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# Line broadening by focusing

Adelsindo Liberato de Brito
and Arthur Jabs

Departamento de Física, Universidade Federal
da Paraíba, 58 000 João Pessoa,
PB, Brasil

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## Abstract

We point out that the spectral width of a quasi-monochromatic light beam broadens when the beam is focussed. A quantitative formula for this broadening is derived from classical wave theory. The effect is shown to explain some experiments on laser beams done by E. Panarella which that author has explained under the ad-hoc hypothesis that the frequency of the photons changes along with the intensity of the light beam. The line broadening by focusing might also contribute to gas ionization by incident light when the ionization potential is well above the mean photon energy. Some remarks are made on some direct applications of the Heisenberg relations in comparison with our treatment. (Author)

## 1. Introduction

Panarella (1-3) has pointed out that ionization of gases induced by incident laser radiation has been observed where it should not occur because the energy $h\nu$ of the incident light quanta is a factor of 5 and more below the energy necessary for ionizing the gas atoms. He has studied the case and has come to the conclusion that the existing theories attempting to account for the effect, namely multiphoton and cascade theory, actually are in serious divergence from experimentation. He has therefore put forward an "effective-photon hypothesis" which states that the photon-energy expression $\varepsilon = h\nu$ should be modified according to the formula

$$[1] \quad \varepsilon = h\nu / [1 - \beta_\nu f(I)]$$

where $\varepsilon$ is the photon energy, $h$ the Planck constant, $\nu$ the light frequency, $I$ the light intensity and $\beta_\nu$ a coefficient whose product with $f(I)$ significantly differs from zero only in the case of very high intensity laser light. Thus, such light may contain photons with energies that greatly surpass hν and even reach values above the ioniza-tion energy of the gas atoms. More specifically, in a light pulse Panarella (2) assumes that: "In the central portion of the pulse, where the intensity is high, the frequency is larger than the initial emitting laser frequency $\nu_o$. In the head and tail of the pulse the frequency is lower than $\nu_o$ because conservation of energy requires that energy gained by photons in some part of the pulse has to be recovered through energy loss in same other part". When we, as usual, do not consider the temporal development but integrate over the time of the pulse we effectively obtain a broadening in the photon-energy distribution, i.e. a line broadening if the peak intensity of the pulse is high enough. Those photons in the distribution whose energies are high enough to ionize Panarella calls "effective photons", and he shows that with them many of the difficulties of the cascade and multiphoton theories can be avoided.

The formula [1] violates Planck's relation $\varepsilon = h\nu$. The violation may be quite negligible numerically in the intensity ranges experimentally investigated at the time when quantum theory was postulated, before lasers where known. Nevertheless, the formula is sufficiently close to the basis of quantum theory to justify a search for an alternative explanation of the above-mentioned "impossible" ionization that would be in accordance with the standard formulas of electromagnetism and quantum theory. Such an alternative is here presented for those cases where the increase in peak intensity of the beam is brought about by focusing the beam. Our formulas do not contain the intensity but only the beam convergence (or divergence) angle $\alpha$.

There are some papers (4) which are concerned with the question of how the photons in the above-mentioned laser pulse might exchange their energy in order to satisfy Panarella's formula [1]. We think these considerations are based on too naive a photon picture.

Another explanation, similar in spirit to ours, has been proposed by Allen (5) and is also discussed by Panarella and Gupta (6). Allen has pointed out that focusing diminishes the uncertainty in position and, due to the Heisenberg relations, increases the uncertainty in momentum, and thus results in a line broadening. We shall return to this in our discussion of the Heisenberg relations in the last section.

In order to support our alternative explanation in this paper we consider a publication of Panarella (3) which does not deal with ionization but describes experiments devised to demonstrate only the existence of the line-broadening effect. The spectrum of the laser-beam light is here considered only in the vicinity of the center of the line and not as far from it as would be necessary for ionization.

Since angular convergence always goes along with an increase in intensity when the focal region is approached, the task of deciding which of these two conditions is really responsible for the line broadening is difficult in general. Here it is made possible by the auxiliary experiments performed by Panarella in order to make sure that the observed effect is really the one searched for. Thus we will show that Panarella's experiments are actually more simply explained by our formulas than by his.

