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Kurzfassung

einer in Zusammenarbeit zwischen dem Institut für Theoretische Physik der Universität Graz (Prof.Dr.N.Pucker) und dem Institut für Reaktorsicherheit (Dr.F.Putz) verfaßten Dissertation

Österreichisches
Forschungszentrum Seibersdorf
Ges.m.b.H.

INSTITUT FÜR REAKTORSICHERHEIT

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MULTIPLE COLLISION SOLUTIONS FOR TIME-DEPENDENT NEUTRON TRANSPORT IN SLABS OF FINITE THICKNESS

Abstract

Multiple collision solutions of the time-, space-, and angle-dependent neutron transport equation in slab geometry are given. Two different monodirectional sources have been used: Firstly, a $\delta(t)$-shaped pulse of neutrons ($\delta(t)$:Dirac delta distribution) impinging on the slab at time $t = 0$, secondly, a "rectangular" source, emitting neutrons for a time interval Δt, describing a somewhat more realistic situation. Detailed results up to collision order three are discussed and exhibited in several figures. Interestingly the "scalar" flux of one times scattered neutrons for the slab problem turns out to be independent of space in the region influenced by the slab boundaries.

ZEITABHÄNGIGER TRANSPORT VON MONOENERGETISCHEN NEUTRONEN DURCH PLATTEN ENDLICHER DICKE

Kurzfassung

"Multiple Collision"-Lösungen der zeit-, raum- und winkelabhängigen Neutronentransportgleichung in Plattengeometrie werden ermittelt. Zwei verschiedene Quellen, die Neutronen in eine bestimmte Richtung aussenden, wurden verwendet: Erstens ein $\delta(t)$-Neutronenpuls ($\delta(t)$: Dirac-Delta-Distribution), der zur Zeit $t = 0$ auf die Platte trifft, zweitens, eine "Rechteckquelle", die Neutronen ein Zeitintervall Δt lang aussendet, um eine etwas realistischere Situation zu beschreiben. Detaillierte Ergebnisse bis zur Stoßordnung drei werden diskutiert und in mehreren Abbildungen dargestellt. Interessanterweise ergibt sich der "skalare" Fluß einmal gestreuter Neutronen als unabhängig vom Ort in dem Bereich, der von den Plattengrenzen beeinflußt wird.

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I. INTRODUCTION

The multiple collision method, also known as "order of scattering theory", "Neumann series solution", or "method of successive approximation", has been successfully applied to time dependent neutron transport problems.[1]

In case of infinite and semi-infinite media the time-, space-, and angle-dependent neutron distribution function may be written as a product of a pure function of time t multiplied by a function of the two variables μ and η, where μ is the particle direction cosine and $\eta = x/vt$, with x and v the particle position and velocity respectively. In this way it is possible to "reduce" the multiple collision equations to simpler ones, which may be solved by integration in the complex plane.[2-6]

If one introduces a finite slab geometry two serious difficulties arise:

1) The above mentioned separation of the neutron distribution function is not possible, since space and time cannot be combined into a single variable η.
2) The reduction of the finite problem to the infinite one via the Lemma of Placzek[7)] leads to unknown neutron fluxes leaving the slab, which are by themselves special solutions of the problem.

Because of fact 1) one cannot "reduce" the multiple collision equations. Difficulty 2) may be overcome if one modifies the Lemma of Placzek in a suitable manner. Two different neutron sources have been assumed: Firstly, a $\delta(t)$-shaped pulse source and secondly, a source emitting neutrons for a time intervall Δt, which in contrast to the first source leads to solutions which are already non-singular for collision order one. In the limit $\Delta t \to 0$, and keeping constant the total number of emitted neutrons, the two sources become identical, while with $\Delta t \to \infty$ one gets a constant source and may study the building up of the stationary problem. For the first source analytical solutions are established up to collision order two. The distribution of three times collided neutrons is evaluated numerically, while for the second source solutions are evaluated for one collision order less. In part IV detailed results are discussed and exhibited by respective figures.

II. THEORY

Using the notation of B.D.Ganapol[3] the monoenergetic time dependent neutron transport equation in plane geometry is

$$[\tfrac{1}{v}\partial/\partial t+\mu\partial/\partial x+\Sigma]\Phi(x,\mu,t)=\frac{c}{2}\Sigma\int_{-1}^{+1}\Phi(x,\mu',t)d\mu'+S(x,\mu,t) \quad . \qquad (1)$$

Two different monodirectional sources are used:

$$S(x,\mu,t) = H(\mu)\delta(\mu-\mu_o)\delta(x)\delta(t) \qquad (2)$$

$$S(x,\mu,t) = H(\mu)\delta(\mu-\mu_o)\delta(x)H(-t+\Delta t)/\Delta t \ , \qquad (3)$$

where $H(s)$ is the unit step function. The appropriate initial and boundary conditions for a slab of finite thickness d are as follows:

1) No neutrons exist in the slab before source emission

$$\Phi(x,\mu,t) = 0 \;, \qquad t < 0 \tag{4}$$

2) No neutrons enter the slab except from the source

$$\Phi(0,\mu,t) = 0 \qquad \text{for} \quad \mu > 0 \tag{5a}$$

$$\Phi(d,\mu,t) = 0 \qquad \text{for} \quad \mu < 0 \;. \tag{5b}$$

As the Boltzmann-equation (1) represents a local balance equation the boundary conditions (5) may be incorporated into the equation. Following directly the prescription of the Lemma of G. Placzek[7] one would have to absorb all the neutrons coming out of the slab by introducing appropriate surface sinks:

$$\left[\frac{1}{v}\partial/\partial t+\mu\partial/\partial x+\Sigma\right]\Phi(x,\mu,t)=\frac{c}{2}\Sigma\int_{-1}^{+1}\Phi(x,\mu',t)d\mu'+S(x,\mu,t) +$$

$$-|\mu|H(-\mu)\Phi(0,\mu,t)\delta(x)-\mu H(\mu)\Phi(d,\mu,t)\delta(x-d) \quad . \tag{6}$$

However one would already have to know special solutions of the problem - the outcoming neutron fluxes $\Phi(0,\mu,t)$ and $\Phi(d,\mu,t)$.

In case of a semi-infinite medium B.D.Ganapol was successful in determining the necessary quantity from the corresponding stationary albedo problem.[4,5] As mentioned in the introduction, for the slab problem an analogous procedure fails, and in this paper a suitable modification of the

Placzek Lemma is used: The medium is extended again over the whole space, however, the neutrons coming out of the slab are not absorbed. These neutrons enter the region surrounding the slab and some of them are scattered back. They are absorbed when crossing the boundaries of the slab in inward direction. In this way the boundary conditions are fulfilled as well. Equation (1), taking now into account the boundary conditions (5) in the form of "correcting" flux terms Φ_{corr} reads:

$$\left[\frac{1}{v}\partial/\partial t+\mu\partial/\partial x+\Sigma\right] \Phi(x,\mu,t) = \frac{c}{2}\Sigma\int_{-1}^{+1}\Phi(x,\mu',t)d\mu'+S(x,\mu,t)+$$

$$- \mu H(\mu)\Phi_{corr}(0,\mu,t)\cdot\delta(x) - |\mu|\, H(-\mu)\Phi_{corr}(d,\mu,t)\delta(x-d) \quad (7)$$

The relevant boundary condition is

$$\Phi(x,\mu,t) = 0 \quad \text{for } |x| \to \infty \quad (8)$$

Substituting the Neumann series

$$\Phi(x,\mu,t) = \sum_{n=0}^{\infty}\Phi_n\,(x,\mu,t) \quad (9)$$

into equation (7) the recursive coupled multiple collision equations are:

$$\left[\frac{1}{v}\partial/\partial t+\mu\partial/\partial x+\Sigma\right]\Phi_0(x,\mu,t) = S(x,\mu,t) \quad (10)$$

$$\left[\frac{1}{v}\partial/\partial t+\mu\partial/\partial x+\Sigma\right]\Phi_n(x,\mu,t) = \frac{c}{2}\Sigma\int_{-1}^{+1}\Phi_{n-1}(x,\mu',t)d\mu'+$$

$$-\mu H(\mu)\Phi_{n\,corr}(0,\mu,t)\delta(x) - |\mu| H(-\mu)\Phi_{n\,corr}(d,\mu,t)\delta(x-d) \quad (11)$$

$$n = 1,2,3...$$

with the initial and boundary conditions

$$\Phi_n(x,\mu,t) = 0 \quad \text{for } t < 0 \quad (12a)$$

$$\Phi_n(x,\mu,t) = 0 \quad \text{for } |x| \to \infty \quad (12b)$$

The solutions of (11) are the distributions of n-times collided neutrons $\Phi_n(x,\mu,t)$ within the slab $(0 \leq x \leq d)$.
The correcting neutron fluxes $\Phi_{n_{corr}}(x,\mu,t)$ may easily be obtained from an appropriate infinite medium problem (for $\Phi_{n_{corr}}(0,\mu>0,t)$ see Fig. 1; analogously for $\Phi_{n_{corr}}(d,\mu<0,t)$).

The respective transport equation is

$$\left[\frac{1}{v}\partial/\partial t+\mu\partial/\partial x+\Sigma\right]\Phi_{n_{corr}}(x,\mu,t) = \frac{c}{2}\Sigma\int_{-1}^{+1}\Phi_{n-1}(x,\mu',t)d\mu' \qquad (13)$$

where $\Phi_{n-1}(x,\mu,t)$ is a solution of equation (11).
The essential point is that only the slab solution $\Phi_{n-1}(x,\mu,t)$ has to be used for evaluating $\Phi_{n_{corr}}(x,\mu,t)$.
Every linear differential equation of first order may be integrated along its characteristic curves [8].
This is done for equation(11); its characteristic curves are

$$t = t' \quad \text{and } x = x_0+\mu vt'. \qquad (14)$$

Using the integrating factor $e^{-v\Sigma t'}$ we now have:

$$\begin{aligned}\frac{d}{dt'}\left[e^{v\Sigma t'}\Phi_n(x_0+\mu vt',\mu,t')\right]&=e^{v\Sigma t'}\,\frac{v}{2}c\Sigma\int_{-1}^{+1}\Phi_{n-1}(x_0+\mu vt',\mu',t')d\mu'\\ &-ve^{v\Sigma t'}\,\mu\, H(\mu)\Phi_{n_{corr}}(0,\mu,t')\delta(x_0+\mu vt')+\\ &-ve^{v\Sigma t'}\,|\mu|H(-\mu)\Phi_{n_{corr}}(d,\mu,t')\delta(x_0+\mu vt'-d) \qquad (15)\end{aligned}$$

