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DYNAMICS OF R.F. CAPTURED ELECTRON COOLED PROTON BEAMS

W. Kells and F. Mills

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Introduction

In the course of electron cooling experiments at the Electron Cooling Ring (ECR) at Fermilab, several peculiar features of the longitudinal phase space of cold protons (200 MeV) captured in R.F. buckets were observed. Here we present the experimental facts, present a simple theory, and summarize computer simulation results which support the theory and facts.

The experimental apparatus and measurement techniques have been described elsewhere.[1,2,3] R.F. bunching was achieved with a single PPA, loaded cavity gap driven at harmonic number 6($\sim$ 7.56 MHz) of the revolution frequency. R.F. voltage could be developed across this gap sufficient to entirely capture even the uncooled circulating proton beam ($\delta p/p$ FWHM = 0.17%).

1. Experiments

All results described were obtained by spectrum analysis or time domain analysis of longitudinal "Schottky" signals.[4] A time domain technique, first suggested by F. Mills was used for initial qualitative observation of the longitudinal electron cooling. This "cooling into buckets" is illustrated in figure 1. We use a vertical energy scale of peak-peak cavity gap volts, the origin being defined as bucket center (which may drift with respect to absolute proton energy due, e.g., to bend current drift or ripple). Proton energy difference from bucket center to bucket half width is:

$$\Delta E_p = \beta \sqrt{\frac{2eV_{RF}E_p}{\pi\eta h}} \qquad (1)$$

we have $\beta = 0.57$, $E_p \sim 1138$ MeV, $\eta = 0.609$, $h = 6$. For comparison to the small buckets actually used (figure 1), we note that V_{RF} = 1.5 KeV corresponds to a bucket whose width equals the initial uncooled proton width ($\delta p/p = 0.17\%$). On this same energy scale we represent the electron beam relative to the bucket center by a line (negligible spread) which would ideally be "tuned" exactly to bucket center. If this were the case, the equilibrium cooled bunches would be Gaussian balls centered on the bucket with width determined by the electron beam temperature.[4,5]

Figure 2 illustrates a good longitudinal pick-up spectrum of the asymptotically cooled beam - a comb (tooth spacing = ω_{RF}) of approximately equal envelope intensity up to the electronic band edge ($\sim$ 500 MHz). The high signal level of this coherent source allows spectra averaged over short time intervals (minimizing the smearing of fluctuations). This spectra is equivalent to a bunch width $\sigma_\psi \leq 1.3\times10^{-2}$ rad. Knowledge of the pp V_{RF} (30 volts here) yields, from eqn. (1), an upper limit on the electron energy spread:

$$\delta E_e = \frac{m_e}{m_p} \Delta E_p \sqrt{\sin\sigma_\psi} < 2.8 \text{ eV} \qquad (2)$$

On the other hand, we know from direct measurements of the electron system high voltage that $\delta E_e \sim 6.5$ volt fluctuations occured.

If we now detune the electron system energy from bucket center, the comb does not simply roll off at lower frequency. Strong high frequency components are evident for detunings up to a point where the electron line becomes tangent to the separatrix ($\Delta E_e \sim 50$ volts for $V_{RF} = 30$ volts). Zeros in the comb envelope were observed, as would be expected in the Fourier transform of, e.g., a ring distribution (figure 1 and below). However, the various jitters made determination of the exact envelope shape impossible (and hence the line density projection, upon transforming back).

A similar experiment with R.F. buckets was performed in order to measure the "drag force".[6] Protons are cooled with no R.F. on. Then a small bucket is switched on (separatrix far from the electron line) and the electron beam voltage is switched to bucket center. Pick up intensity is monitored as a function of time as illustrated in figure 3 (spectrum analyser used as a receiver tuned to some $h\omega_{RF}$). Until the protons are captured the pickup signal is incoherent ($\sim$ 0). Here we focus attention on the sharp transient observed before the steep rise to coherence. Such structure was not typically seen during the cooling into buckets experiment.

