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AN INTRODUCTION TO COMBINATORIAL GEOMETRY*

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The combinatorial geometry package as used in many three-dimensional multimedia Monte Carlo radiation transport codes, such as HETC,[1] MORSE,[2] and EGS,[3] is becoming the preferred way to describe simple and complicated systems. Just about any system can be modeled using the package with relatively few input statements. This can be contrasted against the older style geometry packages in which the required input statements could be large even for relatively simple systems. However, with advancements come some difficulties.

The users of combinatorial geometry must be able to visualize more, and, in some instances, all of the system at a time. Errors can be introduced into the modeling which, though slight, and at times hard to detect, can have devastating effects on the calculated results. As with all modeling packages, the best way to learn the combinatorial geometry is to use it, first on a simple system then on more complicated systems.

The basic technique for the description of the geometry consists of defining the location and shape of the various zones in terms of the intersections and unions of geometric bodies. The geometric bodies which are generally included in most combinatorial geometry packages[+] are: 1) box, 2) right parallelepiped, 3) sphere, 4) right circular cylinder, 5) right elliptic cylinder, 6) ellipsoid, 7) truncated right cone, 8) right angle wedge, and 9) arbitrary polyhedron. The data necessary to describe each of these bodies are given in Table 1. As can be easily noted, there are some subsets included for simplicity. For example, the right parallelepiped and box is a subset of the arbitrary polyhedron.

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Specifying Individual Bodies

As an example of how to define these bodies consider the following:

1. Box (BOX)

To specify a BOX, the vertex $\overline{V}$ (x,y,z) at one of the corners and a set of three *mutually perpendicular vectors,* a_i (x_i, y_i, z_i), representing the height, width, and length of the box are given. This is illustrated in Fig. 1.

2. Rectangular Parallelepiped (RPP)

To specify the RPP, the x, y, and z coordinates that bound the RPP are given as shown in Fig. 2.

3. Sphere (SPH)

To specify the SPH, the vertex $\overline{V}$ (x,y,z) of the center of the sphere and the radius R of the sphere are given as illustrated in Fig. 3.

4. Right Circular Cylinder (RCC)

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[+]The combinatorial geometry package which is described here is the one used with the MORSE-CG code RSIC-CCC-203C, Oak Ridge National Laboratory, Oak Ridge, Tennessee U.S.A.

Table 1. Input Required For Various Geometrical Bodies in Combinatorial Geometry

Card Columns Body Type	ITYPE 3-5	IALP 7-10	Real Data Defining Particular Body						Number of Cards Neede
			11-20	21-30	31-40	41-50	51-60	61-70	
Box	BOX	IALP is assigned by the	Vx H2x	Vy H2y	Vz H2z	H1x H3x	H1y H3y	H1z H3z	1 of 2 2 of 2
Right Parallelepiped	RPP	user or by the code if	Xmin	Xmax	Ymin	Ymax	Zmin	Zmax	1
Sphere	SPH	left blank.	Vx	Vy	Vz	R	-	-	1
Right Circular Cylinder	RCC	blank.	Vx R	Vy -	Vz -	Hx -	Hy -	Hz -	1 of 2 2 of 2
Right Elliptic Cylinder	REC		Vx R1x	Vy R1y	Vz R1z	Hx R2x	Hy R2y	Hz R2z	1 of 2 2 of 2
Ellipsoid	ELL		V1x R	V1y -	V1z -	V2x -	V2y -	V2z -	1 of 2 2 of 2
Truncated Right Cone	TRC		Vx R1	Vy R2	Vz -	Hx -	Hy -	Hz -	1 of 2 2 of 2
Right Angle Wedge	WED or RAW		Vx H2x	Vy H2y	Vz H2z	H1x H3x	H1y H3y	H1z H3z	1 of 2 2 of 2
Arbitrary Polyhedron	ARB		V1x V3x V5x V7x	V1y V3y V5y V7y	V1z V3z V5z V7z	V2x V4x V6x V8x	V2y V4y V6y V8y	V2z V4z V6z V8z	1 of 5 2 of 5 3 of 5 4 of 5
			Face Descriptions (see note below)						5 of 5
Termination of Body Input Data	END								

NOTE: Card 5 of the arbitrary polyhedron input contains a four-digit number for each of the si faces of an ARB body. The format is 6D10.3, beginning in column 11. See the ARB writeup for a example.

To specify the RCC, the vertex $\overline{V}$ (x,y,z) at the center of one base, a height vector, $\overline{H}$, expressed in terms of its x, y, and z components, and a scalar, R, denoting the radius, are given. This is shown in Fig. 4.

5. **Right Elliptical Cylinder (REC)**

To specify the REC coordinates of the center of the base ellipse, a height vector (as in the case of the RCC) and two vectors in the plane of the base defining the major and minor axes are given (see Fig. 5).

