynamic state space [16,17]. Later he defined a correlation volume via the curvature of this Riemannian space [18,19]. So then the correlation volume (or length) can be calculated from the equation of state, which, in some cases, is better known than the details of the field theory. This yields a possibility to determine the initial monopole/entropy ratio from different assumptions, and then the two results can be compared. Unfortunately, Ruppeiner's original method does not work for charge-symmetric situations; when there is only one independent extensive density, the Riemann tensor of the state space vanishes, and then the prediction is $V_{cor} = 0$ [19]. Now, this result olearly indicates that the mentioned formulation cannot yield the correlation lengths of all the existing correlation mechanisms, however, the method can be improved. Here we are going to calculate the correlation volume (and the monopole/entropy ratio) for a GUT continuum from the equation of state.

## 2. THE FLUCTUATION PROBABILITY

Since the metric tensor of the thermodynamic state space in extensive coordinates is just the inverse of the correlation matrix of the fluctuating extensives [20], one can expect an intimate and direct connection between the metric tensor $g_{ik}$ and the fluctuation probability $p(V,x|x_o)$/where V and x denote the volume and densities of a subsystem of a reservoir of density $x_o$. In extensive coordinates the connection seems to be simple. Since the entropy is the logarithm of the number of microstates, one expects $p \sim e^{Vs}$, which, in quadratic approximation, leads to a Gaussian law

$$p_V^G(x|x_o) = \left(\frac{V}{2\pi}\right)^{n/2} \sqrt{g(x_o)}\ e^{-Vd\sigma^2/2}$$

$$d\sigma^2 = \sigma_{ik} dx^i dx^k \tag{2.1}$$

$$dx^i = x^i - x_o^i$$

where for index summation the Einstein convention is used, n denotes the number of independent densities. Now, this formula is simple and covariant, but it is obviously an approximation; it can be correct for infinitesimal fluctuations, but clearly breaks down when x is far from $x_o$. Nevertheless, eq. (2.1) yields the starting point. Namely, let us introduce an evolution variable $\tau = 1/V$ (the underlying physical picture can be found e.g. in Ref. 19). Then, $p^G$, as a function of $\tau$, fulfils the following evolution equation

$$\frac{\partial}{\partial\tau} p^G(\tau;x|x_o) = g^{rs}(x_o) \frac{\partial^2}{\partial x^r \partial x^s} p^G(\tau;x|x_o) \tag{2.2}$$

Now, this formula is to be generalized in such a way that it be

a) covariant (because of the intimate connection between the metric and the fluctuation probability);

b) the same in the leading term and maximally simple (Occam's razor); and

c) consistent with the average extensive density of the total system (the additivity of the extensives).

The first condition means that there must be covariant derivatives on the right hand side (but not that there should be simply the covariant Laplacian; there are no evidences for minimal coupling as for gravity), the second one can be fulfilled by a right hand side which is linear in p, containing a covariant Laplacian, together with first and zeroth derivatives, and then the third condition determines these latter terms [21]. The final result for the fluctuation probability $p(\tau;x|x_o)$ in general coordinates $x^i$ is:

$$p(\tau;x|x_o) = \rho(\tau;x|x_o)\sqrt{g(x)}$$

$$\frac{\partial}{\partial\tau} \rho(\tau;x|x_o) = g^{rs}\rho_{;rs} + (h^r\rho)_{;r} \tag{2.3}$$

$$\varphi^{i|}{}_{;rs} g^{rs} = \frac{\partial\varphi^{i|}}{\partial x^r} h^r$$

where the $\varphi^{i|}(x)$ fields are the extensive densities expressed by general coordinates; the bar indicates that $^{i|}$ is not a vectorial index but a name [21]. This is the simplest consistent covariant equation governing the fluctuations. Starting from the obvious initial condition

$$p(0;x|x_o) = \delta^{(n)}(x-x_o) \tag{2.4}$$

the fluctuation probability of any finite subsystem is uniquely determined.

## 3. THE CORRELATION VOLUME

Ruppeiner observed that the correlation length can be calculated from the informations carried by the metric. He recognized the Ricci scalar R in some path integral formulae for $p(\tau;x|x_o)$ [19], and concluded that there must be some role of R if $|\tau R| \gtrsim 1$; since $R = 0$ for an ideal Boltzmann gas, R is produced by correlations, therefore volumes smaller than cca. $|R|$ cannot be regarded as uncorrelated. Therefore he defined the correlation volume as [19]

$$V_{cor} = -|R|/2 \tag{3.1}$$

Obviously, such volumes are essentially correlated. However, this is only a sufficient condition for the existence of essential correlations. A stronger criterion can be obtained in the following way.