Figure 4 shows drawings of actual spectra (a single high R.F. harmonic, h_{RF} = 60, chosen to maximize frequency dispersion). It is evident that the coherent peak follows any detuning of ω_{RF} from E_e. As detuning increases a larger halo of protons which have slipped outside the separatrix arises. Finally (for detuning $\sim$ separatrix height), the peak "snaps back" to the E_e- equilibrium frequency. It is no longer a coherent peak but a true Schottky band.

2. MODEL

The above findings can all, at least qualitatively, be explained by an elementary consideration of the usual SHO model of R.F. bucket motion plus the influence of a smoothed F_{11} (friction drag) force. The form of F_{11} is approximately[6] as shown in figure 5. The important dynamical feature is the negative slope of F_{11} for $\Delta E > \delta E_{max}$ (which is small on the energy scale of figure 2). This negative slope is equivalent to a dissipation. The total Hamiltonian is nonconservative. the general motion of a proton within the bucket but below the electron mean energy line is a spiral approaching a limit cycle tangent (on the large bucket energy scale) to the electron line.

Notice that a single point ($E_p = 0$, ψ_{EQ}) equilibrium also exists for low enough F_{11};

$$-F_{11}(\Delta E_e-) = F_{RF}\,(\psi_{EQ})$$

however, this is unstable (viz. negative F_{11} slope at $\delta E_e \sim \Delta E_e$). In the limit of $\psi_{EQ} << 1$ (weak cooling compared to R.F.), the limit cycle is nearly an H_{SHO} ring, tanget to the line $\Delta E_e \approx \delta E_{max}$. A larger cooling force (e.g., higher e^- beam current) produces a cycle "ring" more and more squashed against the $\sim \Delta E_e = \delta E_e$ line. Finally, a cooling strength is attainable at which protons permanently adhere to the $\sim \Delta E_e$ line and are siphoned out of the bucket ("halo" in figure 4). For a given V_{RF}, the strength to siphon out is:

$$-\left.\frac{\Delta E_p}{T_o}\right|_{cooling} \geq \frac{eV_{RF}}{T_o}\sin\psi_{INT}$$

where T_o is the rotation period and ψ_{INT} is the bucket phase of the ΔE_e- line separatrix intersection. In terms of $F_{11,max}$:

$$-F_{11,max} \geq \frac{eV_{RF}}{\zeta T_o \beta c}$$

Where ζ = fraction of ring circumference cooled. The RHS is ~ 0.2 eV/cm for the typical small buckets used, which is about 10^3

larger than the $F_{11,max}$ valves observed.[2,7] Thus our experiments should correspond to rings ∼ symmetrical about the bucket center.

Notice that as ΔE_e is increased (corresponding to the experiment of figure 4b, c), passing everywhere outside the separatrix, protons must still spiral out to this value. Thus high frequency components (viz. narrow $\delta\psi$ ring thickness, characteristic of the electron temparature) at $h\omega_{RF}$ will remain for all detuning up to separatrix crossing. At crossing, the protons rapidly lock to the frequency defined by the electron beam energy (still with high frequency components at each Schottky band).

3. Simulations

A monte carlo particle tracking program was developed to display these features in detail (Program RFCOOL). Up to ∼500 "protons" randomly generated in any region of initial longitudinal phase space (all transverse motion ignored) are followed in steps of up to ∼10 T_o per itteration. F_{11} is incorporated as a smooth function (as in figure 5) giving protons a kick, $\Delta P = F_{11} \times T_o$ per revolution. Proper adiabadic turn on/off of V_{RF} was included.

Figure 6 illustrates an equilibrium ring limit cycle for V_{RF} and F_{11} characteristic of our cooling experiments. The initial proton distribution was uncooled in this run (as in figure 1).

A sequence as sketched in figure 3 was also simulated. In this case output was generated at a series of times between the moment of ΔE_e change (to bucket center) and final cooling to a "ball" at bucket center. Histograms at each output were FFT'ed in ψ. The result, as a function of time from ΔE_e - change is plotted in figure 7. Essentially the "interference"-like spikes in figures 3 and 7 are the result of the rapid $\pi/2$ phase change in the charge distribution center of gravity as it is captured. Just before capture, protons congregate at the unstable fixed point. But the final charge "ball" is $\pi/2$ further advanced in phase. Clearly at some intermediate point in time, the $h_{RF} = 1$ moment vanishes while the $h_{RF} = 2$ moment will go through a local maximum.