6. **Ellipsoid (ELL)**

To specify an ELL, two vertices, $\overline{V}_1$ and $\overline{V}_2$, denoting the coordinates of the foci, and a scalar, R, denoting the length of the major axis, are given (see Fig. 6).

7. **Truncated Right Angle Cone (TRC)**

To specify the TRC, a vertex $\overline{V}$ at the center of the lower base, the height vector, $\overline{H}$, expressed in terms of its x, y, z components, and two scalars, R_1 and R_2, denoting the radii of the lower and upper bases, are given (see Fig. 7).

8. **Wedge (WED) or (RAW)**

To specify a RAW, the vertex $\overline{V}$ at one of the corners is given, i.e., the (x,y,z) coordinates. Also, one must specify a set of three *mutually perpendicular vectors, a_i*, with a_1 and a_2 describing the two legs of the right triangle of the wedge. That is, the x, y, and z components of the height, width, and length vectors must be given (see Fig. 8).

9. **Arbitrary Polyhedron (ARB)**

To specify an ARB, assign an index (1 to 8) to each vertex. For each vertex, give the x, y, z coordinates. Each of the six faces are then described by a four-digit number giving the indices of the four vertex points in that face. For each face, these indices must be entered in either clockwise or counterclockwise order (see Fig. 9).

Combining Bodies

These geometric bodies are combined to form the correct model. Special operators noted by the symbols +, −, and OR are used to describe the intersections and unions. These symbols are used by the program to construct information relating material descriptions to the body definitions.

If a body appears in a zone description with a (+) operator, it means that the *zone being described is wholly contained in the body*. If a body appears in a zone description with a ( ) operator, it means that the zone being described is *wholly outside the body*. If the body appears with an (OR) operator, it means that the input zone being described includes all points in the code zone following the "OR." *OR may be considered as a union operator.* In some instances, an input zone may be described in terms of subzones, called code zones, lumped together by (OR) statements. Body numbers after an "OR" describe a code zone. Code zones may be thought of as subsets of an input zone.

Techniques for describing a particular geometry are best illustrated by examples. Consider an object composed of a sphere and a cylinder as shown in Fig. 10. To describe the object, sphere and cylinder are the same, then they can be considered as one zone, say Zone I (Fig. 10, Item c). The description of Zone I would be:

$$I = OR\ +2\ OR\ +3\ .$$

This means that a point is in Zone I if it is inside *either* Body 2 *or* Body 3.

If different materials are used in the sphere and cylinder, then the sphere with a cylindrical hole in it would be given a different zone number (say J) from that of the cylinder (K).

The description of Zone J would be (Fig. 10, Item d):

$$J = +2\ -3\ .$$

This means that points in Zone J are all those points inside Body 2 which are not inside Body 3.

The description of Zone K is simply (Fig. 10, Item e):

$$K = +3\ .$$

In other words, all points in Zone K lie inside Body 3.

Combinations of more than two bodies and similar zone descriptions could contain a long string of (+), (−), and (OR) operators. It is important, however, to remember that every spatial point in the geometry must be located in one and only one zone.

As a more complicated example of the use of the (OR) operator, consider the system shown in Fig. 11 consisting of the shaded zone, A, and the unshaded zone, B. These zones can be described by the two BOXes, Bodies 1 and 3, and the RCC, Body 2. The zone description would be:

$$A = +1\ +2$$

and

$$B = OR\ +3\ -1\ OR\ +3\ -2\ .$$

Notice that the OR operator refers to all following body numbers until the next OR operator is reached.

In combinatorial geometry the most elementary zone description is called a "code zone." The code zone description uses only the "+" and "−" operators to describe a material zone. Code zones may be combined with the "OR" operator to define an "input zone" containing more than one code zone. An input zone may be composed of only one code zone. An input zone is a material zone of only one medium containing one or more code zones. An input zone may describe volumes of space that are not continuous. Figure 12 illustrates the difference between input zones and code zones.

Figure 12 shows the shaded zone, Input Zone 1, as all points in Body 1 "OR" all points in Body 2. Input Zone 1 in Fig. 12-A contains two code zones. The first code zone is +1 and the second code zone is +2. Alternatively, this material zone could have been described as two input zones, each with only one code zone as illustrated by Zones 2 and 3 of Fig. 12-A . In both methods, the set of points shared by Bodies 1 and 2 have been doubly defined; this is valid since both zones contain the same medium. If Bodies 1 and 2 contained different material media, then the two bodies could not be combined into a single input zone, and the overlap between the two bodies would be erroneous. Figure 12-B's shaded zone is all points in Body 1, and the clear zone is all points in Body 2 which are "not" in Body 1. The shaded zone, Zone 1, is described as +1. The clear zone, Zone 2, is described as +2 −1. Figure