Consider an idealized system without spatial correlations. Take a subsystem, there the extensives have a probability distribution. Halving the volume, the resulted two subsystems are of course uncorrelated with each other, consequently the probability distribution of the original subsystem is a convolution of those of the new ones. (This is true only in extensives, being they additive.) Continuing this division process beyond any limit, one gets that the probability distribution of the original subsystem (or that of any subsystem) has to be infinitely divisible [22]. The distributions of this class can be produced by repeated convolutions of Gaussian and generalized Poisson distributions. Now, assume that there is a $V_{cor}$ characteristic volume, below which the spatial correlations are important. Then the probability distribution for the total system is produced by $N=(\ln(V/V_{cor})/\ln 2)$ convolutions, it is not an infinitely divisible distribution, but for great values of N it is near to such a one, because the result of repeated convolutions approaches a Gaussian distribution. Thus $V_{cor}$ can be calculated by solving the evolution equation (2.3-4); at some $\tau_{cor} = 1/V_{cor}$ value of the evolution parameter the probability distribution ceases to be near to an infinitely divisible one.

While this comparison is possible in principle, it cannot be directly done. For getting a straightforward criterion, one can evaluate the momenta. For the first momentum the criterion is automatically fulfilled because of the additivity of the extensives. The second central momenta are additive in convolution, therefore

$$\int (x^i-x^i_o)(x^k-x^k_o)p(\tau;x|x_o)d^nx = \tau g^{ik}_o+(\text{corr. terms.}) \tag{3.2}$$

Let us stop here, then we get again sufficient conditions, but a wider set than eq. (3.1). Assume that $p(\tau;x|x_o)$ can be expanded into a Taylor series, substitute this expansion into the left hand side of eq. (3.2), and express the $\tau$ derivatives via eq. (2.3). Then the result is as follows:

$$\int (x^i-x^i_c)(x^k-x^k_o)p(\tau)d^nx = \tau g^{ik}(\tau_o) + \frac{1}{2}\tau^2 g^{ik}(\tau_o)_{,rs}g^{rs}+\vartheta(\tau^3) \qquad (3.3)$$

Now, eq. (3.2) indicates that we are above $V_{cor}$ if the first term of the right hand side of eq. (3.3) dominates the higher ones. Stopping at the quadratic term, one can evaluate this criterion by introducing the eigenvectors of the matrix

$$M^{ik} = g^{ik}{}_{,rs}g^{rs} \qquad (3.4)$$

Namely, let $v^i_{\underline{a}}$ fulfil the relations

$$M^{ir}v_{\underline{a}r} = m_{\underline{a}}v^i_{\underline{a}}$$
$$g^{rs}v_{\underline{a}r}v_{\underline{a}s} = 1 \qquad (3.5)$$

(The lifting of the indices is made, of course, by the metric $g_{ik}$.) Then, in the usual way, one gets that the eigenvectors belonging to different eigenvalues $m_{\underline{a}}$ are orthogonal to each other. Here we restrict ourselves to the generic case when $M^{ik}$ possesses four eigenvectors. Then, rewriting the right hand side of eq. (3.3) by means of $m_{\underline{a}}$ and $v^i_{\underline{a}}$, one gets

$$<(x^i-x^i_o)(x^k-x^k_o)> = \sum_{\underline{a}} \{1 + \frac{1}{2}\tau m_{\underline{a}}\}v^i_{\underline{a}}v^k_{\underline{a}}\tau+\vartheta(\tau^3) \qquad (3.6)$$

Therefore the first term dominates if

$$1 \gg \frac{1}{2}\tau m_{\underline{a}} \qquad (3.7)$$

Eq. (3.6) clearly indicates that there may exist different correlation mechanisms with different characteristic volumes. One can choose the maximal of them, and then

$$V_{cor} = \frac{1}{2}\max_{\underline{a}}(m_{\underline{a}})$$
$$g^{ir}{}_{,st}g^{st}v_{\underline{a}r} = m_{\underline{a}}v^i_{\underline{a}} \qquad (3.8)$$