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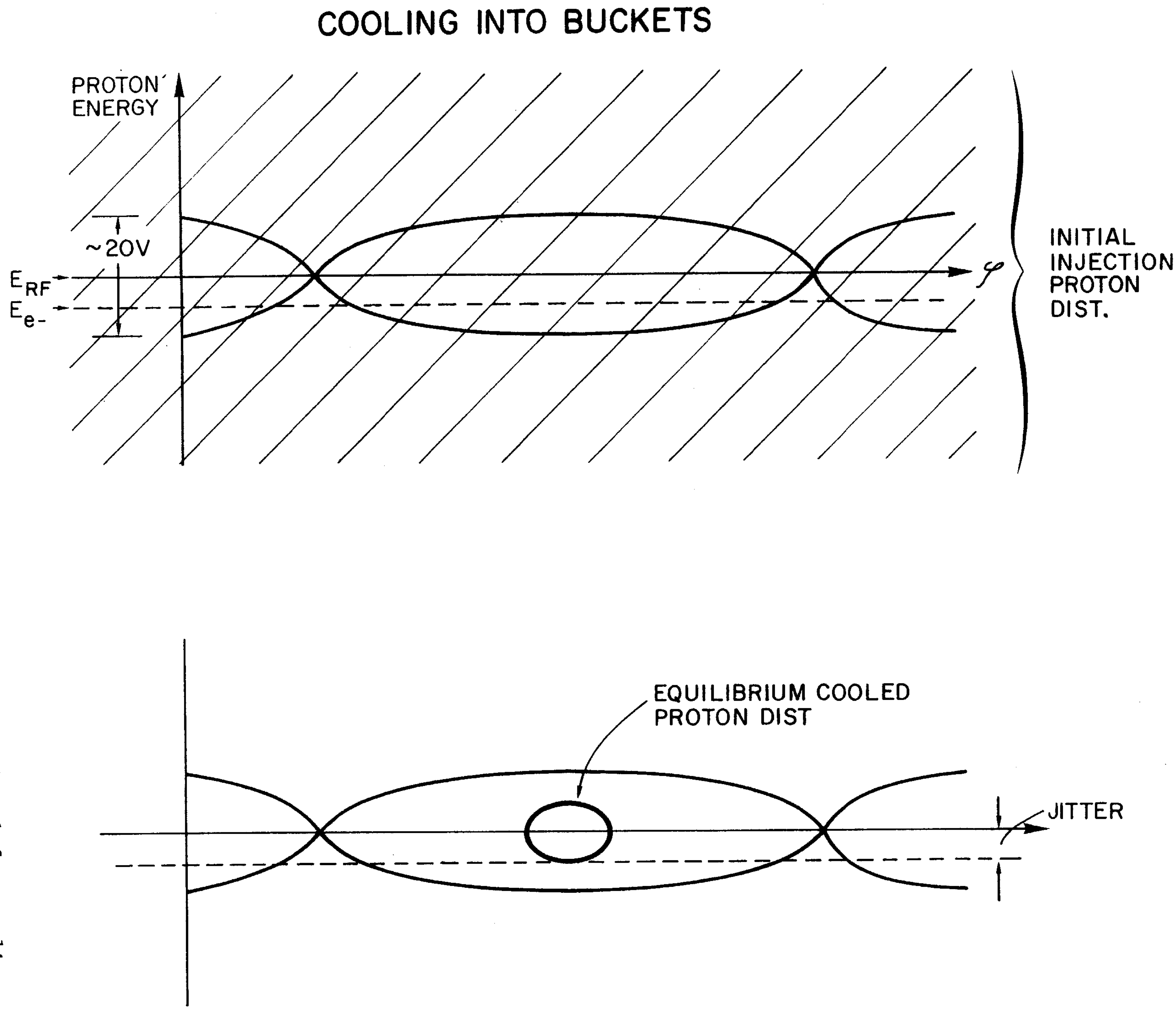
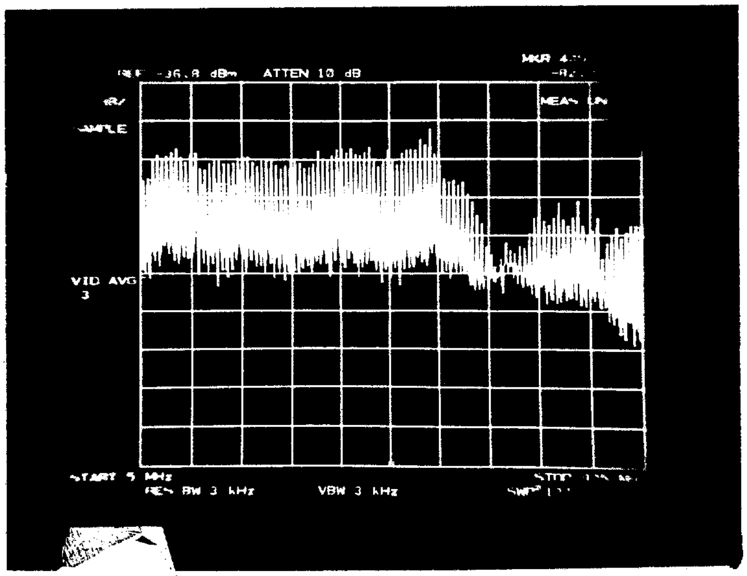