Integration from 0 to t, using the initial condition (12a) and transforming x_0 back to $x_0 = x-\mu vt$ yields:

$$\Phi_n(x,\mu,t)=\underbrace{\frac{v}{2}c\Sigma\int_0^t e^{-v\Sigma(t-t')}\int_{-1}^{+1}\Phi_{n-1}(x-\mu v(t-t'),\mu',t')d\mu'dt'}_{\text{solution of the infinite medium problem}}+$$

$$-H(\mu)\, e^{-\Sigma\frac{x}{\mu}}\, \Phi_{n_{corr}}(0,\mu,t-\frac{x}{\mu v})\, \left[H(\frac{x}{\mu v})-H(-t+\frac{x}{\mu v})\right] +$$

$$-H(-\mu)\, e^{-\Sigma\frac{x-d}{\mu v}}\, \Phi_{n_{corr}}(d,\mu,t-\frac{x-d}{\mu v})\, \left[H(\frac{x-d}{\mu v})-H(-t+\frac{x-d}{\mu v})\right] \quad (16)$$

Using the following properties of the unit step function

$H(ax) = H(x) \quad , \quad a > 0$

$[1-H(-b)] = H(b)$

one may simplify the Heavyside terms of (16):

$$H(\mu)\ \left[H(\frac{x}{\mu v})-H(-t+\frac{x}{\mu v})\right] = H(\mu)\ H(t-\frac{x}{\mu v}) \quad , \ x \geqq 0$$

$$H(-\mu)\ \left[H(\frac{x-d}{\mu v})-H(-t+\frac{x-d}{\mu v})\right] = H(-\mu)\ H(t-\frac{x-d}{\mu v}), \ 0 \leqq x \leqq d.$$

The solution of equation (13) supplies the necessary correcting flux terms:

$$\Phi_{n_{corr}}(\tilde{x},\tilde{\mu},\tilde{t}) = \frac{v}{2}c\Sigma\int_0^{\tilde{t}} e^{-v\Sigma(\tilde{t}-t')}\int_{-1}^{+1}\Phi_{n-1}(\tilde{x}-\tilde{\mu}v(\tilde{t}-t'),\mu',t')d\mu'dt' . \quad (17)$$

Substituting this result into equation (16) yields

$$\Phi_n(x,\mu,t) = \frac{v}{2}c\Sigma\int_0^{t} e^{-v\Sigma(t-t')}\int_{-1}^{+1}\Phi_{n-1}(x-\mu v(t-t'),\mu',t')d\mu'dt' +$$

$$-H(\mu)H(t-\frac{x}{\mu v})\ \frac{v}{2}c\Sigma\int_0^{t-\frac{x}{\mu v}} e^{-v\Sigma(t-t')}\int_{-1}^{+1}\Phi_{n-1}(x-\mu v(t-t'),\mu',t')d\mu'dt'$$

$$-H(-\mu)\ H(t-\frac{x-d}{\mu v})\ \frac{v}{2}c\Sigma\int_0^{t-\frac{x-d}{\mu v}} e^{-v\Sigma(t-t')}\int_{-1}^{+1}\Phi_{n-1}(x-\mu v(t-t'),\mu',t')d\mu'dt'$$

Splitting into forward and backward directions and using the identity

$$H(b)\int_0^{b}\ldots \equiv \int_0^{bH(b)}\ldots$$

one obtains the solution

$$\Phi_n(x,\mu,t) = \frac{v}{2}c\Sigma\int_{\substack{t_0 \text{ for } \mu>0\\ t_d \text{ for } \mu<0}}^{t} e^{-v\Sigma(t-t')}\cdot\int_{-1}^{+1}\Phi_{n-1}(x-\mu v(t-t'),\mu',t')d\mu'dt' \quad (18)$$

where

$$t_0 \equiv (t-\frac{x}{\mu v})\ H(t-\frac{x}{\mu v})$$

$$t_d \equiv (t-\frac{x-d}{\mu v})\ H(t-\frac{x-d}{\mu v}) \qquad 0 \leq x \leq d \quad .$$

Since $\lim\limits_{d\to\infty} |t_d| = 0$ the solution for the semi-infinite medium is also contained in equation (18).

Because the left boundary of the slab is situated at $x=0$ one can not shift this boundary to $-\infty$ in analogy to the previous procedure. But it is evident from the first line of equation (16) that in this case $t_0=0$. In Fig.2 it is shown how the $(n-1)$-times scattered neutrons

are "summed up" (along the dashed line) for $\mu>0$.

For taking into account that neutrons may not enter the slab the condition is

$$x-\mu v(t-t') \geq 0$$

which implies

$$t' \geq t - \frac{x}{\mu v}$$

(compare the definition of t_0).

Analogous considerations lead to t_d.

In this way one may have stated the solution (18) directly from integral transport theory.

III SOLUTIONS [9]

First the slab problem is solved assuming the source (2). Equation (10) has the well known solution

$$\Phi_0(x,\mu,t) = v\ H(\mu)\ e^{-v\Sigma t}\ \delta(\mu-\mu_0)\ \delta(x-\mu vt), \qquad (19)$$

since the distribution of unscattered neutrons is the same within the slab and an infinite medium in the region $0 \leqq x \leqq d$. This is also true for the distribution of neutrons scattered once into the forward direction ($\mu>0$):

$$\Phi_1(x,\mu>0,t) = \frac{v}{2}c\Sigma\ e^{-v\Sigma t}\ \frac{1}{\mu_0-\mu}\ \left[H(x-\mu vt)-H(x-\mu_0 vt)\right] \qquad (20)$$

In backward direction the result of the slab problem is

$$\Phi_1(x,\mu<0,t) = \frac{v}{2}c\Sigma\ e^{-v\Sigma t}\ \frac{1}{\mu_0-\mu}\left[H(t-\frac{x}{\mu_0 v})-H(t-(\frac{x-d}{\mu v}+\frac{d}{\mu_0 v}))\right]. \qquad (21)$$

The difference to the infinite medium solution is pointed out in Fig. 3.

Integration of $\Phi_1(x,\mu,t)$ over all directions

$$\int_{-1}^{0}\Phi_1(x,\mu'<0,t)d\mu' + \int_{0}^{1}\Phi_1(x,\mu'>0,t)d\mu' = \Phi_1(x,t) \qquad (22)$$

yields the "scalar" flux:

$$\Phi_1(x,t) = \frac{v}{2}c\Sigma\ \ e^{-v\Sigma t}.\ \Big\{$$

$$\left.\begin{aligned} &- \ln\left|\frac{1+\mu_0}{1-\mu_0}\right| H(x-\mu_0 vt) + \\ &+ \ln\left|\frac{x-\mu_0 vt}{(1-\mu_0)vt}\right| H(x-vt) + \\ &- \ln\left|\frac{x-\mu_0 vt}{(1+\mu_0)vt}\right| H(x+vt) + \end{aligned}\right\} \text{infinite medium}$$

$$+ \ln\left|\frac{x-\mu_0 vt}{(1+\mu_0)(vt-d/\mu_0)}\right| \quad H(x+vt-d-d/\mu_0)\Big\} \qquad (23)$$

which seems to be quite a surprising result. The space and time dependent scalar flux of neutrons scattered once turns out to be independent of the spatial coordinate for $x \geq d + \frac{d}{\mu_0} - vt$ (Fig. 4).

At long enough times ($vt \geq d+\frac{d}{\mu_0}$) the scalar distribution is homogeneous over the whole slab (Fig.5).

By substituting Eq. (23) into Eq. (18) we find the distribution of twice scattered neutrons in the forward direction:

$$\Phi_{2,slab}(x,\mu>0,t) = \left(\frac{v}{2}c\Sigma\right)^2 \quad e^{-v\Sigma t}\ \frac{1}{(\mu_0-\mu)v} \cdot \Big\{$$

$$\{(x-\mu vt)\ \left[\ln\left|\frac{1+\mu}{1-\mu}\right|\ H(-x+\mu vt)\ -\ \ln\left|\frac{x-\mu vt}{(1-\mu)vt}\right|\ \ H(-x+vt)\right]\ +$$

$$-(x-\mu_0 vt)\left[\ln\ \left|\frac{1+\mu_0}{1-\mu_0}\right|\ \ H(-x+\mu_0 vt)-\ln\left|\frac{x-\mu_0 vt}{(1-\mu_0)vt}\right|\ H(-x+vt)\right]\}$$ infinite medium

$$+(x-\mu vt)\left[\frac{\mu_0}{\mu}\ \ln\left|\frac{1+\mu_0}{\mu_0}\right|\ -\ \ln\left|\frac{1+\mu}{\mu}\right|\right]\ H(-x+\mu vt) \qquad \text{(CI)}$$

$$+\left[(x-\mu vt-d+\frac{\mu}{\mu_0}\ d)\ \ln\left|\frac{x-\mu vt-d+\frac{\mu}{\mu_0}d}{(1+\mu)(\frac{d}{\mu_0}-vt)}\right|-(x-\mu_0 vt)\ \ln\left|\frac{x-\mu_0 vt}{(1+\mu_0)(\frac{d}{\mu_0}-vt)}\right|\right]\cdot$$

$$\cdot H(x+vt-d-\frac{d}{\mu_0}) \qquad \text{(CII)}$$

$$+\left[\frac{\mu_0}{\mu}\ (x-\mu vt)\ \ln\left|\frac{\mu_0\ (x-\mu vt)}{(1+\mu_0)(x-\mu vt+\frac{\mu}{\mu_0}d)}\right|\ -\ (x-\mu vt-d+\frac{\mu}{\mu_0}d)\right.\cdot$$

$$\left.\cdot\ln\left|\frac{\mu(x-\mu vt-d+\frac{\mu}{\mu_0}\ d}{(1+\mu)(x-\mu vt+\frac{\mu}{\mu_0}d)}\right|\right]\ H(-x+\mu vt-\mu d-\frac{\mu}{\mu_0}\ d) \qquad \text{(CIII)}$$

(24)

The solution shows a distinct structure (Fig.6).

$$\Phi_{2_{\text{slab}}}(x,\mu>0,t) = \boxed{\begin{array}{c}\text{semi-infinite medium}\\ \boxed{\begin{array}{c}\text{infinite}\\ \text{medium}\end{array}} + \underbrace{\boxed{\text{CI}}\end{array}} + \boxed{\text{CII}} + \boxed{\text{CIII}}}_{\text{correcting terms}}$$

CI corrects the infinite medium solution to the semi-infinite one: As shown in Fig. 7 neutrons which have their second collision outside the slab may not contribute to the distribution.

CII and CIII are correcting the solution of the semi-infinite medium to the slab solution:

Fig. 8 shows why CII is switched on at $t = \frac{d}{\mu_0 v} + \frac{d-x}{v}$,
while Fig. 9 points out that after a time $t = \frac{d}{\mu_0 v} + \frac{d}{v} + \frac{x}{\mu v}$
one would correct too much with CI and CII.
Therefore CIII is a positive correction which puts the balance right.