Now, observe that while there is some similarity between eqs. (3.1) and (3.8), they are obviously different. First, the right hand side of eq. (3.8) cannot be expressed by algebraic curvature invariants. This is not surprising, for example the algebraic curvature invariants identically vanish in Riemann spaces of Petrov-type N [23]. Second, $M^{ik}$ cannot be expressed purely by the Riemann tensor; it does not necessarily vanish even for flat spaces. This is a consequence of the existence of a preferred frame in the definition of the correlation volume. Namely, the extensives are the only coordinates which are additive when unifying subsystems. Therefore one should expect the appearance of the $\varphi^{i|}$ fields on the right hand side of eq. (3.3). Thus, in general coordinates, the correlation volume can be expressed by covariant derivatives of the fields $\varphi^{i|}$ (see the Appendix). Now we are going to investigate two special systems, which are sufficiently simple, and do not contain any interactions in the usual sense, the ideal Boltzmann and Planck gases.

## 4. IDEAL GASES

Consider an ideal Boltzmann gas. Then the condition (3.1) would give $V_{cor} = 0$, since its Riemann space is flat. Nevertheless, the Cartesian coordinates are not the extensives, therefore $M^{ik}$ does not vanish. The entropy function is of the form

$$s = \frac{3}{2} \ln (\varepsilon/\varepsilon_o) - \frac{5}{2} n \ln n \tag{4.1}$$

where $\varepsilon_o$ is a scale constant, $\varepsilon$ is the energy density, n is the particle density. Hence by using $x^i = (\varepsilon, n)$, one gets

$$g^{ik} = \begin{bmatrix} \frac{5}{3}\frac{\varepsilon^2}{n} & \varepsilon \\ \varepsilon & n \end{bmatrix}$$

$$M^{ik} = \begin{bmatrix} \frac{20}{9}\frac{\varepsilon^2}{n^2} & 0 \\ 0 & 0 \end{bmatrix} \tag{4.2}$$

Then eq. (3.8) yields

$$V_{cor} = \frac{2}{3n} \tag{4.3}$$

The result is simple enough, but cannot be interpreted as a volume in which the local interactions become important, since in our system all interactions have been neglected. However, eq. (4.3) possesses a clear physical meaning. Observe that, apart from the factor 2/3, produced by the choice that the first two terms of the expansion (3.3) are just equal at $V_{cor}$, the correlation volume is the specific volume, occupied by a single particle in average. In the whole formulation it was assumed that the extensives were continuous variables and $p(\tau;x|x_o)$ was a continuous function of its argument. At volumes comparable to 1/n this assumption breaks down; if we want to keep it, and use fractional particle numbers, the physical existence of an indivisible particle itself gives the spatial correlation going beyond 1/n with the division.

The second example is a Planck gas. Consider first a simple charge-symmetric continuum, as e.g. a gas of neutral mesons, or nucleon-antinucleon pairs [24]. Then from thermodynamical viewpoint, there is no particle number in the system, the only independent extensives are V and E, the only independent density is the energy density $\varepsilon$. So the Riemann space is of one dimension, therefore the curvature automatically vanishes, and Cond. (3.1) yields that the correlation volume is 0. But this result is obviously incorrect because there may be interactions in the system. On the other hand, eq. (3.8) can yield nonvanishing correlation volume. As an example, let us consider the ideal Planck gas, where the interactions are neglected. Nevertheless, it is a quantum gas, where the Bose occupation law can lead to correlations. The equation of state for a pure photon gas is

$$\varepsilon = \frac{\pi^2}{30}\frac{1}{(\hbar c)^3}T^4 \tag{4.4}$$

on such a temperature scale where the Boltzmann constant is 1. Then

$$s = \frac{4}{3}\left(\frac{2\pi^2}{30}\right)^{1/4}(\hbar c)^{-3/4}\varepsilon^{3/4} \tag{4.5}$$

whence

$$g^{\varepsilon\varepsilon} = 4\left(\frac{30}{2\pi^2}\right)^{1/4}(\hbar c)^{3/4}\varepsilon^{5/4}$$

$$M^{\varepsilon\varepsilon} = 5\left(\frac{30}{2\pi^2}\right)^{1/2}(\hbar c)^{3/2}\varepsilon^{1/2} \tag{4.6}$$

Then eq. (3.8) leads to

$$V_{cor} = \left(\frac{75}{8\pi^2}\right)\left(\frac{\hbar c}{T}\right)^3 \tag{4.7}$$

But this means that the correlation length is in the order of the wavelength of the dominant component of the radiation, cca $\hbar c/5T$. Thus the result (4.7) is physically correct. Therefore we conclude that eq. (3.8) works satisfactorily, and the produced correlation volume is in fact a physical correlation volume.