## 2. Derivation of the Broadening Formulas

We describe the light pulse, after it has been produced in some way, as a wave packet of the form

$$[2] \qquad \psi(\vec{x},t) = (2\pi)^{-3/2} \int_{-\infty}^{+\infty} \tilde{\psi}(\vec{k}) \exp[i(\vec{k}\vec{x} - \omega(\vec{k})t] d^3\vec{k}$$

where $\tilde{\psi}(\vec{k})$ is the inverse Fourier transform of $\psi(\vec{x},0)$, $\vec{k}$ is the wave vector with $k \equiv |\vec{k}| = 2\pi/\lambda$, $\lambda$= wavelength, and

$$[3] \qquad \omega(\vec{k}) = ck = c\,(k_x^2 + k_y^2 + k_z^2)^{1/2}$$

is the angular frequency depending on the wave vector $\vec{k}$. Formula [3] represents the dispersion law for a free packet with positive frequencies only. $\tilde{\psi}(\vec{k})$ is assumed to be normalized according to $\int |\tilde{\psi}|^2 d^3k = 1$ but this is in no way essential for the considerations in this paper. We do not consider polarization effects, so we use the scalar field variable $\psi(\vec{x},t)$ which might be regarded as one component of the electric field or as any function that satisfies the wave equation $\Box\psi = 0$. The description of the light pulse by one single wave packet of the type [2] simplifies our argument but is not really necessary. All our results remained unchanged if we used a superposition of such wave packets with randomly distributed phases (in $\psi(\vec{K})$). The moderate restrictions which we shall put on $\psi(\vec{k})$ are then to be put on each of the wave packets of the ensemble.

Our formulas will be derived by calculations on a classical wave pulse. Our semi-classical treatment is sufficient when the threshold of gas ionization is considered in the sense of Einstein's light quantum explanation of the photo-electric effect.

We associate the convergence or divergence of the beam of light pulses with the contracting or spreading, respectively, of the wave packet [2] in the directions normal to the velocity of its center. Of course, the two cases of convergence (contracting) and divergence (spreading) are completely equivalent. The spreading of a such a wave packet has been treated by Bradford (7) in a very general way. Let the velocity of the center be along the y axis and let us define the mean value of x and the width normal to y, i.e. the transverse width $\Delta x$ of a packet by the formulas familiar from (but in no way restricted to) quantum mechanics

$$[4] \qquad \langle x \rangle (t) = \int x \left| \psi (\vec{x}, t) \right|^2 d^3 x$$

[5] $$\Delta x(t) = \int (x - \langle x \rangle)^2 |\psi(\vec{x},t)|^2 d^3x$$

The time dependence of $\Delta x$ is then given by Bradford as

$$(\Delta x(t))^2 = (\Delta x(0))^2 + t^2(\Delta v_{gx})^2$$

[6] $$= -\int \mathrm{Re}\left[\tilde{\psi}^* \partial^2 \tilde{\psi}/\partial k_x^2\right] d^3k$$

$$+ t^2 \int |\tilde{\psi}|^2 (\partial\omega/\partial k_x - \langle v_{gx} \rangle)^2 \, d^3k$$

where $\partial\omega/\partial k_x = ck_x/k$ according to [3], and $\langle v_{gx} \rangle =$ $\int |\tilde{\psi}|^2 (\partial\omega/\partial k_x) d^3k$ is the mean value of the x component of the group velocity, in the same sense as formula [4], only in k (momentum) space instead of x space. Everything is quite analogous for $\Delta z(t)$. Formula [6] refers to a packet that is timed (by means of the phases) so that its minimum transverse extension occurs at t=0. We may then define the spreading velocity along the x axis by $\partial\Delta x(t)/\partial t$ which for large t tends to

[7] $$v_{sx\infty} = \Delta v_{gx}$$

From Figure 1 we see that we may define the beam convergence (or divergence) angle $\alpha_x$ by means of

[8] $\sin\alpha_x = \Delta v_{gx}/c$

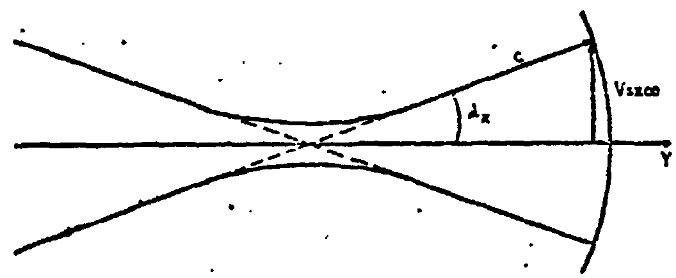


Fig. 1 Transverse spreading of a wave packet.