Fig. 10 shows the typical time dependence of the neutron flux $\Phi_2\,(x,\mu>0,t)$. The solution for backward direction is:

$$\Phi_{2,\text{slab}}\,(x,\mu<0,t) = \left(\frac{v}{2}\,c\,\Sigma\right)^2 e^{-v\Sigma t}\ \frac{1}{(\mu_0-\mu)v}\cdot\Big\{$$

$$(x-\mu vt)\ \left[-\ln\left|\frac{x-\mu vt}{(1-\mu)vt}\right|\ H(-x+vt)\ \right]$$

$$-(x-\mu_0 vt)\ \left[\ln\left|\frac{1+\mu_0}{1-\mu_0}\right| H(-x+\mu_0 vt)-\ln\left|\frac{x-\mu_0 vt}{(1-\mu_0)vt}\right|\ \ H(-x+vt)\right]$$

(infinite medium)

$$+\left[\ln\left|\frac{1+\mu}{\mu}\right| - \frac{\mu_0}{\mu}\ln\left|\frac{1-\mu_0}{\mu_0}\right|\right]\ (x-\mu vt-d+\frac{\mu}{\mu_0}d)\ H(x-\mu vt-d+\frac{\mu}{\mu_0}d)$$

$$+\left[(x-\mu vt-d+\frac{\mu}{\mu_0}d)\ \ln\left|\frac{\mu_0(x-\mu vt-d+\frac{\mu}{\mu_0}d)}{(1+\mu)(d-\mu_0 vt)}\right| -(x-\mu_0 vt)\ \cdot\right.$$

$$\left.\ln\left|\frac{\mu_0(x-\mu_0 vt)}{(1+\mu_0)(d-\mu_0 vt)}\right|\right] H(x+vt-d-\frac{d}{\mu_0})$$

$$+\left[\ (x-\mu vt)\ \ln\left|\frac{\mu(x-\mu vt)}{(1-\mu)(x-\mu vt-d)}\right| - \frac{\mu_0}{\mu}(x-\mu vt-d+\frac{\mu}{\mu_0}d)\ .\right.$$

$$\left.\ln\left|\frac{\mu_0(x-\mu vt-d+\frac{\mu}{\mu_0}d)}{(1-\mu_0)(x-\mu vt-d)}\right|\right]\ H(x-\mu vt-d+\mu d)\qquad\Big\}$$

(25)

Only corrections due to the right slab boundary occur. The distribution of three times scattered neutrons is obtained by numerical integration.

Analytical solutions assuming the "rectangular" source (3) were evaluated up to collision order one:

$$\Phi_{0\ \text{slab},\Delta t\text{-source}}(x\geq 0,\mu,t) = \frac{H(\mu)}{\mu}\ \ \delta(\mu-\mu_0)\ e^{-\Sigma\frac{x}{\mu}}\ \frac{1}{\Delta t}\ \left[H(x-\mu v(t-\Delta t)-H(x-\mu vt)\right]$$

(26)

$$\Phi_{1,\text{slab},\Delta t\text{-source}}\ (x\geq 0,\mu>0,t) = \frac{1}{2}\ \frac{1}{\mu_0-\mu}\ \frac{1}{\Delta t}\ \ c.\ \Big\{$$

$$+\left[e^{-v\Sigma(t-\Delta t)} - e^{-\Sigma\frac{x}{\mu}}\right]\ \ H\ (x-\mu v(t-\Delta t))$$

$$-\left[e^{-v\Sigma(t-\Delta t)} - e^{-\Sigma\frac{x}{\mu_0}}\right]\ \ H\ (x-\mu_0 v(t-\Delta t))$$

$$- \left[e^{-v\Sigma t} - e^{-\Sigma\frac{x}{\mu}}\right] \quad H(x-\mu vt)$$

$$+ \left[e^{-v\Sigma t} - e^{-\Sigma\frac{x}{\mu_0}}\right] \quad H(x-\mu_0 vt) \quad \Big\} \qquad (27)$$

Numerical integration was used to obtain $\Phi_2(x,\mu,t)$. With increasing order of scattering the main differences between the solutions for the two sources (i.e. the singularities) vanish.

IV. DISCUSSION OF THE NUMERICAL RESULTS

The following assumptions have been used for each plot:

$\mu_0=1$: neutrons from the source enter the slab in perpendicular direction

$v=\Sigma=1$: this represents the choice of a special system of units (length unit = 1 mean free path, time unit = time to cover a distance of 1m.f.p.)

$c=1$: pure scattering (no absorption, $\Sigma_a = 0$)

$d=1$: thickness of the slab is 1 m.f.p.

The spatial and angular dependent distribution of once collided neutrons at time t = 1.0 is shown in Fig. 11.

The neutron front is just reaching the right slab boundary. At $x = \mu_0 vt = \mu vt$ the solution has a simple pole. At time t = 1.5 the distribution of once collided neutrons is affected by the right slab boundary (Fig. 12).

Due to the Heavyside step function $H(t-(\frac{x-d}{\mu v} + \frac{d}{\mu_0 v}))$ of equation (21) no neutrons exist in the region $x > \mu vt + d - \frac{\mu}{\mu_0} d$.

As shown in Fig. 13 the distribution of twice scattered neutrons $\Phi_2(x,\mu,t=1.0)$ is influenced by the left boundary of the slab up to $x=\mu vt$. As expected from the integration involved in Eg. (18) $\Phi_2(x,\mu,t)$ has a logarithmic singularity at the point $x=\mu_0 vt = \mu vt$. According to the boundary conditions for a slab the neutron flux $\Phi_2(x,\mu,t=1.5)$ (Fig.14)

vanishes at x=0 in forward direction and at x = d in backward direction. The distribution of three times scattered neutrons (Fig.15) is continuous for all (x,μ,t). At a time, when the neutron front has left the slab, the distribution of three times scattered neutrons (Fig.16) is similiar to the

distribution of twice scatterd neutrons of Fig.14.
For the "rectangular" source (3) already the distribution of once collided neutrons is continuous (Fig. 17, compare with Fig. 11).

As the rectangular source emits neutrons for a time intervall Δt the boundaries of the slab do not affect the neutron distribution (Fig.18) in such a sudden manner as with

with the $\delta(t)$ - shaped pulse source shown in Fig. 12. Since the main difference between solutions for the two different neutron sources (2) and (3) is their singular behaviour, with increasing collision order the solutions are becoming more similiar. See Fig. 19, 20 and compare with Fig.13,14 respectively.

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Fig.1. Evaluation of the correcting neutron flux $\phi_{n_{corr}}(x,\mu>0,t)$ from the appropriate infinite medium problem.

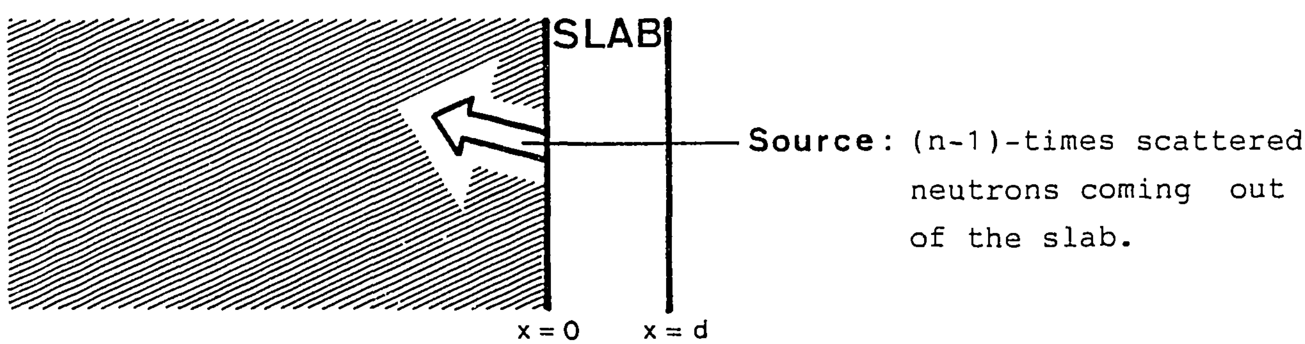


Fig.2. The (n-1)-times scattered neutrons are "summed up" along the dashed line.

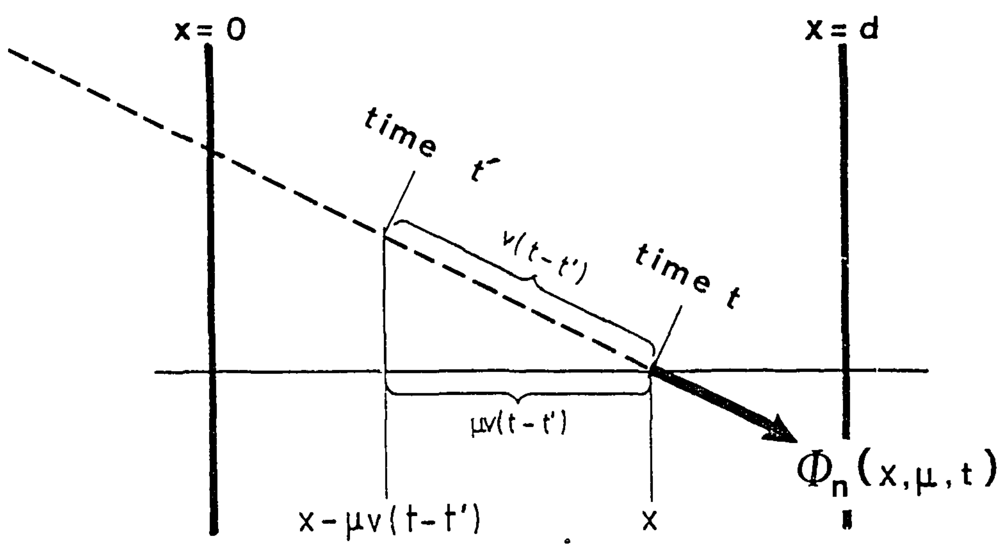


Fig.3. The distribution of once collided neutrons for the slab problem compared with the infinite medium solution.