## 5. THE GUT CONTINUUM

In Grand Unified Theories there are fermion, boson and Higgs fields, the bosons produce the interactions between the fermions, and the Higgs self-interaction is described by the quartic potential [3] (henceforth, for simplicity's sake, $\hbar=c=k_B=1$)

$$V(\varphi) = V_o - \frac{1}{2} M^2 \mathrm{Tr}\varphi^2 + \frac{1}{3} \varepsilon M \mathrm{Tr}\varphi^3 + + \frac{1}{4} [\lambda_1 (\mathrm{Tr}\varphi^2)^2 + \lambda_2 \mathrm{Tr}\varphi^4] \quad (5.1)$$

Here M is the scale parameter of the symmetry breaking, $\varepsilon$, $\lambda_1$ and $\lambda_2$ are some totally unknown numbers. The $\lambda$ parameters are sometimes guessed to be proportional with $g^2 = 4\pi\alpha$ [12], or with $g^4$ [13,14], obviously very large or very small values are improbable and unaesthetic. The effective potential, which is the negative of the pressure, contains thermal and quantum corrections too, in one loop approximation these corrections can be simply evaluated [9], it is of course a question, if the one-loop terms are sufficient or not. The pure quantum corrections give a logarithmic contribution, while the thermal part is

$$\delta V(\varphi,T) = - \frac{1}{6\pi^2} \mathrm{Tr} \int_0^\infty \frac{x^4 dx}{\sqrt{m^2+x^2}\, \exp\{\frac{1}{T} \sqrt{m^2+x^2} - 1\}} \quad (5.2)$$

Here $m^2$ is the self-consistent effective mass [9]

$$m^2(\varphi,T) = \mathrm{Tr} \frac{\partial^2}{\partial\varphi_i \partial\varphi_k} V_{eff} \quad (5.3)$$

(see also Ref. 6). So, in this approximation, the equation of state for the pressure P of the symmetric phase in the simplest GUT is [6]

$$P(T) = [\frac{142}{90} \pi^2 - \frac{5}{288} (7\lambda_1 + 3\lambda_2)]T^4 + \frac{5}{\text{[illegible]}} M^2 T^2 - V_o \quad (5.4)$$

(the constant $V_o$ is to be chosen in such a way that the equation of state in the deeper symmetry-broken phase give $P(0) = 0$). If one wants to get the equation of state in extensives, a cubic equation is to be solved, and the result is not too transparent. Nevertheless, eq. (5.4) is obviously some expansion in M/T, and not true for low temperatures. In a similar expansion

$$s = \frac{4}{3} (3A)^{1/4} \varepsilon^{3/4} (1 + \frac{1}{4} 3^{1/2} A^{-1/2} B \varepsilon^{-1/2})$$
$$T = (3A)^{-1/4} \varepsilon^{1/4} (1 - \frac{1}{4} 3^{-1/2} A^{-1/2} B \varepsilon^{-1/2}) \quad (5.5)$$

where

$$A = \frac{142}{90}\pi^2 - \frac{5}{288}(7\lambda_1 + 3\lambda_2)$$
$$B = \frac{5}{24}M^2 \qquad (5.6)$$

and then, from eq. (3.8):

$$V_{cor} = \frac{5}{24A}\frac{1}{T^3}\left(1 - \frac{2}{5}\frac{B}{A}\frac{1}{T^2}\right) \qquad (5.7)$$

The leading term is similar to the correlation volume of a Planck gas, because the $T^4$ term in eq. (5.4) imitates a free Planck gas when $T/M \gg 1$; of course the multiplicative factor is different, because of the different number of kinds of particles (the first term in A) and of the interactions (the second one).