Fig. 1 Top: Initial conditions before cooling; protons mostly outside of bucket. Bottom: Equilibrium "ring" limit cycle distribution of protons after they have completely cooled into the bucket.



<u>Fig. 2</u> Spectrum analyser trace of $h\omega_{RF}$ combs for "tuned" bucket cooling (ω_{RF} <=> E_e-). Horizontal scale spans ∿ 0 - 850 MHz. Roll-off at ∿ 500 MHz due principally to electronics bandwidth limit.

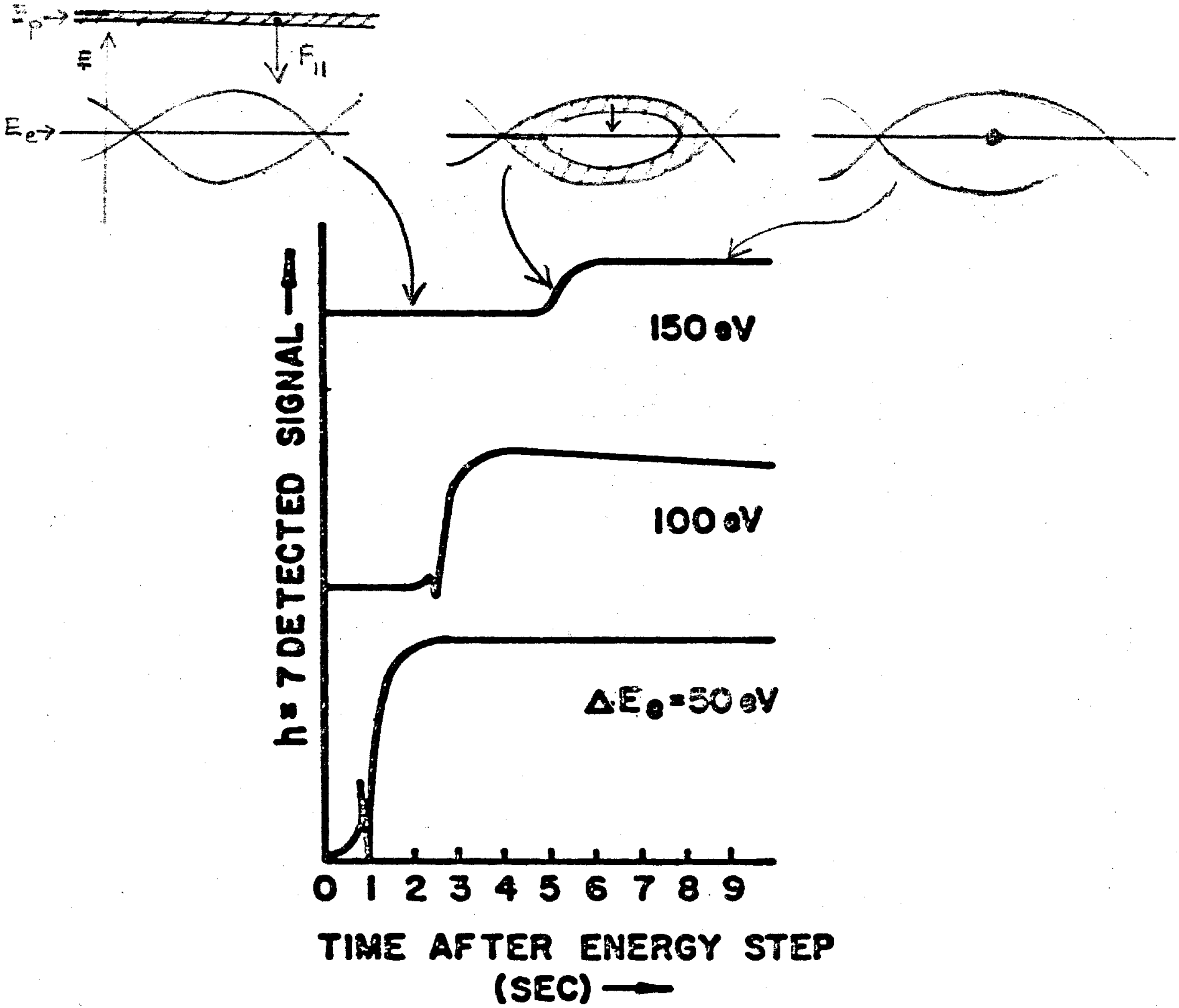


Fig. 3 Time sequence of a narrow momentum spread proton band (cold) being dragged into an R.F. bucket. Three experimental traces are illustrated (with different initial offsets). The transient at capture is to be noted.