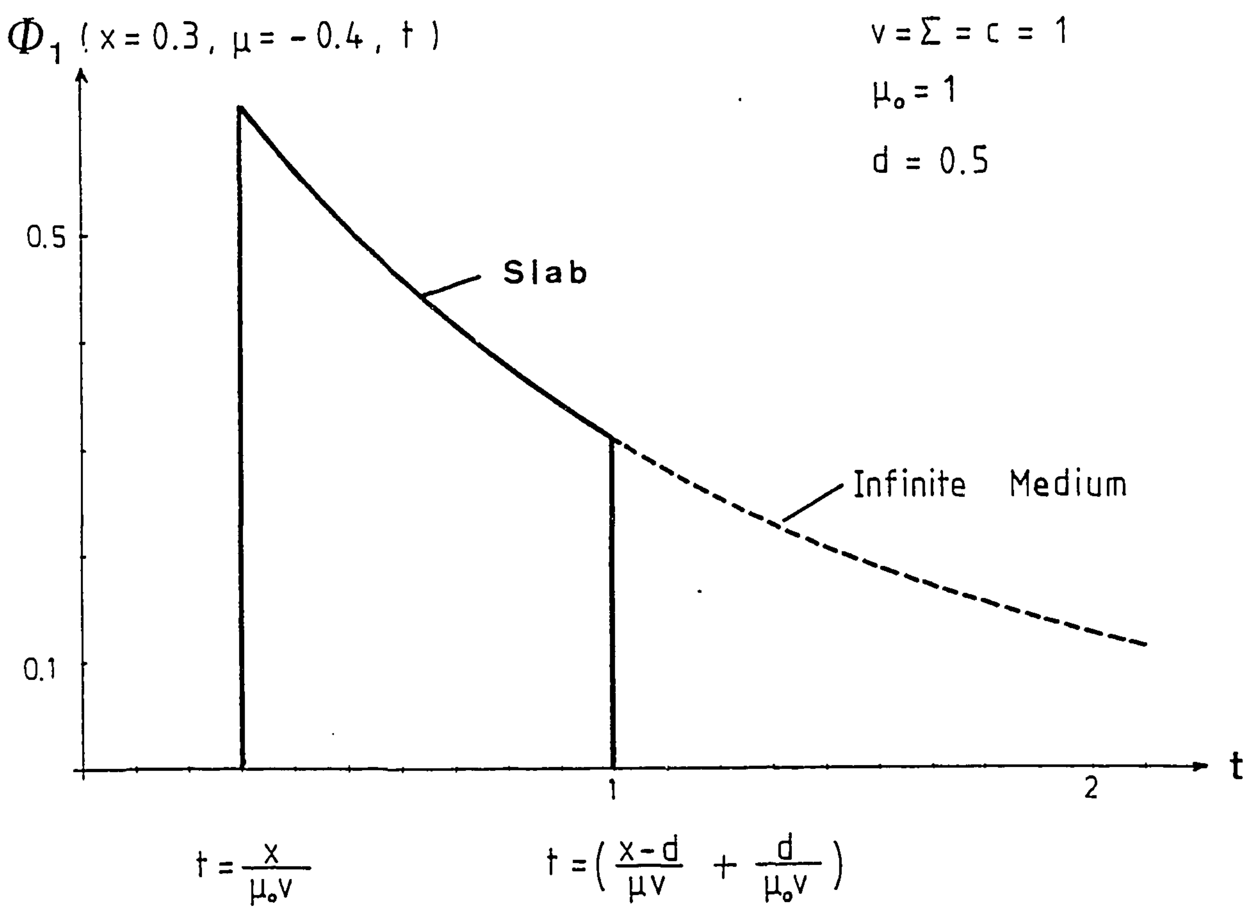


Fig.4. Space and time dependent scalar flux of neutrons scattered once at the time t = 1.4 .

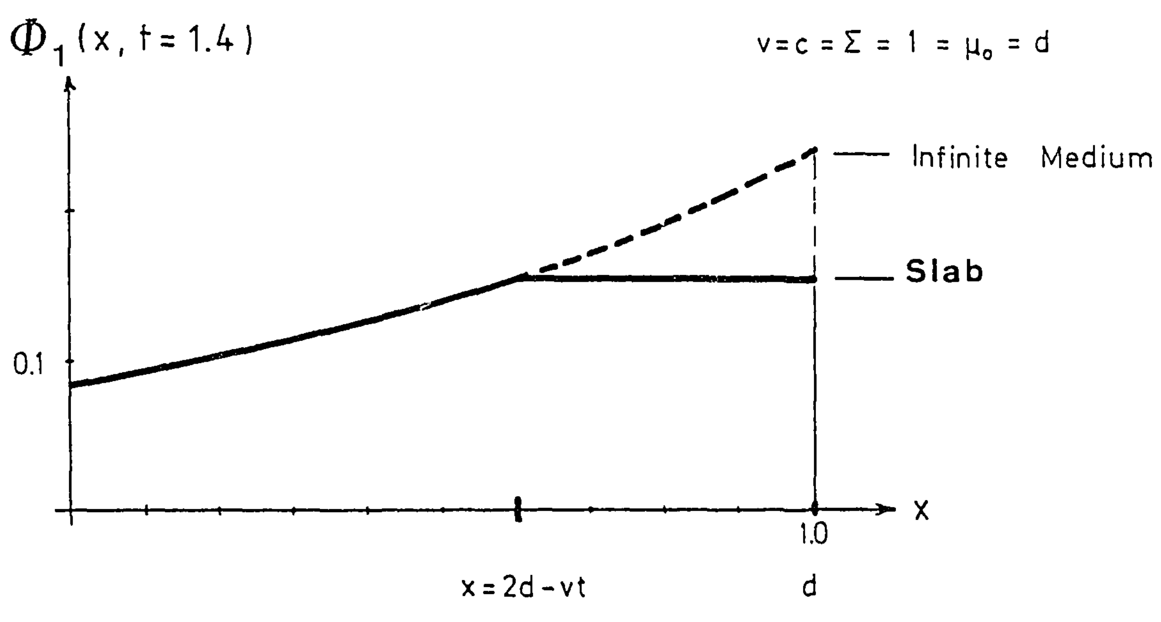


Fig.5. The scalar distribution of once collided neutrons at time t = 2.0 .

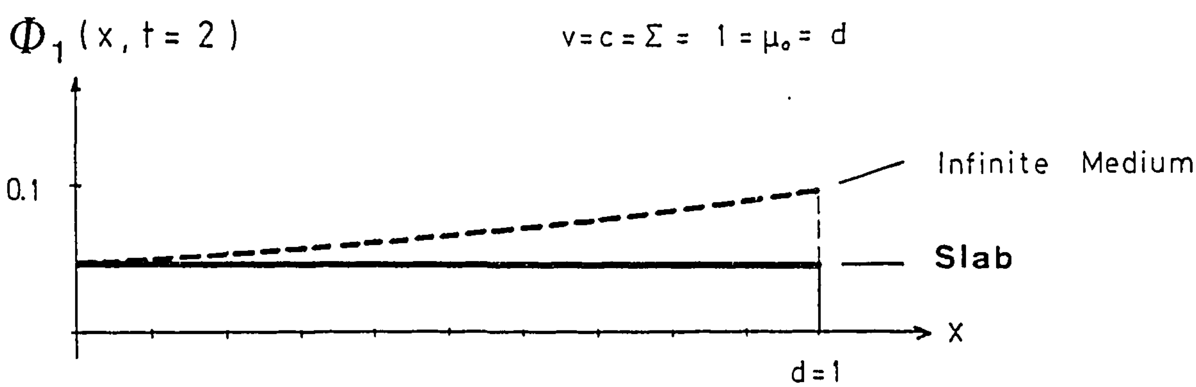


Fig.6. Structur of the solution $\Phi_{2\,\mathrm{slab}}(x,\mu>0,t)$.

$$\Phi_{2\,\mathrm{slab}}(x,\mu>0,t) = \boxed{\begin{array}{c}\text{semi-infinite medium}\\ \boxed{\begin{array}{c}\text{infinite}\\ \text{medium}\end{array}} \qquad + \ \boxed{\text{CI}}\end{array}} + \boxed{\text{CII}} + \boxed{\text{CIII}}$$

correcting terms

Fig.7. Neutrons which have their second collision outside the slab may not contribute to the distribution.

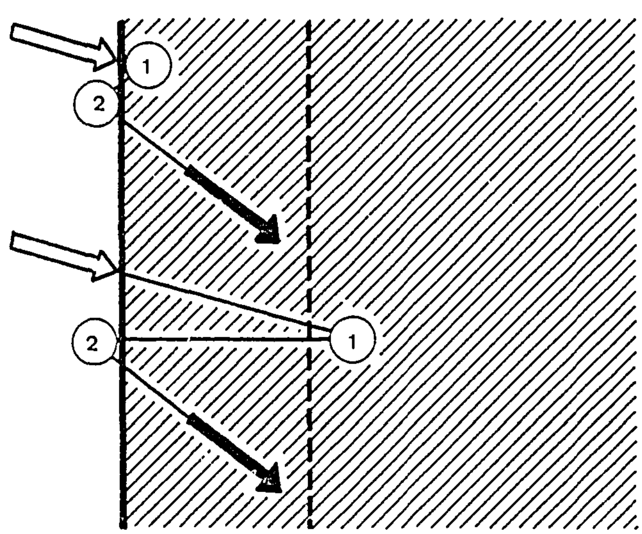


Fig.8. The correcting term CII is switched on at $t = \frac{d}{\mu_0 v} + \frac{d-x}{v}$.

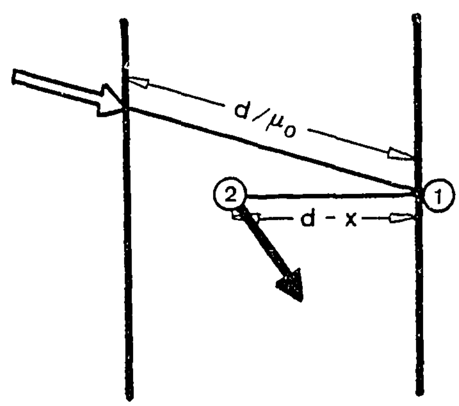


Fig.9. After a time $t = \frac{d}{\mu_0 v} + \frac{d}{v} + \frac{x}{\mu v}$ CIII puts the balance right.

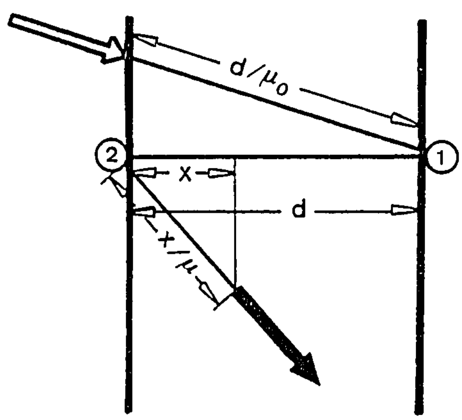


Fig.10. Time dependence of the neutron flux $\Phi_2(x,\mu>0,t)$ with $\delta(t)$-pulsed source.