What we finally want is a relation between the beam convergence angle $\alpha_x$ and the line width $\Delta\omega$ of the wave packet. This we achieve in two steps. In the first step we derive a relation between $\Delta v_{gx}$ ($= c \sin\alpha_x$) and $\Delta k_x$, $\Delta k_y$, $\Delta k_z$, and in the second step we derive a relation between $\Delta k_x$, $\Delta k_y$, $\Delta k_z$ and $\Delta\omega$. In order to simplify the formulas we restrict ourselves to nearly unidirectional and quasi-monochromatic wave packets, that is to packets that are closely centered around the value $\vec{k}_o = (o, k_o, o)$ in wave-vector (momentum) space:

[9] $\Delta k_x,\ \Delta k_y,\ \Delta k_z << |\vec{k}_o| \equiv k_o$

In order to obtain the relation between the spread $\Delta v_{gx}$ in the x component of group velocity and the spreads, $\Delta k_x$, $\Delta k_y$, $\Delta k_z$ in the components of the wave vector we expand $v_{gx}=\partial\omega/\partial k_x$ in a three-dimensional Taylor series about $\vec{k}_o$ and break the series off after the quadratic terms

[10] $v_{gx}\equiv\partial\omega/\partial k_x=ck_x/k_o - ck_x(k_y-k_o)/k_o^2$

We then apply the formulas analogous to [4]

[11a] $(\Delta v_{gx})^2 = \langle v_{gx}^2\rangle - \langle v_{gx}\rangle^2$ ,

[11b] $\langle v_{gx}\rangle = \int v_{gx}(\vec{k})|\bar{\Phi}(\vec{k})|^2 d^3k$ ,

[11c] $\langle v_{gx}^2\rangle = \int (v_{gx}(\vec{k}))^2|\bar{\Phi}(\vec{k})|^2 d^3k$ ,

to each single term on the right-hand side of [10] and to the terms in the corresponding expression for $(v_{gx}(\vec{k}))^2$ . In order to simplify further our formulas we suppose that the distribution $\bar{\Phi}(\vec{k})$ separates

[12] $\bar{\Phi}(\vec{k}) = \phi_1(k_x)\phi_2(k_y)\phi_3(k_z)$.

and that the $\phi_i$'s are symmetrical about their respective maximum values. This may be considered a good approximation to the actual cases. We then have for example $\langle k_x(k_y - k_o)\rangle = \langle k_x\rangle\langle(k_y - k_o)\rangle = 0$, and the result is

$$\Delta v_{gx} = c\ \Delta k_x/k_o \cdot (1 + (\Delta k_y/k_o)^2)^{1/2}$$

[13] $$= c\ \Delta k_x/k_o$$

The relative error in formula [13] is in the order of $\frac{1}{2}(\Delta k_y/k_o)^2$. The values for $\Delta k_y/k_o$ found in the experiments to be described below are always less than $10^{-2}$, so the error is always less than $0.5\text{x}10^{-4}$. Inserting then [13] into [8] we obtain, with the same relative error

[14] $$\sin\alpha_x = \Delta k_x/k_o$$

The second step is the calculation of $\Delta\omega$ as a function of $\Delta k_x$, $\Delta k_y$, $\Delta k_z$. This is again done for nearly unidirectional and quasimonochromatic wave packets by expanding $\omega(\vec{k})$ in a three dimensional Taylor series about $\vec{k}_o = (o,k_o,o)$. Terms up to the sixth order have been retained. In the averaging process we again assume the separation [12] and the symmetry of the $\phi_i$'s. The final result of the lengthy calculation keeping all terms up to the order $\Delta^2(\Delta/k_o)^4$

($\Delta$ = $\Delta k_x$ or $\Delta k_y$ or $\Delta k_z$) is

[15]
$$
\begin{aligned}
(\Delta\omega/c)^2 = (\Delta k_y)^2 \Big\{ 1 - \big[(\Delta k_x)^2 + (\Delta k_z)^2\big] / k_o^2 \\
+ \big[2(\Delta k_x)^2 (\Delta k_z)^2 + ((\Delta k_x)^4 + (\Delta k_z)^4)(3\chi-1)/2\big] / k_o^4 \\
- (\Delta k_y)^2 \big[(\Delta k_x)^2 + (\Delta k_z)^2\big] \chi / k_o^4 \Big\} \\
+ \big[(\Delta k_x)^4 + (\Delta k_z)^4\big] (\chi-1)/(4 k_o^2) \\
+ \big[(\Delta k_x)^2 (\Delta k_z)^4 + (\Delta k_x)^4 (\Delta k_z)^2\big] (1-\chi)/(4 k_o^4) \\
+ \big[(\Delta k_x)^6 + (\Delta k_z)^6\big] (\chi-\zeta)/(8 k_o^4).
\end{aligned}
$$

$\chi$ and $\zeta$ are numbers determined by the relations

[16] $$\langle k_x^4 \rangle = \chi (\Delta k_x)^4, \qquad \langle k_x^6 \rangle = \zeta (\Delta k_x)^6$$

and analogously for the other components. $\chi$ and $\zeta$ thus depend on the type of the distributions $\phi_i$. Not all types allow relationships of the above form. Usually the distributions are unknown. Therefore in Table I we present the values of $\chi$ and $\zeta$ for some distributions which may be considered representative and which do allow relationships of the above form. A pure Lorentzian distribution would not fit in our framework since the averaging integrals $\langle k_x^n \rangle$ would diverge. Therefore we considered an exponentially damped Lorentzian and a cut-off Lorentzian distribution. The same type of function (though not the same parameter values) has been assumed for

