LYCEN / 8403
January 1984

HIGH ENERGY SPIN ISOSPIN MODES IN NUCLEI

G. CHANFRAY[+]

Institute für Theoretische Physik, Universität Heidelberg,
Philosophenweg 12, D-6 900 HEIDELBERG 1

and

M. ERICSON[++]

Institut de Physique Nucléaire (and IN2P3), Université Claude Bernard Lyon-1
43, Bd du 11 Novembre 1918, 69622 VILLEURBANNE Cedex, France

Abstract :

The high energy response of nuclei to a spin-isospin excitation is investigated. We show the existence of a strong contrast between the spin transverse and spin longitudinal responses. The second one undergoes a shadow effect in the Δ region and displays the occurence of the pionic branch.

+ on leave from IPN Lyon, 69622 Villeurbanne, France

++ also at CERN, 1211 Genève 23

The (p,n) reactions have proved to be a precious tool to explore the *Gamow-Teller (G.T) strength in the low energy region* (E ≤ 30 MeV) of nuclear excitations. There, a quenching of the strength has showed up, attributed to the interplay between nuclear and nucleonic degrees of freedom (in particular the Δ). This gave a strong incentus to the exploration of the G.T strength at high energy Experiments on (^{3}He, T) have been undertaken for that purpose [(1)]. However in this process the energy transfer necessary for the excitation of the Δ goes along with momentum transfer ($\omega = 0.812\ q$). Thus it is not strictly the G.T strength (defined as the one at $q = 0$) which is explored. The kinematics is instead closer to that of ph ton or pion absorption.

It is then natural to inquire what new informations these experiments can bring as compared to the other ways of excitation the Δ, like by photons or pions. Is hadron scattering expected to reveal new features wich are not already available ? The answer to this question is affirmative. Indeed photons excite the Δ through a transverse spin coupling $\vec{S} \times \vec{q}$ which is unable to probe the longitudinal magnetisation. Pions with their $\vec{S}.\vec{q}$ coupling can instead perform this exploration. Nevertheless for physical pions the energy momentum relation is restricted to $\omega^2 = q^2 + m_\pi^2$, which strongly limits the information available. Relaxing this condition by allowing the pion to be virtual as occurs in hadron scattering, new and interesting features emerge. We will show that the response, which is very collective, displays a remarkable contrast with the transverse one. It was pointed our by Alberico et al. [(2)] that a contrast is expected in the low energy sector owing to the influence of the pion exchange force. The aim of this work is to show that a similar situation prevails in the high energy sector with an even more pronounced contrast.

The formalism that we use is the RPA approximation with neglect of antisymetrisation. In a first stage we will explore the response for infinite nuclear matter, where the collective state of the pion branch appears. The nuclear matter description however fails for finite nuclei in the region of this collective state. In order to have an estimate for finite nuclei we use a simplified model. We have tried to keep in touch with reality by insuring that this model is consistent with the existing data obtained with photons or pions.

Nuclear matter description

The RPA expression for the polarization propagator $\pi(q,\omega)$ is related to the bare one $\pi^0(q,\omega)$ by

$$\pi(q,\omega) = \frac{\pi^0(q,\omega)}{1-V(q,\omega)\ \pi^0(q,\omega)}$$

where V is the ph interaction. The quantity $\pi^0(q,\omega)$ incorporates all the excitations of the systems, $N N^{-1}$, 2p2h and ΔN^{-1}. We are here interested in the high energy domain ($\omega \gtrsim$ 200 MeV), well above the NN^{-1} excitations. In that situation the NN^{-1} response can be ignored since in symmetric nuclear matter both the imaginary and real parts of the NN^{-1} polarization propagator vanish.

As for the 2p2h response, it has been studied by Alberico et al. [3]. They have shown that the imaginary part of the bare quantity $\pi^{\circ\ 2p2h}$ is a smooth function of the energy and momentum for which they have proposed the ansatz.

$$\mathrm{Im}\ \pi^{0\ 2p2h}(q,\omega) = 4\pi\ \mathrm{Im}\,C_0\ \rho^2\ \nu^2(q^2-\omega^2) \tag{2}$$

where ρ is the nuclear density, ν is the form factor of the nucleon, $\nu = \Lambda^2/(\Lambda^2+q^2-\omega^2)$. The quantity $\mathrm{Im}\,C_0$ is related to the bare expression for the p wave absorptive potential. In this work we have used $\mathrm{Im}\,C_0 = 0.13\ m_\pi^{-6}$. The expression (2) holds for energies ranging between 100 and 200 MeV. In this work we have extended it in the region of the Δ resonance where it is certainly less reliable.