By using this value for $V_{cor}$ one can argue as follows. The domain structure is a consequence of the fact that the phase transition has started from uncorrelated condensation nuclei, spontaneously created in the symmetric phase. If two nuclei were too close to each other, then they were correlated, and there will be only one domain. Therefore, at least at the beginning of the phase transition process, the volume of a domain will be approximately the correlation volume of the symmetric phase. (If the transition is of strongly first order, then a proper dynamical treatment may be necessary.) Then, counting again the topological knots leading to monopoles, and using eqs. (5.5-7), one gets

$$\frac{n_m}{s} = \frac{6}{5}\tilde{p}\left(1 - \frac{1}{10}\frac{B}{A}\frac{1}{T^2}\right) \qquad (5.8)$$

for the monopole/entropy ratio. The leading term is independent of T, and quite high.

## 6. CONCLUSIONS

Now we are in the position to compare the predictions of the field theory and thermodynamics. For a GUT continuum, composed of fermions, vector bosons and Higgs scalars, field theoretical methods predict that the Higgs scalars are correlated in a volume [2]

$$V_{cor} \simeq 1/(\lambda^3 T^3) \qquad (6.1)$$

where $\lambda$ is the coefficient of the quartic terms in the Higgs self-interaction. Then this volume becomes the average volume of the domains, and if there are no energetic obstacles to the monopole production (which is not obvious, being the monopoles heavier than the temperature at the phase transition [15]), in the simplest approximation the monopole/entropy ratio is

$$\frac{n_m}{s} \simeq \tilde{p}\,\lambda^3 \frac{45}{2\pi^2 N} \tag{6.2}$$

N is an effective number of particle degrees of freedom, occurring in P(T) in the free gas limit [2], for photons N=2, in the simplest GUT $N \simeq 142$ [6]; $\tilde{p}$ is 1/8 for SU(5) [2].

On the other hand, it is possible to define a correlation volume in phenomenologic thermodynamics; this is a volume in whose different parts the fluctuations are not independent. This seems to mean that such a volume tends to develop a common single domain. Evaluating the equation of state in one-loop approximation [6,9], and calculating the fluctuations [17-19,21], one gets:

$$\begin{aligned} V_{cor} &\simeq \frac{5}{24A}\frac{1}{T^3} \\ A &= N\frac{\pi^2}{90} - \frac{5}{288}(7\lambda_1 + 3\lambda_2) \end{aligned} \tag{6.3}$$

Hence, in the same geometric approximation, the monopole/entropy ratio is

$$\frac{n_m}{s} \simeq \frac{6}{5}\tilde{p} \tag{6.4}$$

Comparing the results of the two different approximations, one can observe that the T dependence is the same, but this can be expected for Planck-like gases. On the other hand, the multiplicative number constants are different in eqs. (6.1) and (6.3); the thermodynamic approach predicts a smaller correlation volume (being N>>1, an generally $\lambda \lesssim 1$ is assumed too). Now it would be difficult to decide here which approximation is the better. Namely, at moderate $\lambda$ values the equation of state (5.4) simulates a free gas, although in a quartic potential a particle is not free even at asymptotic temperatures $T/M \to \infty$, so it would not be surprising to get $\lambda$-dependence in the leading term too. There are some grounds of suspicion about the self-consistency of the one loop approximation, since $T^4$ terms may have multiple origin [6]. On the other hand, eq. (6.3) gives the maximal volume in which the fluctuations of the extensives are essentially correlated. The expectation values of the Higgses are not thermodynamic variables, so it is possible that the Higgses are correlated even at greater distances, however it is strange if this correlation is not reflected in the thermodynamic variables. Furthermore, in Sec. 3 we emphasized that we evaluated only the correlations of the second momenta of $p(\tau;x|x_o)$; it is possible that higher momenta would show correlations in a greater volume. Nevertheless, the $N\lambda^3$ difference in the multiplicative factors seems to be more than a technical difference.

In any case, the initial monopole/entropy ratio is predicted by both approximations to be temperature-independent and high enough. Since $\lambda$ is guessed as $g^2$ [12] or $g^4$ [13], it is not expected to be below $10^{-1}$, when the field theoretical prediction is $n_m/s \sim 10^{-6}$. The thermodynamic approach yields $n_m/s \sim 10^{-1}$ in the best case. Both values are above Preskill's limit [5], so the annihilation is still effective and will reduce $n_m/s$ to $10^{-10}$, but this is highly insufficient compared to the present observational limit.