TABLE I. Values for $\chi$, $\zeta$ and $\eta = \frac{1}{4}(\zeta+\chi-2)/(\chi-1)$ appearing in formulas [15] - [26], for various types of the distribution $\phi_1(K_x)$.

| $\phi_1(K_x)$ | | $\chi$ | $\zeta$ | $\eta$ |
|---|---|---|---|---|
| $(2\pi b^2)^{-1/4}\exp[-K_x^2/(4b^2)]$ Gaussian | | 3 | 15 | 2 |
| $(\pi b)^{1/2}\,\mathrm{sech}^{1/2}(K_x/b)$ Hyperbolic secant | | 5 | 61 | 4 |
| $(2bf)^{-1/2}\frac{\exp[-\frac{1}{2}\varepsilon\|K_x\|/b]}{(1+(K_x/b)^2)^{1/2}}$ Exponentially damped Lorentzian $f=f(\varepsilon)=\mathrm{Ci}(\varepsilon)\sin\varepsilon-\mathrm{si}(\varepsilon)\cos\varepsilon$ (Ref. 8) | | $\frac{f(f+2/\varepsilon^3-1/\varepsilon)}{(1/\varepsilon-f)^2}$ | $\frac{f^2(24/\varepsilon^5-2/\varepsilon^3+1/\varepsilon-f)}{(1/\varepsilon-f)^3}$ | |
| | $\varepsilon$ = 0.7 | 8.2 | 240 | 8.6 |
| | 0.8 | 7.7 | 204 | 7.8 |
| | 1.0 | 7.0 | 15 9 | 6.8 |
| | 2.0 | 5.8 | 93 | 5.0 |
| | $\infty$ | 6 | 90 | 4.7 |
| $(2ab)^{-1/2}(1+(K_x/b)^2)^{-1/2}$, $\|K_x\| \le K$ Cut off Lorentzian $a=a(K)=\arctan(K)$ | | $\frac{(K^3/3-K+a)a}{(K-a)^2}$ | $\frac{(K^5/5-K^3/3+K-a)a^2}{(K-a)^3}$ | |
| | K = 0 | 1.8 | 3.8 | 1.1 |
| | 4 | 3.0 | 16.6 | 2.2 |
| | 8 | 5.5 | 47.4 | 2.8 |
| | 12 | 7.6 | 94.2 | 3.7 |
| | 34 | 19.5 | 665 | 9.2 |

all $\phi_i$'s so that only one $\chi$ and one $\zeta$ apear in formula [15] and not one for each component.

Formula [15] can be simplified in several respects. First, in most experimental cases the beam has axial symmetry, $\Delta k_x = \Delta k_z$. Using then [14] with $\sin\alpha_x = \sin\alpha_z = \sin\alpha$ and dividing both sides of [15] by $\omega_o^2 = c^2 k_o^2$ we have

[17]
$$\left(\frac{\Delta\omega}{\omega_o}\right)^2 = \left(\frac{\Delta k_y}{k_o}\right)^2 \left\{1-2\sin^2\alpha\left[1-\frac{1}{2}(3\chi+1)\sin^2\alpha + \chi\left(\frac{\Delta k_y}{k_o}\right)^2\right]\right\} + \frac{1}{2}(\chi-1)\sin^4\alpha\left\{1-2\eta\sin^2\alpha\right\}$$

where

[18] $\eta = (\chi+\zeta-2)/(4(\chi-1))$.

From the Taylor expansion it is clear that we must have

[19a] $\sin\alpha \ll 1$

[19b] $\Delta k_y/k_o \ll 1$.

These conditions are satisfied in many experiments, in particular in those of Panarella (3) with which we shall compare our formulas.

In order to obtain the value of $\Delta k_y/k_o$ after focusing in a particular experiment we consider the original, unfocussed beam. This beam, when it originates from a Laser, usually shows very little angular divergence. Let us denote the original beam divergence (half) angle by $\alpha_o$, and the original line width by $\Delta_o\omega$. Resolving [17] for $\Delta k_y/k_o$ and under the conditions

[20a] $$\frac{\Delta_o\omega}{\omega_o} << 1/\sqrt{2}$$

[20b] $$\sin\alpha_o << 1/\sqrt{4\eta}$$

which essentially are the conditions [19] for our Taylor series to be cut off, we obtain