The behavior of $\mathrm{Re}\ \pi^{2p2h}(q,\omega)$ is not known, we have totally ignored it in this work.

For the ΔN^{-1} polarization propagator we have used the following expression*

$$\pi^{0\Delta}(q,\omega) = \frac{4}{9}\ \frac{f^{*2}}{m_\pi^2}\int\frac{d\vec{k}}{(2\pi)^3}\ \theta(k_F-k)\left[\frac{1}{\omega-\left(\omega_\Delta+\frac{|\vec{k}+\vec{q}|^2}{2m_\Delta}-\frac{k^2}{2m}\right)-\frac{i\,\Gamma(\omega)}{2}}\right.$$
$$\left.-\ \frac{1}{\omega+\omega_\Delta+\frac{|\vec{k}+\vec{q}|^2}{2m_\Delta}-\frac{k^2}{2m}}\right] \tag{3}$$

* Notice that we incorporate the factor $\frac{f^{*2}}{m_\pi^2}$ in our definition of π^Δ.

where f^* is the $\pi N\Delta$ coupling constant ($f^* = 2f$), $\omega_\Delta = m_\Delta - m = 2.2\ m_\pi$ and the width $\Gamma(\omega)$ is

$$\Gamma(\omega) = \Gamma_0\left(\frac{\omega^2 - m^2}{\omega_\Delta{}^2 - m_\pi{}^2}\right)^{3/2} \qquad \text{with } \Gamma_0 = 110 \text{ MeV} \qquad (4)$$

We have neglected in our simplified approach the modification of the Δ position and width by the nuclear medium.

The ph interaction is different in the spin longitudinal and spin transverse channels. In both cases it contains a phenomenological short range piece embodied in the Landau Migdal parameter g'. Beside this it contains pion exchange for the longitudinal case and ρ exchange in the transverse one (we have used for the ρ N coupling constant $C_\rho = \frac{f_\rho^2}{m_\rho^2} \times \frac{m_\pi{}^2}{f^2} = 2$).

The transverse response in the Δ region has been explored through γ absorption for a variety of nuclei. The effects of the medium is weak, no appreciable modification with respect to the free nucleon response has been found [5]. This may be the result of several delicate cancellations. In this work where we do not aim at a fully quantitative description we have insured that the transverse nuclear response at $\omega = q$ remains close to the nucleonic one by choosing a value of g' such as to keep the distortion of the transverse response small. This is achieved with a value g' = 1/3.

While no distortion occurs in the transverse response, the longitudinal one is completely modified. An example is shown in Fig. 1 where the responses are given as a function of ω along the line $\omega = q$. A striking contrast emerges between the longitudinal and transverse response. For the first one the Δ region is shadowed and the strength is concentrated near the pionic branch. This branch corresponds to the singularity of the pion propagator i.e. to the condition

$$\omega^2 - m_\pi{}^2 - q^2 - q^2 \mathrm{Re}\ \tilde{\pi}\ (q,\omega) = 0 \qquad (5)$$

with $$\tilde{\pi}(q,\omega) = \frac{\pi^0(q,\omega)}{1 - \pi^0(q,\omega)\frac{m_\pi{}^2}{f^{*2}}\ g'}$$

Sawyer [5] has drawn the attention to the interest of this branch and he suggested its exploration through the weak interaction process. This branch is accessible only if the probe explores the longitudinal magnetizations as is the case for hadron scattering. The fact that the collective pionic branch does not appear in the transverse channel generates the contrast.

Notice that the amount of shadowing in the Δ region for the longitudinal response sensitively depends on the off shell character of the pion. It disappears for strongly off shell pions. It is instead quite pronounced for nearly on shell ones, as is the case for instance along the line $\omega = q$.