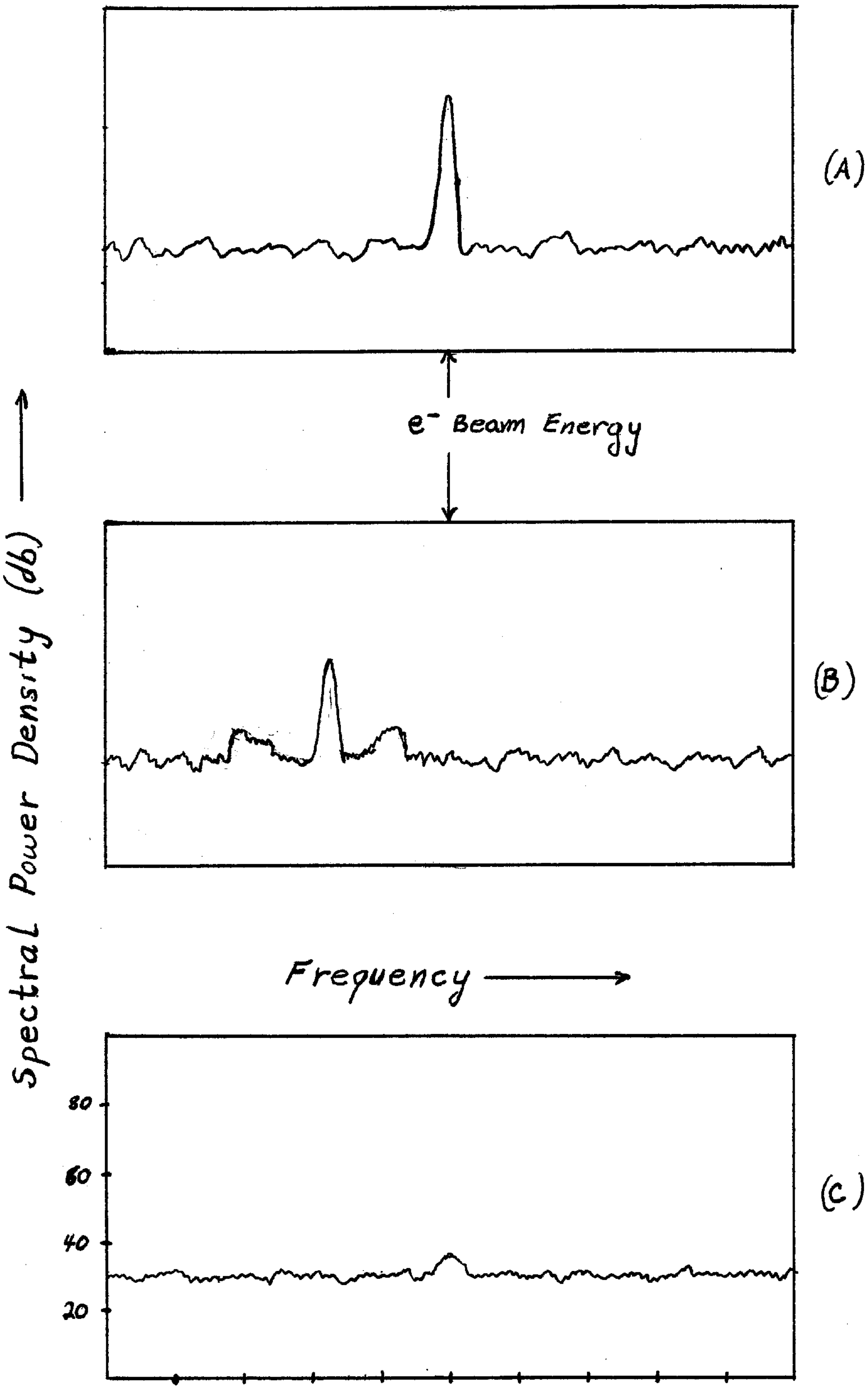


Fig. 4 a) tuned (E_e - <=> ω_{RF}) case: peak is at 60 ω_{RF} = 453.97 MHz. Trace is 16 KHz/Div. b) ω_{RF} detuned by -310 Hz. c) ω_{RF} detuned > 1 KHz.