[21] $$\left(\frac{\Delta k_y}{k_o}\right)^2 = \left(\frac{\Delta_o\omega}{\omega_o}\right)^2 - \frac{1}{2}(\chi-1)\sin^4\alpha_o$$

and under the further condition

[22] $$\sin^4\alpha_o << \left(\frac{1}{2}(\chi-1)\right)^{-1}\left(\frac{\Delta_o\omega}{\omega_o}\right)^2$$

we arive at

[23] $$\frac{\Delta k_y}{k_o} = \frac{\Delta_o\omega}{\omega_o} .$$

The condition [22] and the result [23] correspond to eq. (2.13) in the article of Mandel and Wolf (9). The conditions [19], [22] are in fact satisfied for the unfocussed beam in (3) where $\alpha_o = 4 \times 10^{-3}$ rad (Ref.3, p. 675), and $\Delta_o\omega/\omega_o \approx 10^{-5}$ (Ref. 10). We then assume that the focusing does not change the value of $\Delta k_y/k_o$. To justify this assumption we point out that there is no connection between $\Delta k_y$ and the spreading angle $\alpha$ as it exists for $\Delta k_x$ in formulas [8] and [15]. Only in the higher approximations, which we do not consider here, would $\Delta k_y$ enter into such a relation (see the line above formula [13]). A change in $\Delta k_y$ would inply a change in $\Delta y$ via the Heisenberg (Fourier reciprocity) relation $\Delta k_y \Delta y \geq 1/2$. Such a change in the length of the wave packet is usually only achieved by devices that are more complicated than our weakly focussing lenses, e.g. media with time dependent index of refraction (11). Even if the value of $\Delta k_y/k_o$ was to increase somewhat by the focusing process our final result [26] would be left almost unchanged due to the greater influence of the other terms.

The condition under which our formula [17] shows line broadening by focusing ($\partial(\Delta\omega/\omega_o)^2/\partial\alpha > 0$) can be expressed by the simple relation

$$[24] \qquad \sin\alpha < (3\eta)^{-1/2}$$

provided the conditions

$$[25a] \qquad \Delta k_y/k_o \ll \chi^{-1/2}$$

$$[25b] \qquad \Delta k_y/k_o \ll (3 + 1/\chi)^{1/2}\sin\alpha$$

$$[25c] \qquad \Delta k_y/k_o \ll (\tfrac{3}{8}(\zeta+\chi-2))^{1/2}\sin^2\alpha$$

are satisfied. Actually, conditions [25] as well as [24] are satisfied in (3) up to $\alpha = 11^o$ for all values of $\chi$, $\zeta$ and $\eta$ shown in the Table I.

Under the same conditions ([19a], [25b,c]) the formula [17] simplifies considerably

$$[26] \qquad \frac{\Delta\omega}{\omega_o} \approx \left[\tfrac{1}{2}(\chi-1)(1 - 2\eta\sin^2\alpha)\right]^{1/2}\sin^2\alpha .$$

which means just the second line on the right-hand side of [17]. Here, the problem of knowing $\Delta k_y/k_o$ has disappeared. In order to obtain an upper limit for the error in formula [26] due to the neglection of the terms higher than $\Lambda^2(\Delta/k_o)^4$ in the Taylor series [15] we calculate the error that would arise if we neglected already the terms higher than $\Delta^2(\Delta/k_0)^2$, where we now consider [15] to be exact. This error, under the conditions [25], is $\eta \sin^2\alpha$ which gives 33% with the most unfavorable values of $\eta=9.2$, $\alpha=11^{\circ}$. This is still very much less than the experimental uncertainty in the results of (3). The true error of formula [26] is still smaller than this. We can expect it to be proportional to $\sin^4\alpha$, but we do not know which coefficient is to replace $\eta$.

Formula [26] is the basic formula for our comparison with the experiments of (3) in section 3.

## 3. Comparison with the Experiments (3)

Panarella (3) actually has carried out a series of experiments. In order to convince himself that the effect observed is the effect sought for he repeated the main experiment several times, each time varying one of the experimental parameters. In addition he used different methods of frequency measurement. We do not doubt that Panarella has found a real line broadening. For us, some of those variations serve as valuable additional information to discriminate between his explanation and ours.