Finite nuclei

The previous results refer to infinite nuclear matter. The question then arises. *Do they hold for finite system and under which conditions ?* Indeed the pion branch is special in the sense that it is a singularity (or a near singularity when the 2p-2h absorption is considered) and this raises a conceptual problem. This singularity determines the energy momentum for an asymptotic pion which can propagate to infinity. In a finite system, this means outside the nucleus. Therefore the genuine singularity is always the free pion line $\omega^2 = q^2 + m_\pi^2$ and no other one can appear. In the infinite nuclear matter treatment the external propagation is totally ignored but we do not expect this to be a good approximation in finite systems near the singularity.

For the static case Delorme et al. [6] who have pointed out this problem have utilized a square well model to study the static pion field in a finite system. We will use the same model to investigate what remains of the pion branch in a finite system.

In a finite system the polarization propagator is a function of two momenta q and q'. The response is :

$$R(q,\omega) = -\frac{1}{\pi} \, \mathrm{Im} \, \pi(q,q,\omega) \tag{6}$$

The bare ΔN^{-1} part of π is :

$$\pi^0(q',q,\omega) = \sum_{\Delta h} F^{*}_{\Delta h} (\vec{q}\,') \, G^0_{\Delta h}(\omega) \, F_{\Delta h}(\vec{q}) \tag{7}$$

where $G^0_{\Delta h}(\omega)$ is the first order ΔN^{-1} Green function

$$G^0_{\Delta h}(\omega) = \frac{1}{\omega-(\varepsilon_\Delta-\varepsilon_h) + \frac{i\Gamma(\omega)}{2}} - \frac{1}{\omega+\varepsilon_\Delta-\varepsilon_h} \tag{8}$$

and the matrix element of the spin-isospin operator is defined according to :

$$F_{\Delta h}(q) = \frac{f^{*}}{m_\pi} < \Delta \, h \Big| \sum_{i=1}^{A} \vec{S}_i.\vec{q} \; T_i \quad e^{i\vec{q}.\vec{x}_i} \Big| > 0 \tag{9}$$

We relate the nuclear response to π^0 by an RPA approximation ignoring antisymmetrization †. The ΔN^{-1} interaction is taken as :

$$V(q,\omega) = \frac{f^{*2}}{m_\pi^2} \, v^2(q,\omega) \left[\frac{q^2}{q^2 + m_\pi^2 - \omega^2} \; \vec{S}_1.\hat{q} \; \vec{S}_2.\hat{q} + g' \; \vec{S}_1.\vec{S}_2^{\,+} \right] \vec{T}_1.\vec{T}_2^{\,+}$$

The dressed polarization propagator is searched in the form :

$$\pi(\vec{q}\,',\vec{q},\omega) = \Sigma_{\Delta h} \overline{F^{*}_{\Delta h}}(q',\omega) \, G^0_{\Delta h}(\omega) \, F_{\Delta h}(q)$$

where the dressed matrix element $\overline{F^{*}}$ satisfies the following integral equation :

$$\overline{F^{*}_{\Delta h}}(q,\omega) = F_{\Delta h}(\vec{q}) + \int \frac{d\vec{k}}{(2\pi)^3} \, \tilde{\pi}^0(\vec{q},\vec{k},\omega) \, v^2(k,\omega) \; g' \; F^{*}_{\Delta h}(\vec{k})$$

$$+ \int \frac{d\vec{k}}{(2\pi)^3} \, \tilde{\pi}^0(\vec{q},\vec{k},\omega) \, v^2(\vec{k},\omega) \; \frac{k^2}{\omega^2-k^2-m_\pi^2} \; \overline{F^{*}_{\Delta h}}(\vec{k},\omega) \tag{11}$$

---

† We neglect the moderate coupling between the transverse and longitudinal responses which exists in finite nuclei.

with :

$$\tilde{\pi}^{0}(\vec{q}',\vec{q},\omega) = \pi^{0}(\vec{q}',\vec{q},\omega) + \int \frac{d\vec{k}}{(2\pi)^3} \pi^{0}(\vec{q}',\vec{q},\omega)\; g'\; v^{2}(\vec{k},\omega)\; \tilde{\pi}^{0}(\vec{q},\omega) \tag{12}$$

From now on we will make the following approximations :

i) we ignore the form factor $v(q,\omega)$

ii) we make a static approximation in $G^{0}(\omega)$ by replacing $\varepsilon_{\Delta}-\varepsilon_{k}$ by ω_{Δ}
This approximation is the one used in the Kisslinger potential for low energy pion scattering.