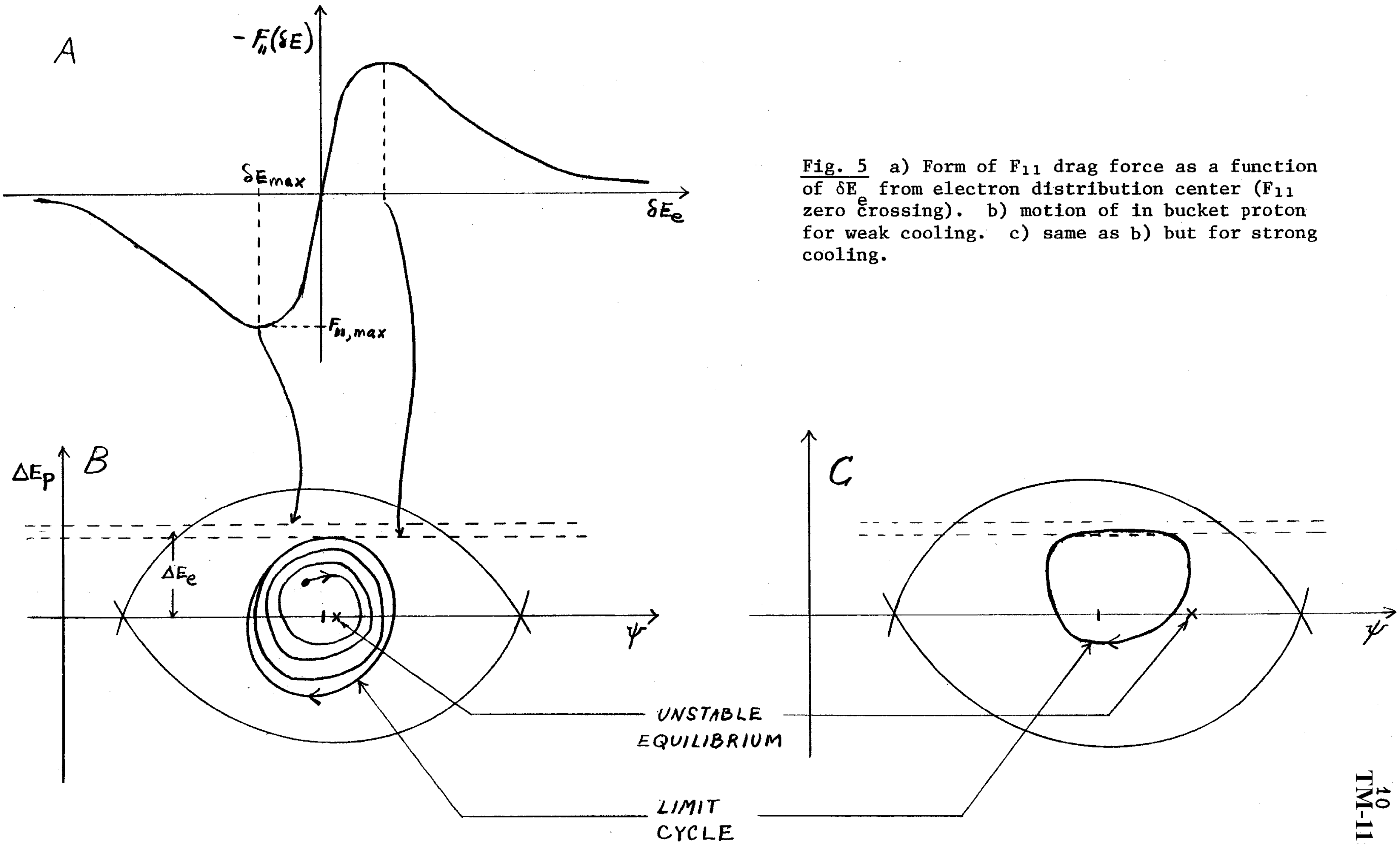


Fig. 5 a) Form of F_{11} drag force as a function of δE_e from electron distribution center (F_{11} zero crossing). b) motion of in bucket proton for weak cooling. c) same