(i) The underlying idea of the first of Panarella's experiments has been to generate an intense beam of pulsed ruby laser light, then to focus it in vacuum by means of a lens, and finally by means of a dielectric mirror filter out of the focussed beam any frequency different from the ruby frequency ($\lambda_0$ = 6943 Å). Figure 2 gives a schematic sketch. If any light is observed on the sensitive layer behind the

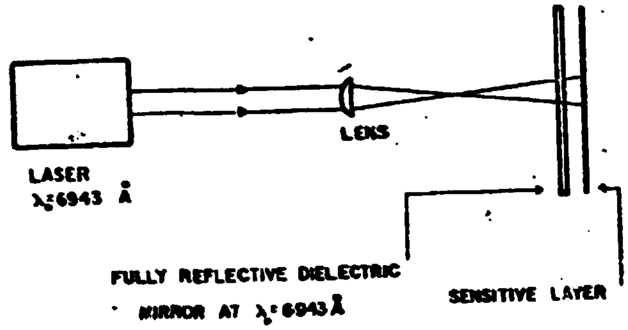


Fig. 2. Schematic Sketch of the experimental arrangement.

dieletric mirror this means that the laser beam has acquired frequencies different from the original one. In fact, light has been detected even 1640Å away from the fundamental ruby wavelength, with the help of selective interference filters (not shown in the figure) inserted between the dielectric mirror and the sensitive layer. The original laser beam has a spectral width of the order of 0.1 Å (10). This figure is concluded from (9) and from the remark found in Panarella's paper that a pulsed ruby laser has a coherence lenght of only a centimeter at most, and using the relations $\Delta E \Delta t = \hbar/2$, $1cm = \Delta l = c\Delta t$.

Drawing a rough curve through the four data points given by Panarella in the last line of his Table I and observing that the total laser peak power is also given we conclude a broadened laser line of width

[27] $\Delta\lambda = 200\ldots800$Å.

The great uncertainty in this width is due to the fact that the points only permit the drawing of a curve that is very asymmetric about the central ruby wavelength. We do not see any reason for this large asymmetry and conclude that the data points carry considerable errors. This is confirmed by Panarella himself in his discussion of the results. Accordingly, Panarella does not use the data to adjust the parameter $\beta_v$ or the function $f(I)$ of his formula [1].

Let us see what our formula [26] says. With $\alpha = 8.5^{o}$, $\lambda_o = 6943$Å, $\chi$ and $\eta$ within the ranges shown in Table I, and observing that $\Delta\lambda/\lambda_o = \Delta\omega/\omega_o$, we get a range after focusing of

[28] $\Delta\lambda = 94\ldots367$ Å.

We think that there is sufficient overlap with the experimental range [27] to justify our explanation. - Panarella gives also some data for $\alpha = 3^{o}$ but these are even less quantitative so that they cannot be used as a check of our formulas.

(ii) In one of the variations of the experimental conditions, the focusing lens was displaced backward by an amount ≈ 1 focal length. No variation in the signal on the sensitive layer was found. In our explanation this is to be expected since the only important parameter is the convergence angle $\alpha$, which was not changed. With Panarella's hypothesis the result is difficult to explain, for the illuminated area on the dielectric mirror and the sensitive layer is larger when the lens is displaced backward, hence the intensity of the light is less. Consequently, upon Panarella's hypothesis that lower intensity implies narrower lines one would expect less light to pass the dieletric mirror (with its finite bandwidth). Panarella tries to remedy this shortcoming of his effective-photon hypothesis by introducing still another ad hoc hypothesis: the intensity I in his formula [1] does not mean the intensity of the light at the place where it interacts with mirrors etc., but the intensity in the focal region. It is in the focal region that the line broadening occurs, and it then survives for some time. Im Panarella's words (4) "it seems that photons of shifted wavelengths do not return immediately to the original frequency when the intensity decreases, but have a certain "lifetime", which is a function of the overall intensity distribution in the pulse".

(iii) Another variation was to position the lens in such a way that the illuminated area on the dieletric mirror (and on the sensitive layer) was the same as with the unfocussed beam. It was found that a signal appeared only when the lens was present. In our explanation this is to be expected since now the relevant focusing angle $\alpha$ was subject to a change. On the other hand, the intensity of the light on the dielectric mirror was the same in both cases, thus Panarella's hypothesis would predict the same signal. Only with his additional "effective-photon lifetime" hypothesis can Panarella save his explanation.

(iv) Calibrated neutral density filters were inserted along the beam path immediately after the laser. It was found in all cases that the signal response was linear with light intensity. Again this is obvious from our formulas whereas Panarella's formula [1], as it stands, predicts a non-linear response. Here, Panarella points out that the raw nature of the data does not allow any quantitative statement. While we would agree with that, we are not convinced that it is due to the reasons put forward by Panarella. He argues that the interference filters between the dielectric mirror and the sensitive layer which were used to select the differente wavelengths in first approximation behave more like short-wave passes rather than narrow-band devices when light does not impinge perpendicular to the filter surface and deviates by an angle larger than, say $\pm 10^{\circ}$. Since the beam aperture is at most $8.5^{\circ}$ in the experiment considered it is difficult for us to understand the meaning of Panarella's statement that: " In our case photons undergoing a frequency shift do so after scattering from the focal region. Consequently, their direction of travel is likely to be outside the $\pm 10^{\circ}$ - aperture central cone having apex at the focal point". We suspect that this conclusion is based on too naive a photon picture. Moreover, a short-wave pass characteristic of the interference filters would result in an asymmetry of the apparent line profile mentioned in Section II in the sense of favoring long wavelengths whereas actually the short wavelengths are favored.