With these assumptions it is more convenient to work in x space. We define :

$$\overline{\phi}_{\Delta h}(\vec{q},\omega) = \frac{q^2}{q^2-K^2}\,\overline{F}^{*}_{\Delta k}\,(\vec{q},\omega) \qquad \text{with } K^2 = \omega^2 - m_{\pi}^{2} \tag{13}$$

The Fourier transform of $\phi_{\Delta h}$ satisfies a Klein-Gordon equation.

$$-\left\{\vec{\nabla}\left[1+\tilde{\alpha}_{0}(\vec{x},\omega)\right]\vec{\nabla} + K^2\right\}\phi_{\Delta h}(\vec{x},\omega) = \frac{f^{*}}{m}\,\vec{\nabla}.\left\{\eta(\vec{x},\omega)\langle 0\left|\Sigma_{i=1}^{A}\vec{S}_{i}.T_{i}\,\delta(\vec{x}-\vec{x}_{i})\right|\Delta h\rangle\right\} \tag{14}$$

where

$$\alpha_{0}(\vec{x},\omega) = \frac{4}{9}\,\frac{f^{*2}}{m_{\pi}^{2}}\,\rho(x)\left[\frac{1}{\omega-\omega_{\Delta}+i\Gamma(\omega)|/2} - \frac{1}{\omega+\omega_{\Delta}}\right] \tag{15 a}$$

$$\tilde{\alpha}_{0}(\vec{x},\omega) = \alpha_{0}(\vec{x},\omega)\left[1-g'\alpha_{0}(\vec{x},\omega)\right]^{-1} \tag{15 b}$$

$$\eta(\vec{x},\omega) = 1+g'\tilde{\alpha}_{0}(\vec{x},\omega) \tag{15 c}$$

The solution of the equation (14) can be written as

$$\overline{\phi}_{\Delta h}(\vec{x},\omega) = -\frac{f^{*}}{m_{\pi}}\langle 0\left|\Sigma_{i=1}^{A}\,\eta(x_i)\Sigma_{L=0}^{\infty}\Sigma_{\lambda=L\pm1}\,b_{L\lambda}\,\overline{g_{L\lambda}}(x,x_i)Y_{LM}(\hat{x}_i)\right. \tag{16}$$

$$\times\left[Y_{\lambda}(\hat{x}) \otimes S_{i}\right]_{LM}\; T_{L}^{\alpha}\left|\Delta h\right\rangle$$

with

$$b_{L\lambda} = \left[(L+1)/(2L+1)\right]^{1/2} \quad \text{for } \lambda = L+1 \ , \ b_{\lambda L} = L/(2L+1)^{1/2} \quad \text{for } \lambda = L-1$$

In expression (16) $g_L(x,x_i)$ is the solution of a radial equation for a pion field emitted at x_i which can be obtained by direct insertion of the expression (16) of $\Phi(x,\omega)$ in the equation (14).

In q space the renormalized response function then becomes :

$$R(\vec{q},\omega) = -\frac{1}{\pi}\,\mathrm{Im}\left\{\frac{q^2-K^2}{q^2}\ \Sigma_{L\lambda}(2L+1)\ b^2_{L\lambda}\int_0^\infty dx_i\ 4\pi x_i^{\,2}\eta(x_i)\overline{g}_{L\lambda}(q,x_i)\,j_L(qx_i)\right\} \tag{17}$$

with

$$\overline{g}_{L\lambda}(q,x_i) = \int_0^\infty dq'\,q^2\ j_\lambda\ (qx)\ \overline{g}_{L\lambda}\ (x,x_i)$$

where we have used closure over single Δh states.

This expression incorporates only 1p-1h type of excitations. We must now include 2p-2h excitations by replacing α_0 by :

$$\alpha_0(x) \rightarrow \alpha_0\ (x) - i\ 4\pi\ \rho^2(x)\ \mathrm{Im}C_0$$

a procedure currently used in pion scattering.