One can observe that, although the presented calculation is thermodynamical, Bais and Rudaz's Boltzmann suppressing factor has not been obtained. But this factor cannot be expected in the symmetric phase, where the massive monopoles still are not created. The obtained $n_m/s$ ratio is meant rather for the precursors of the monopoles; if there are energetic obstacles, some monopole embryos may be aborted during the transition.

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## APPENDIX: THE COVARIANT FORM OF THE MATRIX $M^{ik}$

In the derivation of the correlation volume extensive coordinates were used; a matrix

$$M^{ik} = g^{ik}{}_{,rs}g^{rs} \tag{A.1}$$

was obtained. The maximal eigenvalue of $M^{ik}$ defines the correlation volume $V_{cor}$ as

$$V_{cor} = \frac{1}{2}\max(m_{\underline{a}})$$
$$M^{ir}v_{\underline{a}r} = m_{\underline{a}}v_{\underline{a}}{}^{i} \tag{A.2}$$

(as far as one is restricted to the spatial correlations in the second central momenta of extensives). The expression for $M^{ik}$ has definitely not been cast into a covariant form; eq. (A.1) does not yield anything about the tensorial behaviour of the matrix, being valid only in a special coordinate system. Nevertheless, $V_{cor}$ must be calculable in any coordination, therefore the formula yielding $V_{cor}$ must possess definite tensorial character with proper transformation laws. Here we are manufacturing such a formula, containing a matrix which is scalar with respect to coordinate transformations, and possesses the same value as $M^{ik}$ in extensive coordinates. Using this matrix in eq. (A.2), $m_{\underline{a}}$ and thus $V_{cor}$ are scalars too.

As it was mentioned, the existence of balance laws leads to a somewhat preferred role of the extensives, in spite of the full covariance. Consider an arbitrary coordination; then there are n scalar fields $\varphi^{i|}$ giving the extensive densities

$$x^{i} = \varphi^{i|}(x^{k}) \tag{A.3}$$

where $^{i|}$ is not a vectorial index but a name. Let us introduce n vector fields $a_{i|}{}^{k}$ as

$$a_{i|}{}^{r}\varphi^{k|}{}_{;r} = \delta_{i|}{}^{k|} \tag{A.4}$$

A consequence of eq. (A.4) is

$$a_{r|}{}^{i}\varphi^{r|}{}_{;k} = \delta^{i}_{k} \tag{A.5}$$

Since $\varphi^{i|}$ is a scalar, in extensive coordinates

$$\varphi^{i|}{}_{;k} = \delta^{i|}{}_{k} \tag{A.6}$$

Now, introducing a derivative operator $\partial_{m|}$ producing scalars from scalars

$$\partial_{m|} = a_{m|}{}^{r}\partial_{r} \qquad \text{(A.7)}$$

one can build up a matrix $M^{ik|}$ which is scalar with respect to coordinate transformations

$$\begin{aligned} M^{ik|} &= g^{mn|}\partial_{m|}\partial_{n|}g^{ik|} \\ g^{ik|} &= g^{rs}\varphi^{i|}{}_{;r}\varphi^{k|}{}_{;s} \end{aligned} \qquad \text{(A.8)}$$

Using eqs. (A.4-7) one can see that in extensive coordinates $M^{ik|}$ possesses the same value as $M^{ik}$ in eq. (A.1); since its transformation laws are clear, it can be calculated in any coordinate system. By performing the derivations in eq. (A.8) a not too compact final result is obtained, namely

$$\begin{aligned} M^{ik|} = {} & 2\varphi^{(i|;sw}{}_{w}\varphi^{k|)}{}_{;s} + 2\varphi^{i|;sw}\varphi^{k|}{}_{;sw} - \\ & - 2\varphi^{(i|;s}{}_{t}\varphi^{k|)}{}_{;s}a^{t}{}_{m|}\varphi^{m|;w}{}_{w} \end{aligned} \qquad \text{(A.9)}$$

where the bracket denotes symmetrization in the name indices:

$$b^{(ik|)} = \frac{1}{2}\left(b^{ik|} + b^{ki|}\right) \qquad \text{(A.10)}$$

One can see that the curvature tensor plays a secondary enough role in the expression of $V_{cor}$; in this form of $M^{ik|}$ $R_{iklm}$ does not even occur.

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