as b) but for strong cooling.

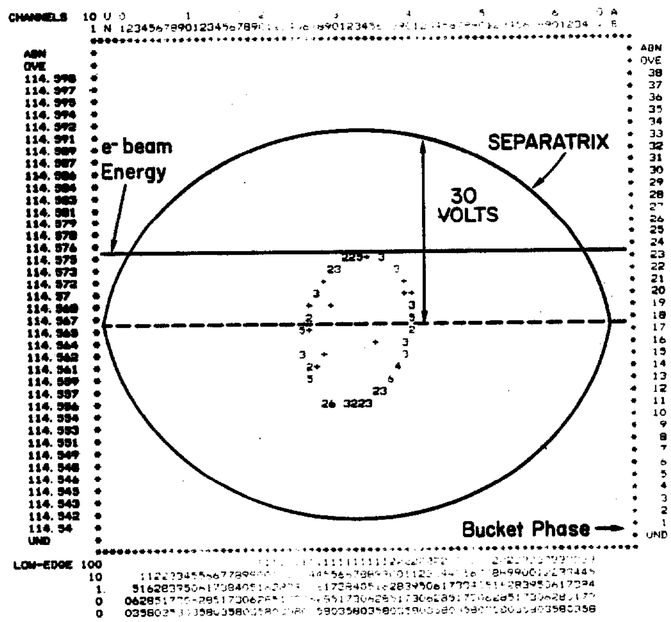


Fig. 6 Simulation result showing proton equilibrium for detuned electron energy. Verticle scale in absolute proton energy (MeV).