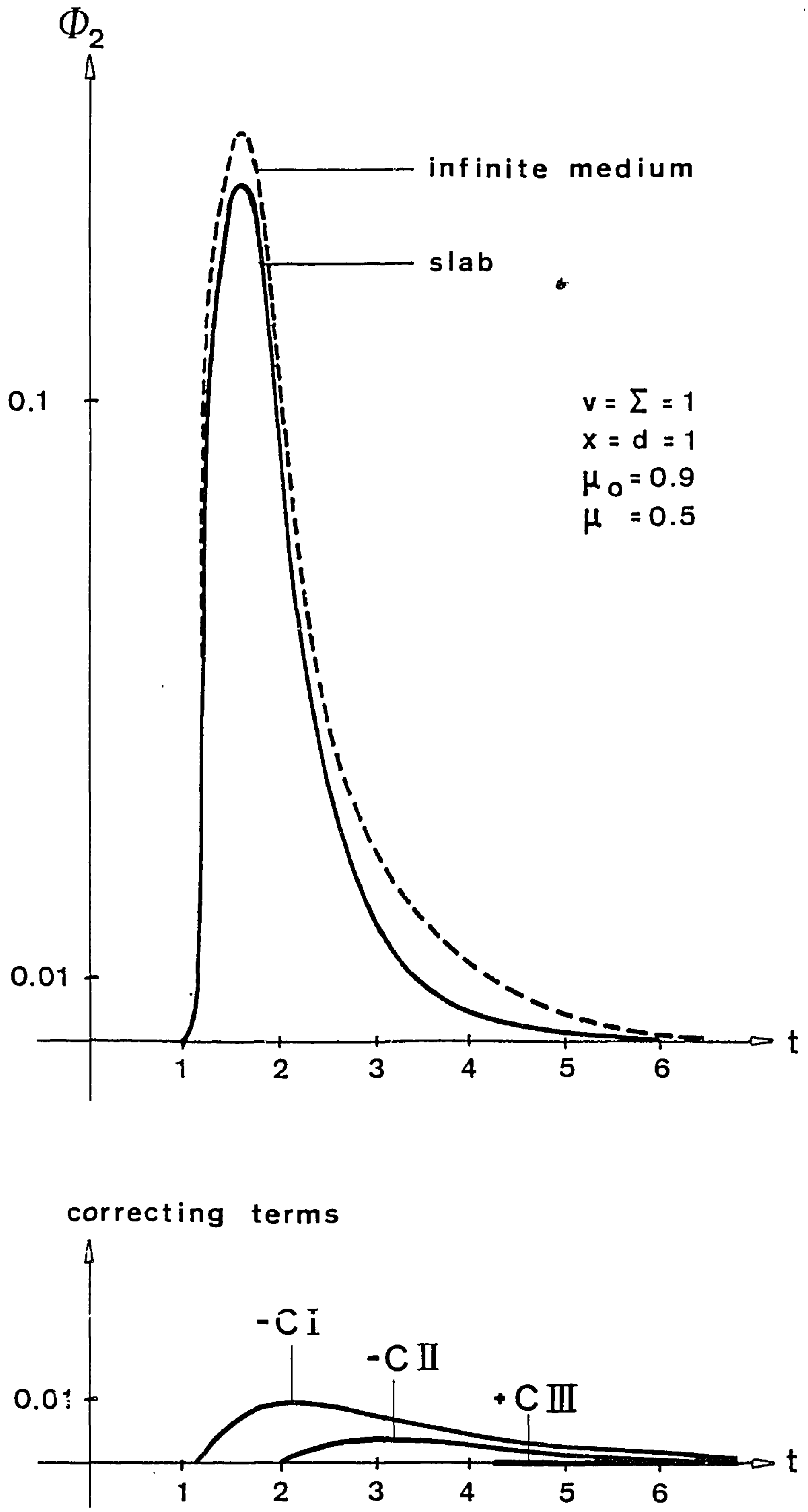


Fig.11. $\Phi_1(x,\mu,t)$ at time t=1.0 with $\delta(t)$-shaped pulse source.

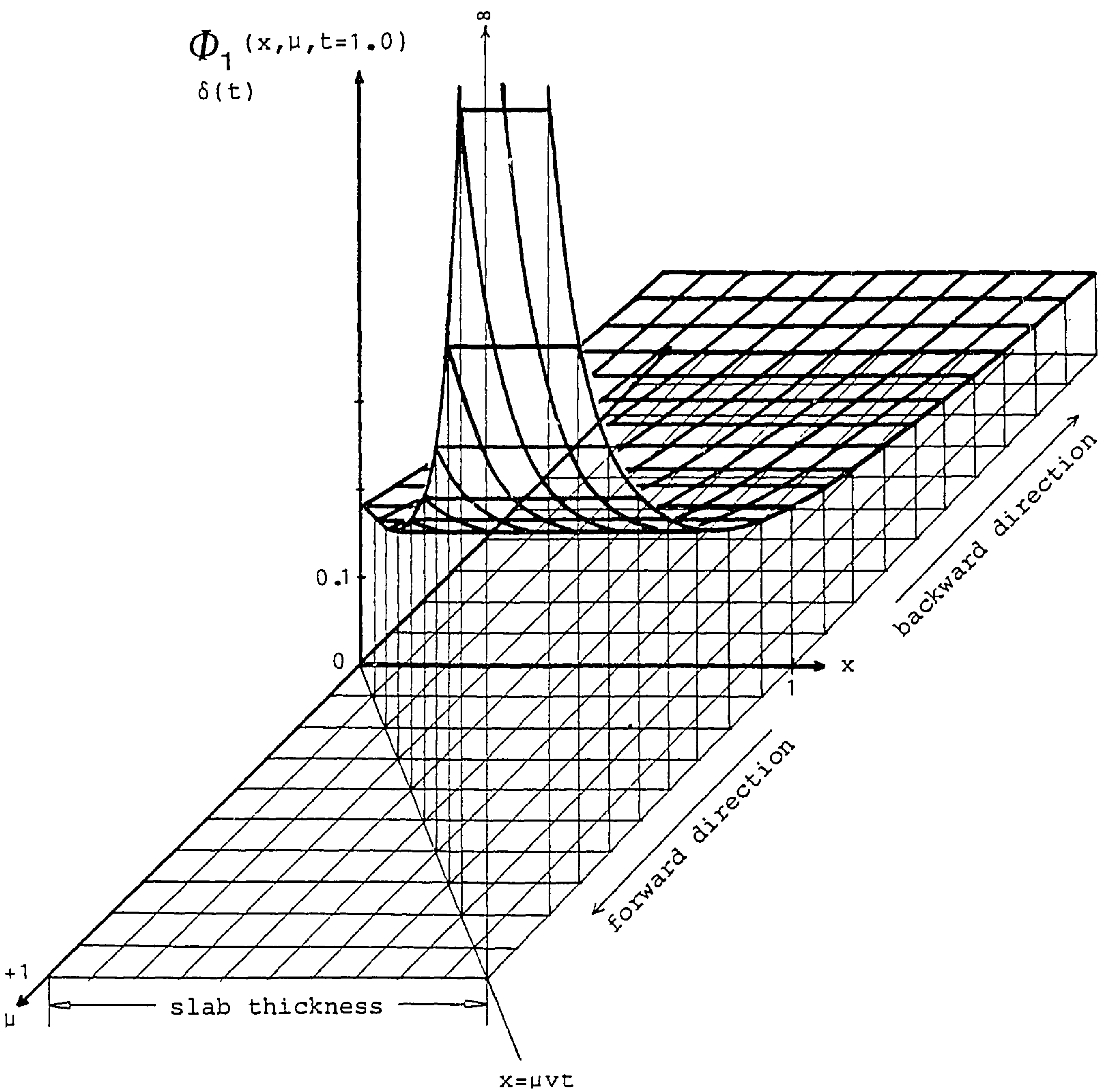


Fig.12. $\Phi_1(x,\mu,t)$ at time t=1.5 with $\delta(t)$-shaped pulse source.

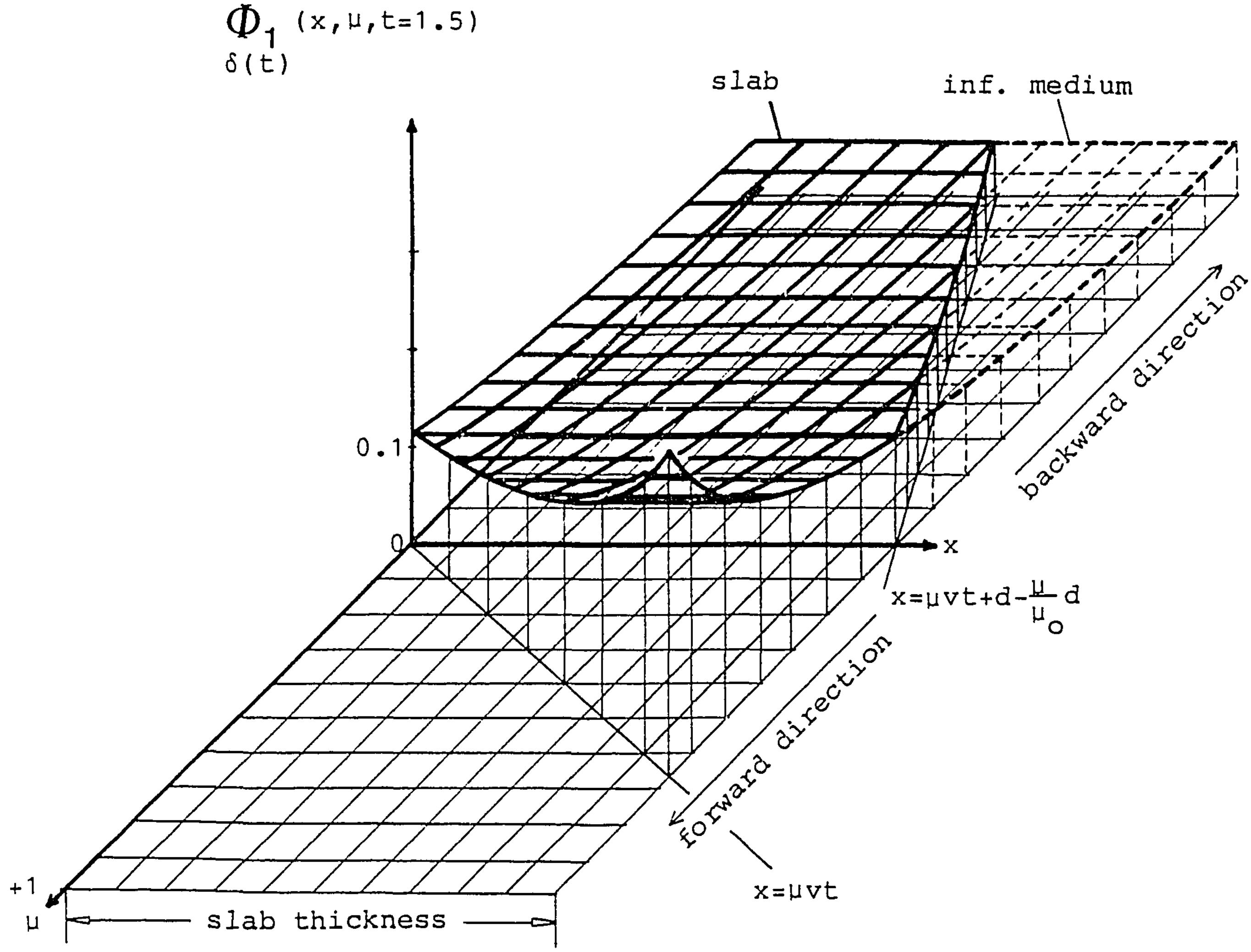


Fig.13. $\Phi_2(x,\mu,t)$ at time t=1.0 with $\delta(t)$-shaped pulse source.