In order to exhibit the finite size effects, we give the exact analytical solution for a square-well potential of radius R (i.e $\rho(x) = (3A/4\pi R^3)\ \theta(R-x) = \rho_0\ \theta(R-x)$

$$R(\vec{q},\omega) = -\frac{A}{\pi\rho_0}\,\mathrm{Im}\ \frac{\tilde{\alpha}_0}{1-\dfrac{q^2\tilde{\alpha}_0}{\omega^2-q^2-m_\pi^{\,2}}}$$

$$\left[\ 1-\Sigma_{L\lambda}\,\frac{G_\lambda(q,\tilde{\alpha}_0)}{G_\lambda(K',\tilde{\alpha}_0)}\ \frac{K'}{q}\int_0^R dx_i\ \frac{3x_i^{\,2}}{R^3}\,b^2_{L\lambda}j_L(K'x_i)\ j_L(q\ x_i)\right.$$

$$\left.+\tilde{\alpha}_0\Sigma_{L\lambda}\frac{G_\lambda(q,0)}{G_\lambda(K',\tilde{\alpha}_0)}\ \frac{K'}{q}\int_0^R dx_i\ \frac{3x_i^{\,2}}{R^3}\ b^2_{L\lambda}\ j_L(K'x_i)\,j_L(qx_i)\right] \tag{18}$$

with

$$\tilde{\alpha}_0 = \tilde{\alpha}_0\ (\rho_0) \qquad\qquad K'^2 = K^2(1+\tilde{\alpha}_0)^{-1}$$

$$G_\lambda(q,\tilde{\alpha}_0) = (1+\tilde{\alpha}_0)\,j'_\lambda(qR)\,h_\lambda(KR) - j_\lambda(qR)\,h'_\lambda(KR) \tag{19}$$

In the last expression the derivatives are taken with respect to the variable x at $x = R$.

The first term $\mathrm{Im}\left\{\tilde{\alpha}_0\left[1-\frac{q^2\tilde{\alpha}_0}{\omega^2-q^2-m_\pi^2}\right]^{-1}\right\}$ represents the renormalized response of infinite nuclear matter. In this piece the pion branch corresponds to its (near) singularity which occurs at $\omega^2 - q^2(1+\tilde{\alpha}_0) - m_\pi^2 = 0$. However it turns out that the strength for this branch is suppressed by the second term which corresponds to a reflexion at the nuclear surface. In fact in the case where α_0 is purely real the residue of the singularity vanishes and the collective pion branch carries no strength whatever the radius. However when the absorption is introduced the suppression is not total but only partial and it depends on the nuclear radius. As expected it is more effective in small systems.

The results of our calculation are shown in Fig. 2 which gives the longitudinal response per nucleon as a function of ω for a fixed momentum $q = 1\ \mathrm{fm}^{-1}$ for some values of the radius R. The main features of the nuclear matter result are preserved, namely the shadowing of the Δ region and the bump at the position of the pion branch. However even in the largest nuclei the nuclear matter limit is not reached : the optical branch is a factor two smaller than the nuclear matter valeur. The scale which governs the approach to the nuclear matter result is the product $(\mathrm{Im}\ K')R$.

We have tested our square well model on the total cross sections for physical pions where the shadow shows up and the pionic branch is absent. In spite of the crudeness of the model the π nucleus total cross sections are roughly consistent with the experimental ones and our model predicts the proper amount of shadowing of the total π nucleus cross sections.

We have therefore found worth to set the conditions of the (^{3}He,T) experiment, $\omega = 0.812\ q$, for nuclei which similate the cases of ^{12}C and ^{56}Fe. The energy momentum relation is such that the shadowing effect in the Δ region fully applies. In both cases we find that the longitudinal response is shifted at lower energies, by $\approx$ 100 MeV. This displacement is consistent with the experimental findings [7]. However a direct comparison is not allowed because

i) the (^{3}He,T) reaction does not probe exclusively the spin longitudinal response but also the spin transverse one which is not shifted

ii) the distortion of the ^{3}He and T waves has to be taken into account.

However these problems do not seem to be untractable and hadron scattering is likely to become a tool for the study of the pion branch. In particular by going to non forward direction the momentum transfer can be increased sufficiently to suppress the shadow effect of the Δ, the pion becoming appreciably virtual. This would allow a check of our interpretation of the observed shift. On addition one could study in this way the propagation of more or less virtual particles in the nuclear medium and the approach to the shadowing regime.