(v) In a different series of experiments a spectroscopic method of frequency measurement was employed. The same ruby laser was fired into a grating spectrograph and the spectra were recorded on photographic film. The beam had been focussed onto the entrance slit of the spectrograph by means of a cylindrical lens of focussing angle $\alpha_x = 11^{\circ}$ . For the cylindrical lens it is $\alpha_z = 0$ and by the same reasoning that led us from formula [15] to formula [26] we are now led to

$$[29] \qquad \Delta\omega/\omega_o = \left\{\frac{1}{4}(\chi-1)\left[1-\frac{1}{2}\left(\frac{\zeta-\chi}{\chi-1}\right)\sin^2\alpha_x\right]\right\}^{1/2}\sin^2\alpha_x$$

Formula [29] with $\alpha_x=11^o$, $\lambda_o = 6943Å$, $\chi$ and $\zeta$ within the ranges shown in Table I gives a broadened line of (half) width $\Delta\lambda=110...328Å$ which is of the order of the photographic records and thus confirms our explanation. More quantitative results are not available.

We now consider the two experiments whose results seem to contradict our formulas.

(vi) In a variation of the experiment in (v) neutral density filters were again inserted along the beam path. With decreasing filter transmission a narrowing of the blackened region was observed. This is interpreted by Panarella as a narrowing of the spectrum itself, according to the spirit of his formula [1]. Since the convergence angle $\alpha_x$ is not changed we expect neither a broadening nor a narrowing. In fact, we think that the reason why the spectrum on the film appears short when filters of lower transmission are inserted, is underexposure of the outer part of the spectrum, and we think that the narrowing effect would have disappeared if the exposure time had been extended (accumulating many pulses on the same film) in inverse proportion to the transmission so as to deposit the same total energy on the film in any of the filtering cases.

(vii) Finally, an interferometric method was employed. Pulses of 25-MW peak power, 80-nsec duration from a Q-spoiled Nd:glass laser ($\lambda_o$ = 10600 Å) illuminated a Mach-Zender interferometer, and the time history of the interference fringes was observed with an image converter camera. The observed fringe spacing is proportional to the light wavelength, so from the contraction of the fringes a decrease in the light wavelength of ≈ 2000 Å was concluded. The intensity variation is here given by the time history of the laser pulse itself and not by beam focusing , so our formulas, [15] - [26], would not give any line broadening. On the other hand, Panarella (2) quotes reference (12), which shows that beam divergence may vary within the interval $10^{-3}...10^{-2}$ rad during one pulse. This may give rise to a variation in line width $\Delta_o\omega/\omega_o$ if we use the tentative value of $\Delta k/k_o = 7\times10^{-5}$ for the Q-spoiled Nd:glass laser (13). We then may employ formula [21] because conditions [20a,b] but not [20c], are fulfilled. Formula [21] with $\chi$=3, gives an increase in $\Delta\omega/\omega_o$ by a factor of 1.7 when $\alpha$ goes from $10^{-3}$ to $10^{-2}$. For still lower values of $\Delta k_y/k_o$ we may altogether neglect $\Delta k_y/k_o$ in [21]. We would then get a broadening by the factor $\sqrt{10}$ = 3.2. Nevertheless, we do not see how this broadening could explain the details in the experimental results under discussion. Neither do we think that Panarella's explanations of these results are convincing in all points. According to Panarella's interpretation of his formula [1] as explained in the introduction, the laser pulse of increasing and then decreasing intensity contains photons of frequencies higher than $\nu_o$ in its central portion, whereas in its head and tail the frequency is lower than $\nu_o$. Now, in all cases a filter with peak transmission at $\lambda_o$ = 10600 Å and half bandwidth 85 Å had been placed in front of the camera. Thus, during the build-up phase of the pulse the filter would first block the light, then at a certain moment, when the momentary light frequency coincides with $\nu_o$, let it pass and then block it again. In the decay phase of the pulse the same would occur in reverse order of the frequency sequence. On the photographic film one therefore expects an increase and then a decrease of fringe intensity twice in every pulse. Another effect of the filter is to make one wonder how the reported shift of 2000 Å could be measured when the front filter precludes all light outside the narrow band of 10600 ± 85 Å from reaching the camera.- Actually, the observation shows one continous increase in intensity up to nearly the end of the pulse. The explanation offered by Panarella is to point out that the filter still allows a transmission of $10^{-4}$ outside the passband. He then calculates that the intensity of the light of the central portion of the pulse is high enough to effect a blackening of the film. However, the increase in the intensity of the pulse from the moment when the frequency was equal to $\nu_o$ was limited to a factor of 3, so there occurs an overall decrease of intensity by a factor of $3x10^{-4}$ which should show up on the film. Moreover, the absence of a decrease of film blackening in the decay phase of the pulse is still left unexplained. We would therefore conclude that other effects are responsible for what is seen on the film. We have in mind effects of intrinsic laser operation like filaments and self-induced phase modulation which are discussed and judged negligible by Panarella but which might yet be bigger than expected. It has even been shown (14) that the spectral composition of the Nd:glass laser varies within a single flash, though perhaps not to such a degree as to fully explain the above effects.