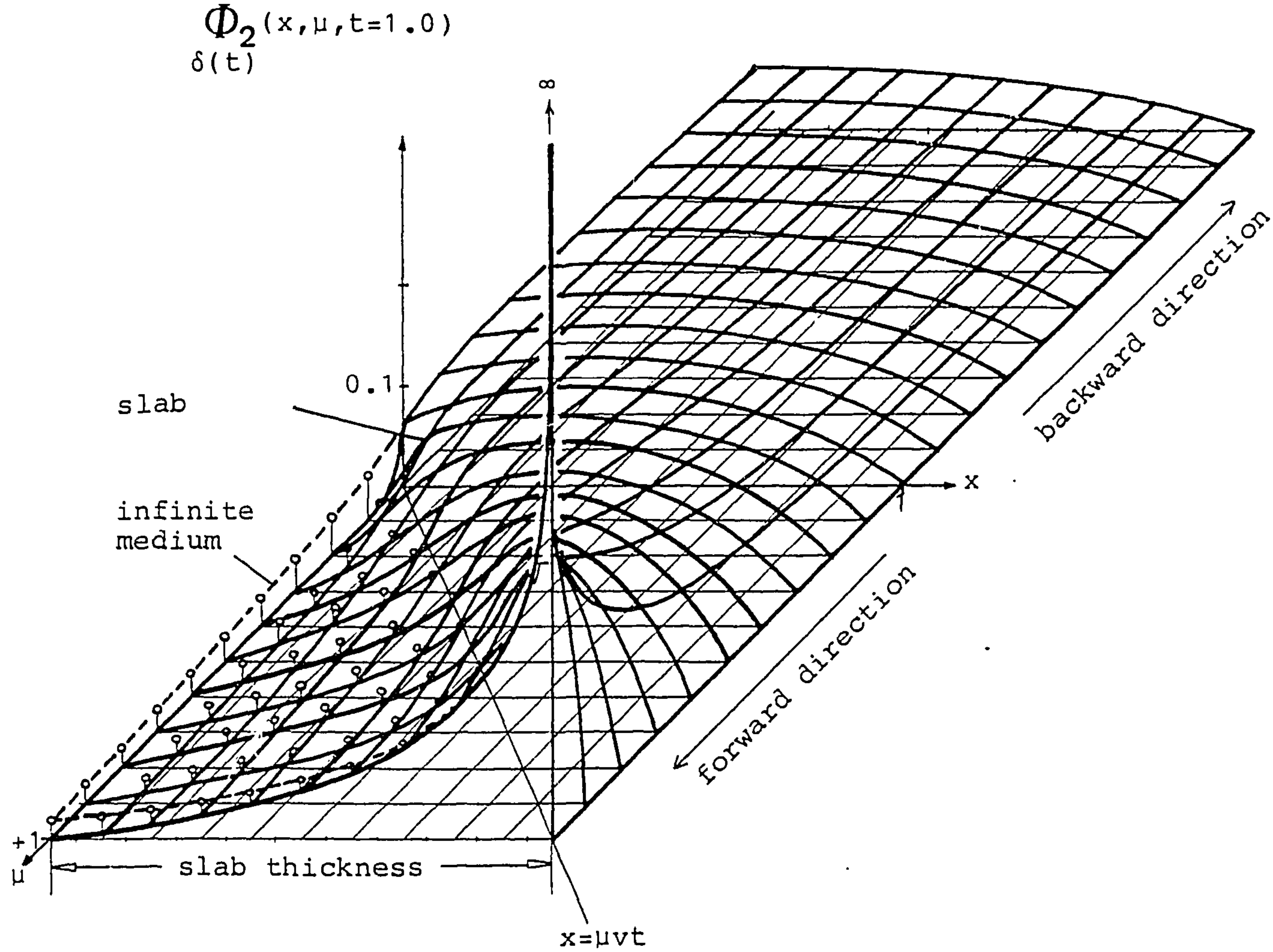


Fig.14. $\Phi_2(x,\mu,t)$ at time t=1.5 with $\delta(t)$-shaped pulse source.

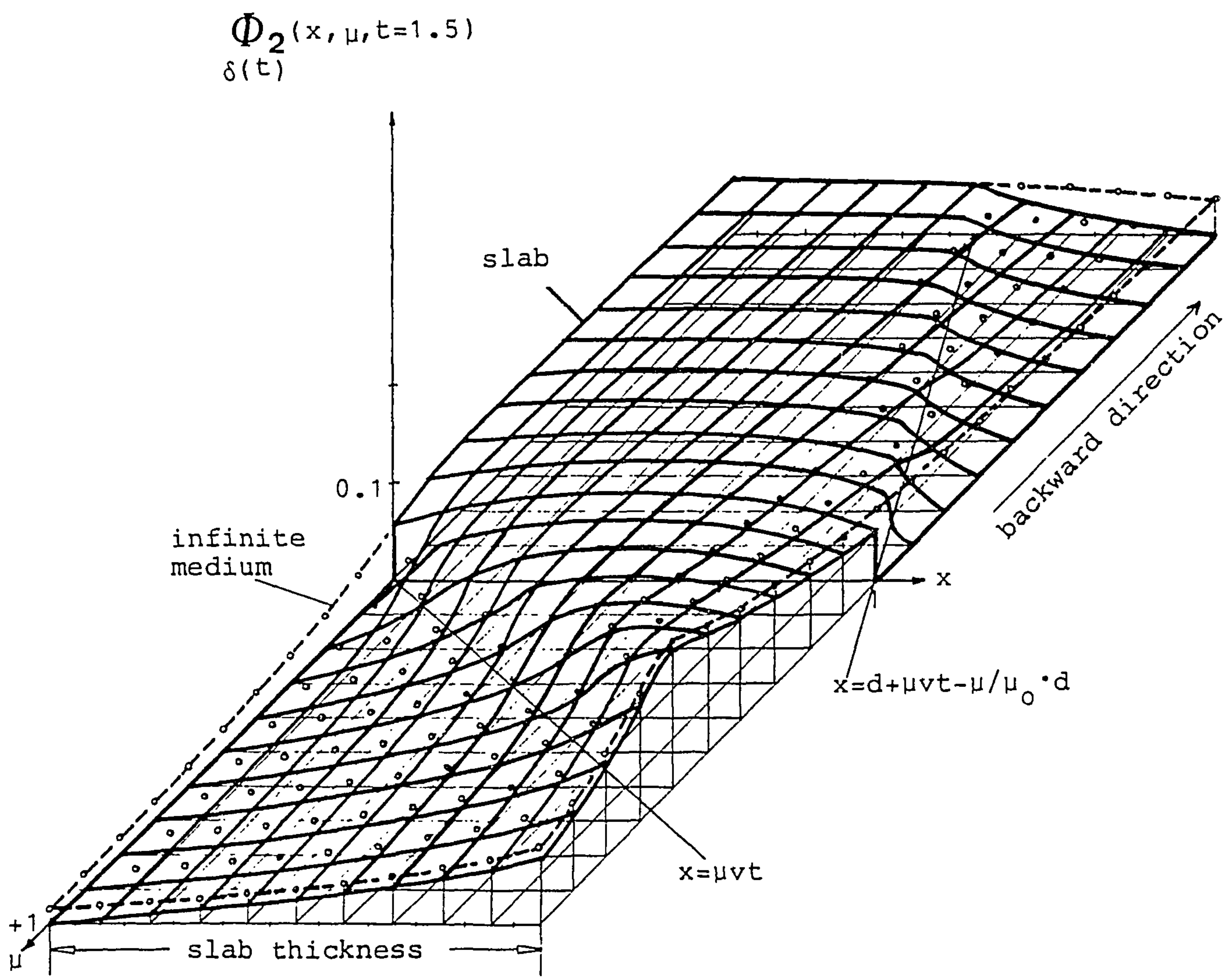


Fig.15. $\Phi_3(x,\mu,t)$ at time t=1.0 with $\delta(t)$-shaped pulse source.