Acknowledgements

It is a pleasure to thank Dr. J. DELORME for useful discussions.

References

1 C. Ellegaarde et al.
Phys. Rev. Lett. 50, (1983) 1745.

2 W.M. Alberico, M. Ericson and A. Molinari,
Nucl. Phys. A 379, (1982) 429.

3 W. Alberico, M. Ericson and A. Molinari,
CERN Preprint 3635 (1983).

4 C. Chollet, J. Arends, H. Beil, R. Bergère, P. Bourgeois,
P. Carlos, J.L. Fallou, J. Fagot, P. Garganne, A. Leprêtre and
A. Veyssière,
Preprint 1983.

5 R.F. Sawyer,
Nucl. Phys. A 335 (1980) 315.

6 J. Delorme, M. Ericson, A. Figureau and C. Thévenet,
Ann. Phys. 102, (1976) 273.

7 C. Gaarde,
Preprint Argonne, May 1983.

Figure Caption

Figure 1 : The spin isospin response of infinite nuclear matter along the line $\omega = q$ in arbitrary units. Dot dashed line : unormalized response continuous line : longitudinal response in RPA ; dashed line : transverse response in RPA. The value if g' is set at $g' = 1/3$.

Figure 2 : The longitudinal spin isospin response at a fixed momentum q = 200 MeV/c Continuous line : infinite nuclear matter.
The other lines represent the finite nuclei responses for various values of the radius R (in fm).

Figure 3 : The RPA longitudinal spin isospin response along the line $\omega = 0.812\ q$ in the absence of renormalization (continuous line) and with renormalization in finite nuclei which simulate ^{12}C and ^{56}Fe.

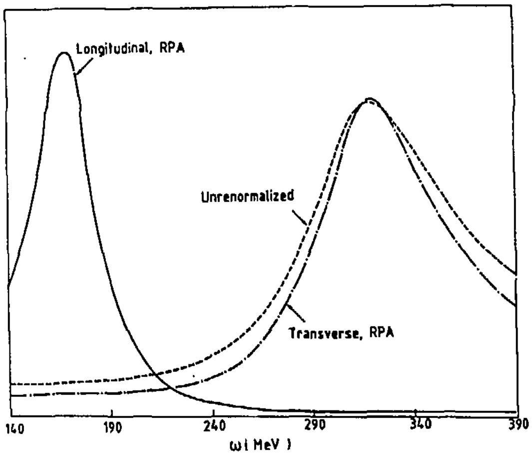

Figure 1

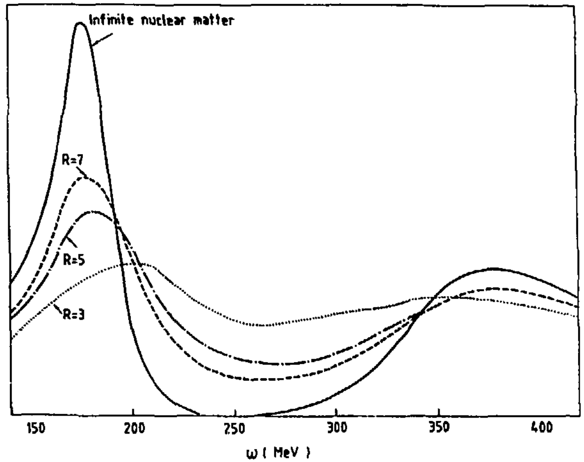

Figure 2

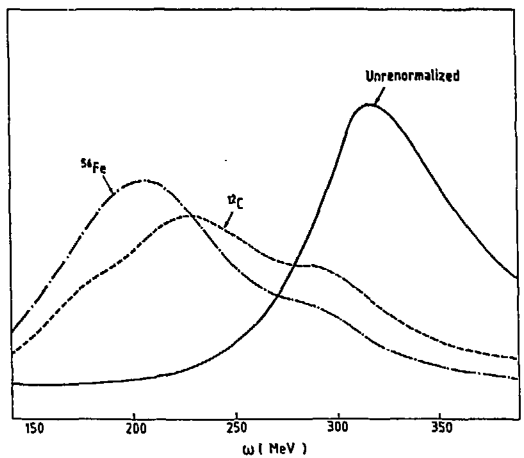

Figure 3