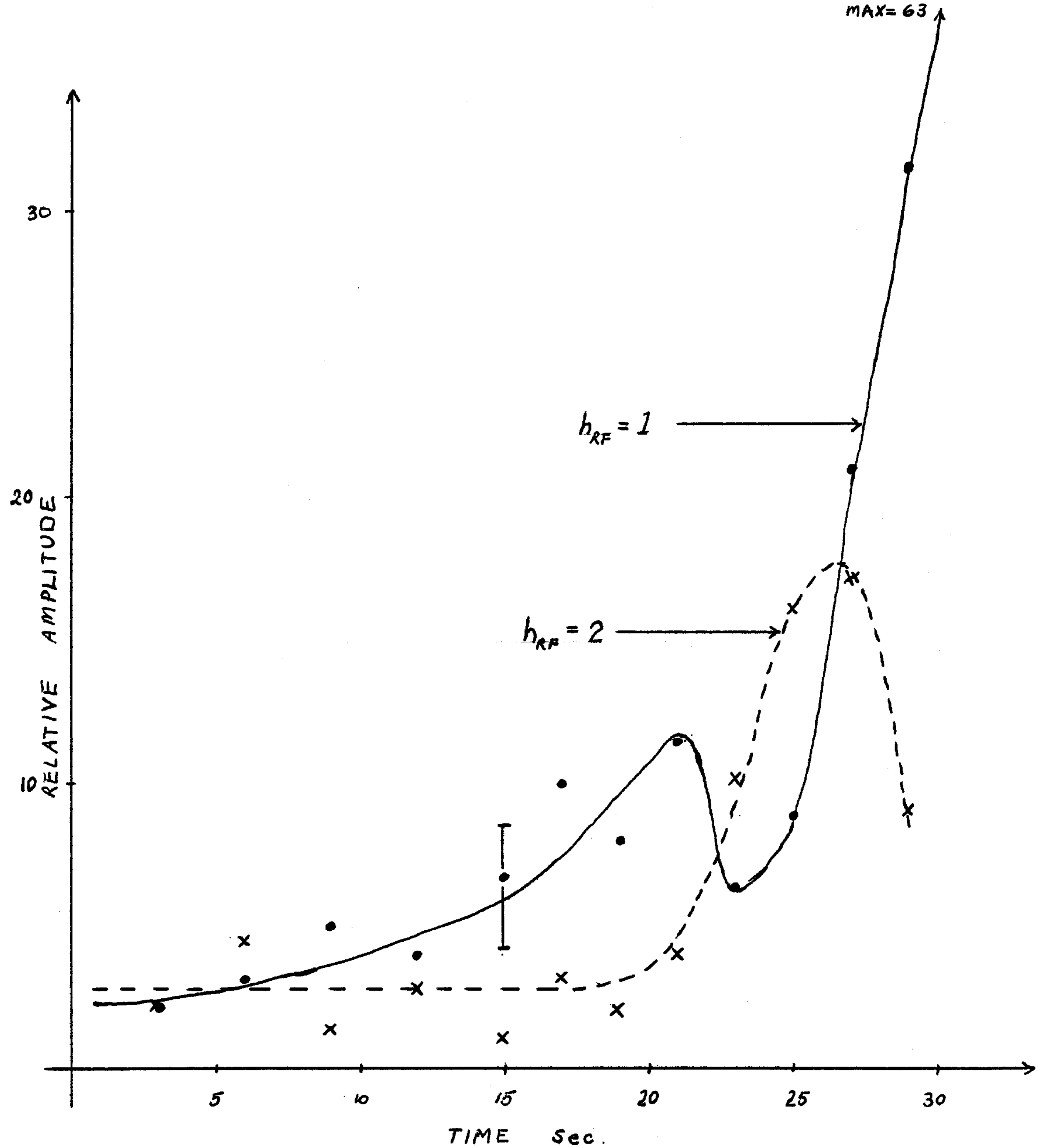


Fig. 7 Plot of h = 1 and h = 2 FFT moments of protons cooling into a bucket as a function of time. Two σ error bars (500 particles) are indicated.