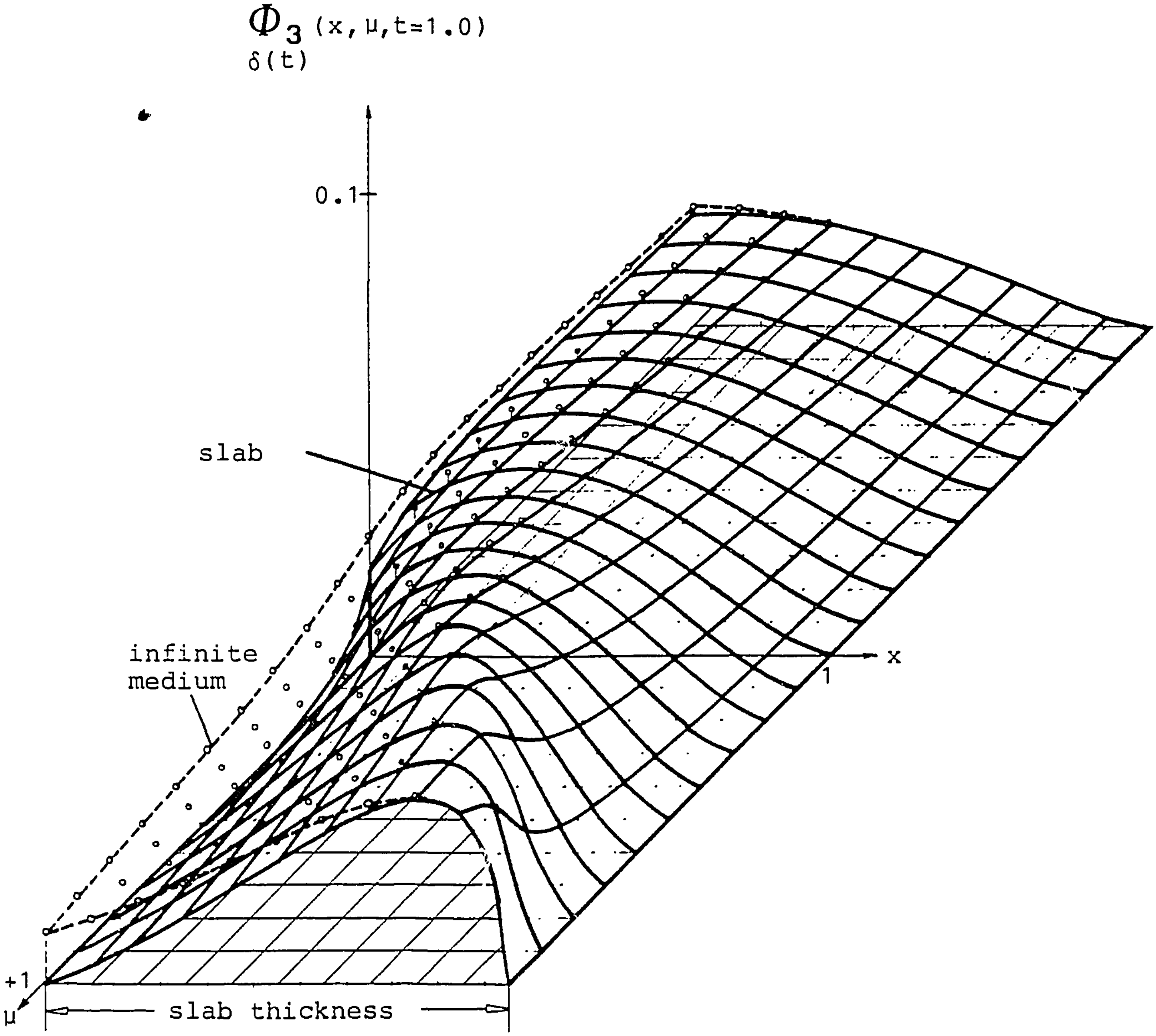


Fig.16. $\Phi_3(x,\mu,t)$ with $\delta(t)$-pulsed source at time t=1.5.

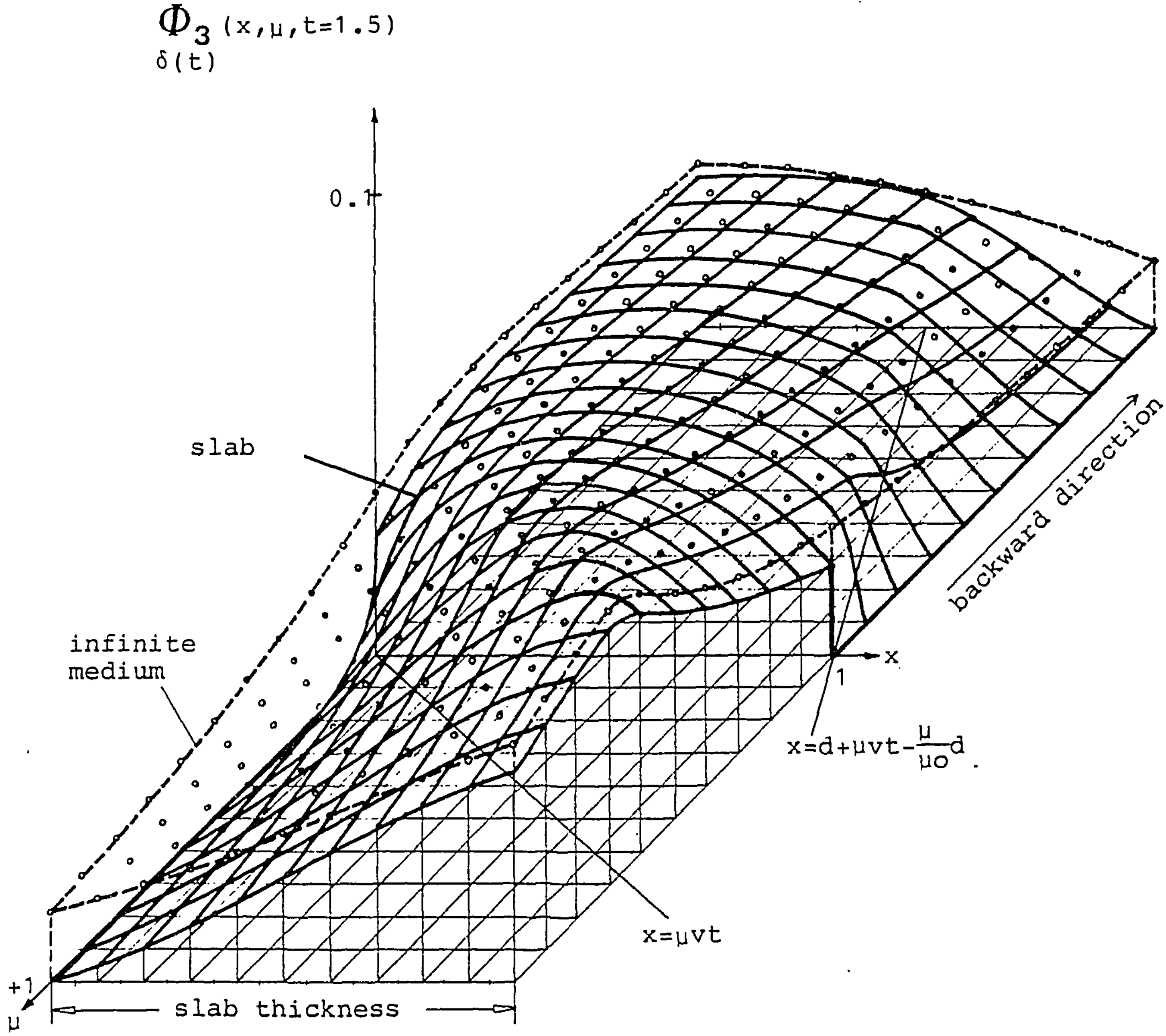


Fig.17. $\Phi_1(x,\mu,t)$ at time t=1.0 with Δt=0.3 rectangular neutron source.

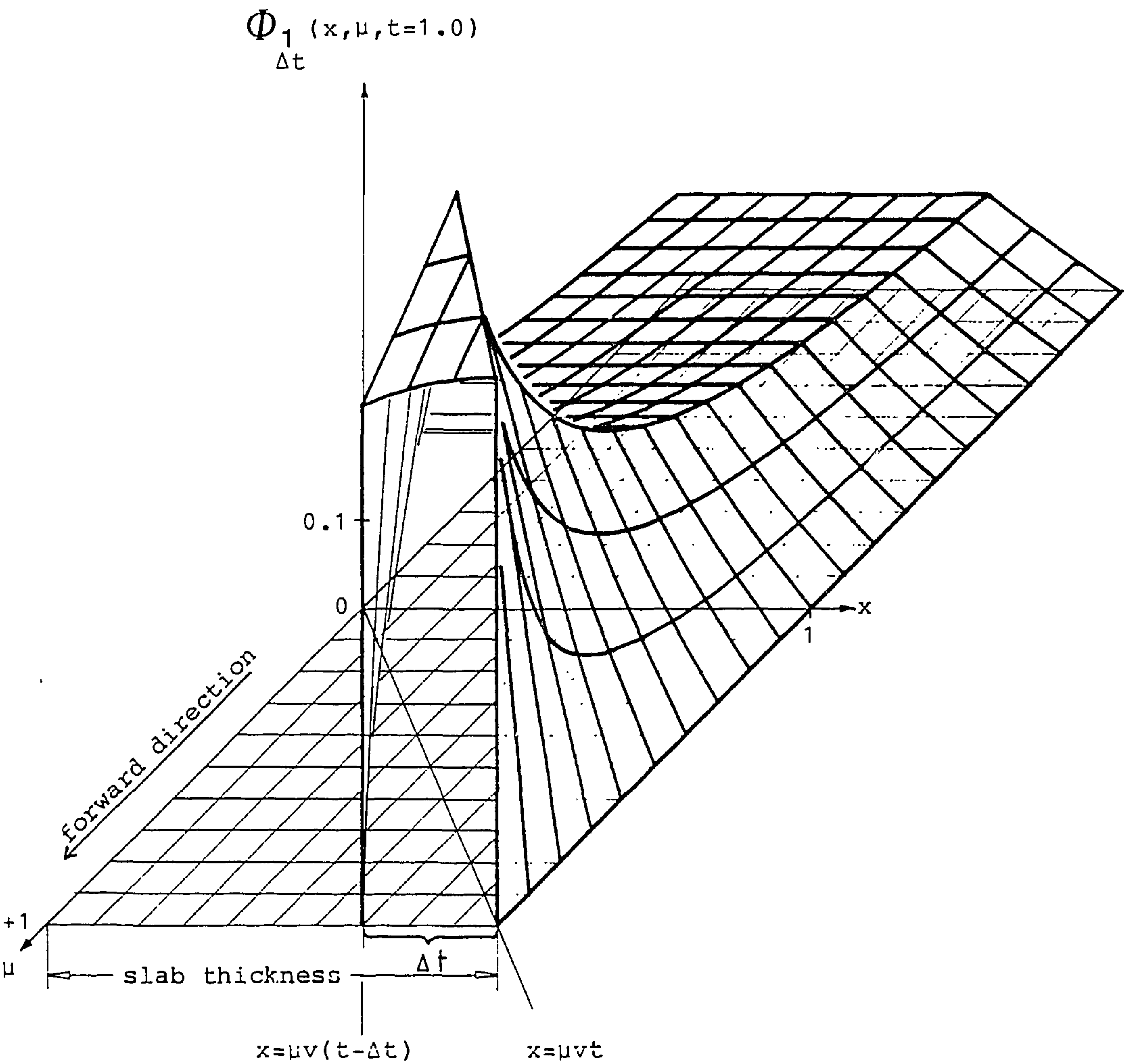


Fig.18. $\Phi_1(x,\mu,t)$ at time t=1.5 with Δt=0.3 rectangular source.