## 4. Discussion

Our formulas for line broadening by focusing are shown to explain most of the experiments of Panarella (3). In the two cases where they do not, we have given some reasons for the discrepancy. Therefore we think that Panarella's hypothesis of photon-energy increase with intensity can be avoided.

As to the ionization of gases whose ionization energy is a factor of 5 and more above the mean photon energy, we do not think that line broadening gives <u>the</u> explanation, but we think that it represents an effect that should be taken into account and will reduce the difficulties encountered in the cascade and multiphoton theories. When the line width broadens, the line intensity far from the center wavelength increases considerably, the exact increase depending strongly on the line shape. Preliminary calculations indicate that with an exponentially damped Lorentzian line shape the effect may be large enough for high intensity laser beams.

We conclude with some remarks on the applications of the Heisenberg relations $\Delta x \Delta k_x \geq \frac{1}{2}$ etc. In fact, some of our results could have been obtained more simply by arguing directly with the "uncertainties" $\Delta_x$, $\Delta k_x$ etc. For example, formula [14] may be obtained by considering the absolute value $k_o$ of the wave vector (momentum) to be well defined, but the direction to be uncertain within the angle $\alpha_x$, so that the x component $k_x$ has an inherent uncertainty of $\Delta k_x = k_o \sin\alpha_x$ (15). However, the "uncertainties" are defined as widths (second central moments) of wave packets and we prefer to argue directly in terms of these wave packets. Formula [15] could not have been obtained by only considering the widths, and there are pitfalls for an anwary use of $\Delta x$ and $\Delta p_x$.

For example, for a focussed beam with axis in y direction the quantity $\Delta x$ is proportional to the beam diameter, and $\Delta x$ decreases when the focal point is approached. Now, sometimes it seems to be assumed (at least implicitely) that, due to the Heisenberg relation $\Delta x \Delta p_x \geq \hbar/2$, the momentum range $\Delta p_x$ and with it the energy range $\hbar\Delta\omega$ would increase when the focal point is approached, and decrease again after it, so that the focal point is the point of maximal $\Delta\omega$. Thus, Allen (5) in his demonstration that the enlarged energy range may enable some photons to ionize a gas, places his detectors just at the focal point However, our formulas [17], [26] only contain the convergence angle $\alpha$ and not the beam diameter. Accordingly, the energy range is the same at any point in front of and behind the focal point. The experiments of Panarella are very interesting for a clarification of these questions. For example, Panarella displaced the focusing lens backward, as described in section 3 (ii), and found no variation in the signal on the sensitive layer. This is in accordance with our formulas, but it may come as a surprise to some readers; it certainly was unexpected by Panarella who in order to account for it introduced the additional hypothesis of his "effective-photon lifetime".

The reader may also check whether his intuitive expectation coincides with our formulas in the following three cases:

(i) Insert another lens between the first lens and the dielectric mirror (cf. Fig. 2) so that behind the second lens the beam is again (almost) parallel. All registered photons have passed through the focal region. Nevertheless, our formulas predict that the broadening disappears since $\alpha = 0$ at the sensitive layer. There is never any broadening (to the approximation of our formulas) in this case even if the diameter of the final beam is varied by a variation of the position of the second lens between the first one and the mirror.

(ii) Displace the lens of Fig. 2 forward instead of backward so that the mirror and sensitive layer are between the lens and the theoretical focal point. In this case no registered photon has passed through the focal region when it reaches the sensitive layer. Our formulas still give a broadening, $\alpha$ being unchanged.

(iii) Still another interesting case is the use a concave lens instead of a convex one so that the beam behind the lens diverges by the angle $\alpha$. Our formulas being equally valid for convergent and divergent beams predict line broadening also in this case.

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## CAPTIONS

FIG. 1. Transverse spreading of a wave packet.

FIG. 2. Schematic sketch of the experimental arrangement.

TABLE I. Values for $\chi$, $\zeta$ and $\eta = \frac{1}{4}(\chi+\zeta-2)/(\chi-1)$ appearing in formulas [15]- [26] for various types of the distribution $\phi_1(k_x)$.