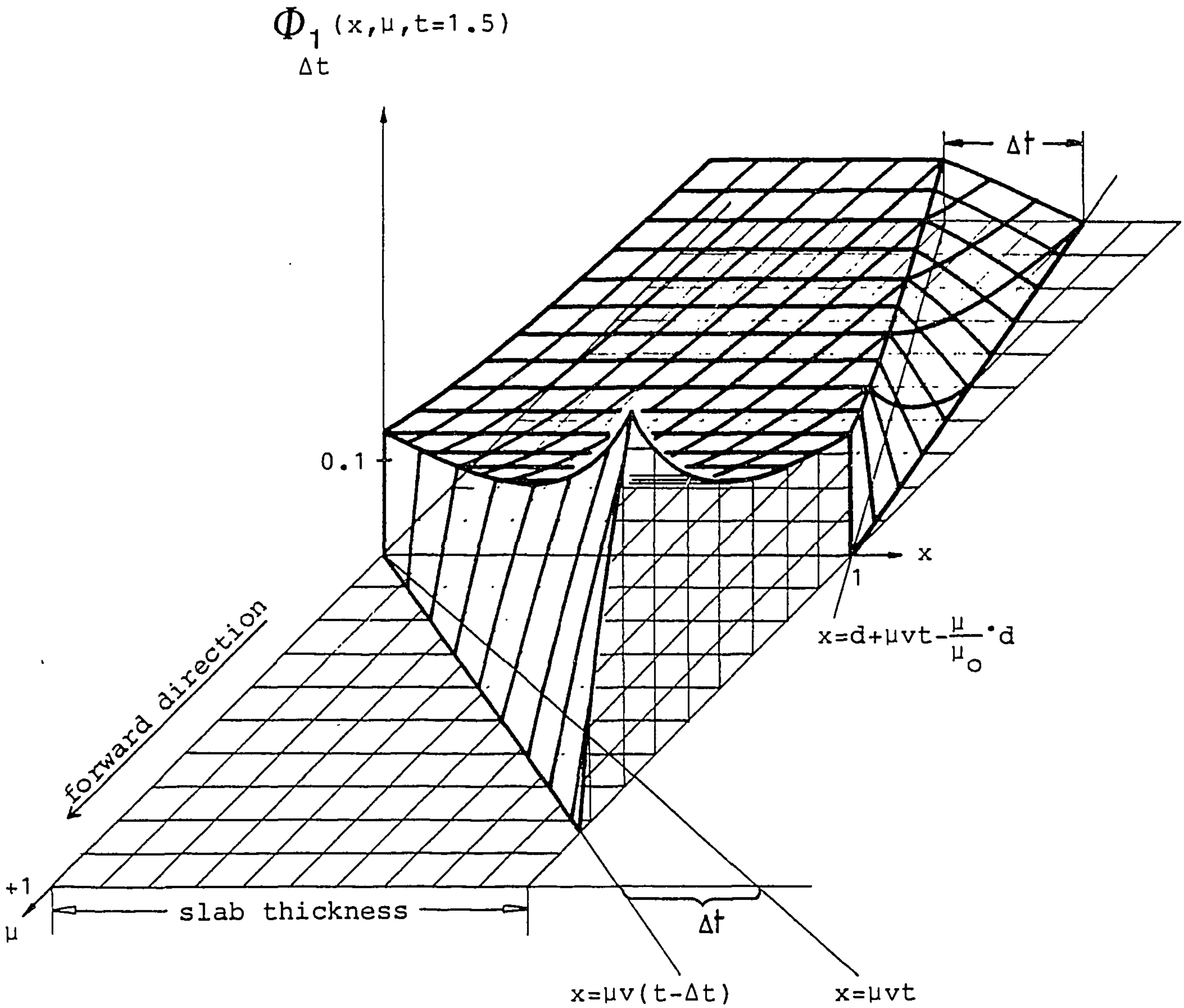


Fig.19. $\Phi_2(x,\mu,t)$ at time t=1.0 with Δt=0.3 rectangular source.

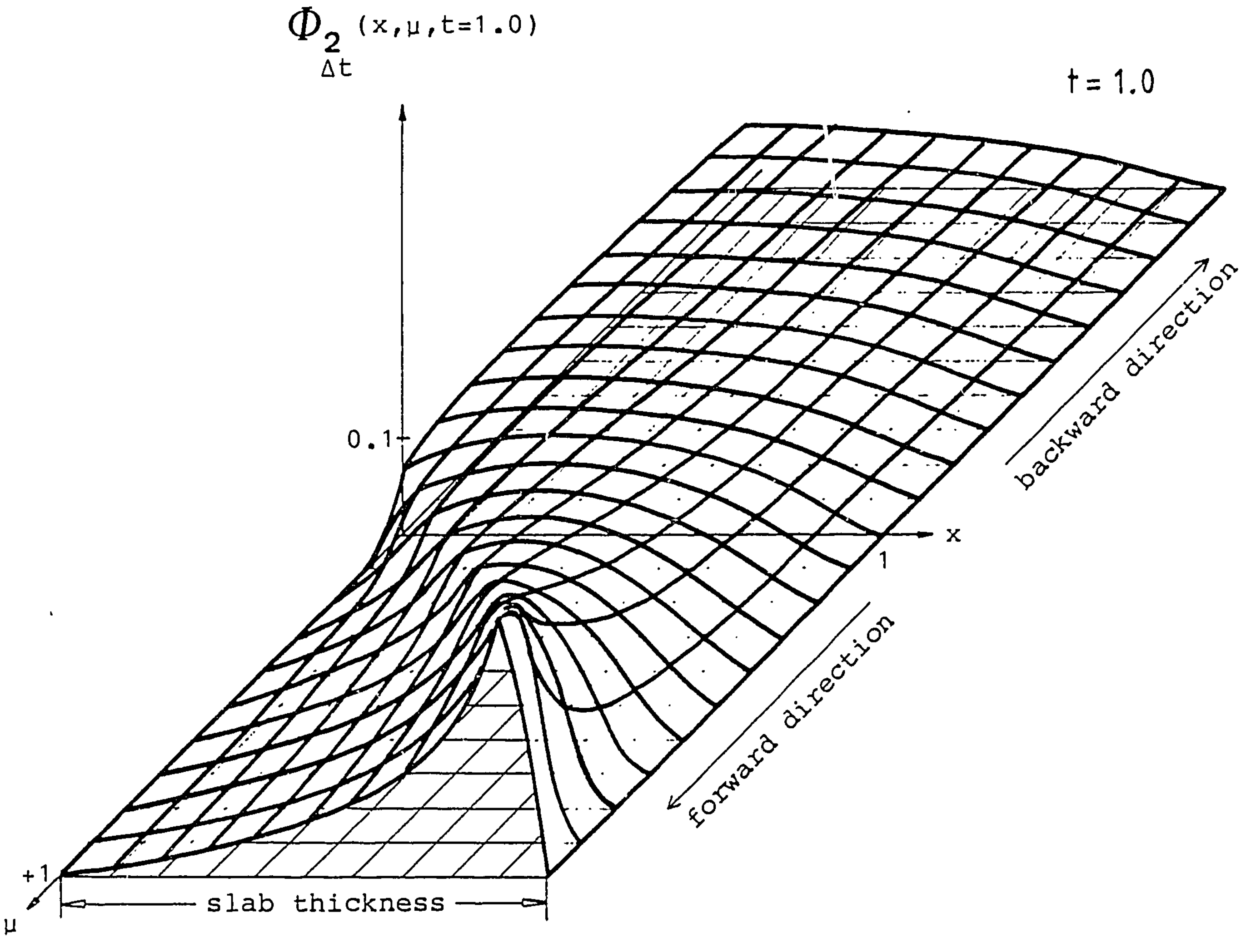
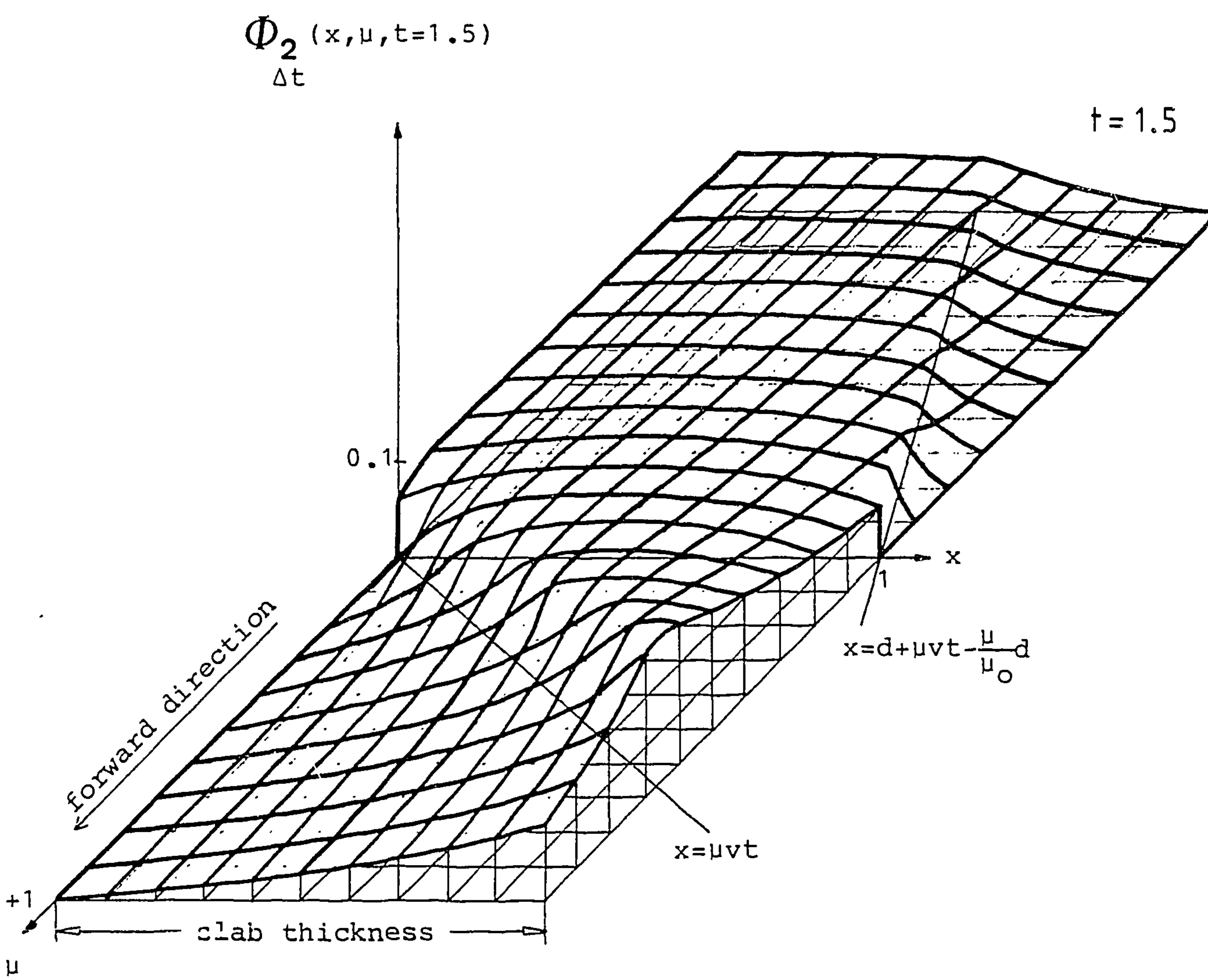


Fig.20. $\Phi_2(x,\mu,t)$ at time t=1.5 with Δt=0.3 rectangular neutron source.



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