12-C's shaded area is all points in Body 2 and is the opposite description of Fig. 12-B's. Figure 12-D's shaded zone is all points in Body 1, which are also in Body 2, that is, points which are common to both bodies. This zone is described as +1 +2. Figure 12 demonstrates the use of the "OR," the "+," and "−" operator. Note the user is not required to use the "OR" operator (see Fig. 12-A). The "OR" operator gives the user a shorthand input notation to lump "code zones" logically together into a single "input zone." All combinatorial geometry tracking is done with "code zones." Tracking across code zones is transparent to the user. Program action occurs only when a particle track crosses an input zone boundary. Input zones are a user convenience for consolidating code zones in a more logical, understandable manner. Each input zone must be defined as a material medium.

Three-dimensional picture programs are often supplied with a combinatorial geometry package, which can be extremely useful in debugging the input data.[4]

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FIGURE CAPTIONS

Fig. 1. Box (BOX).

Fig. 2. Rectangular Parallelepiped (RPP).

Fig. 3. Sphere (SPH).

Fig. 4. Right Circular Cylinder (RCC).

Fig. 5. Right Elliptical Cylinder (REC).

Fig. 6. Ellipsoid (ELL).

Fig. 7. Truncated Right Angle Cone (TRC).

Fig. 8. Right Angle Wedge (WED).

Fig. 9. Arbitrary Polyhedron (ARB).

Fig. 10. Examples of Combinatorial Geometry Method.

Fig. 11. Use of OR Operators.

Fig. 12. Illustrative Input Zone Descriptions.

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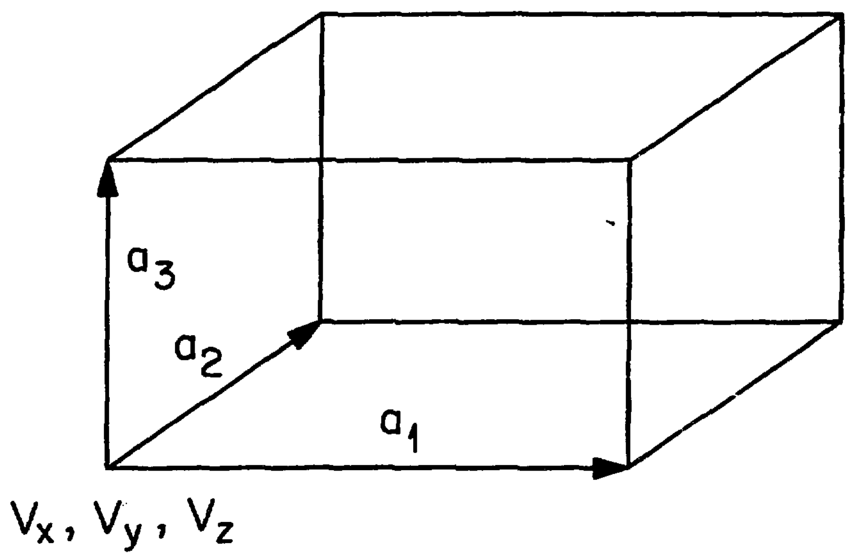

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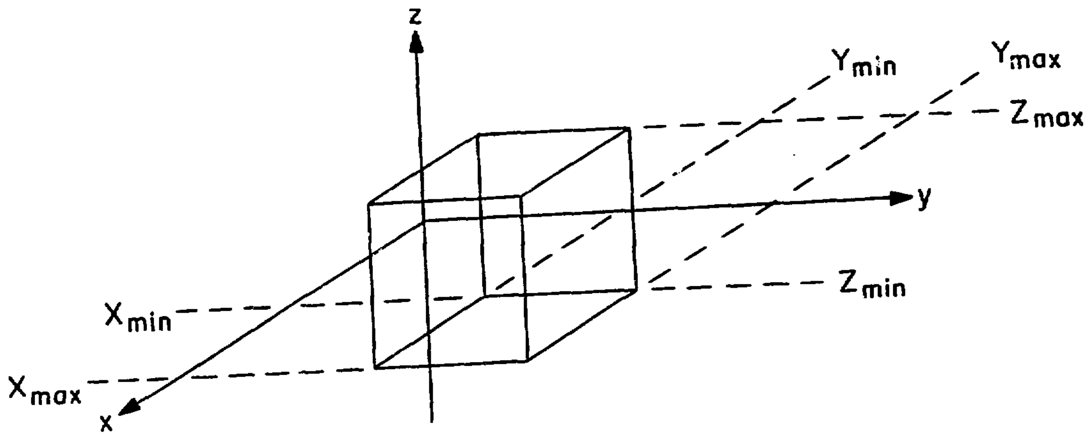

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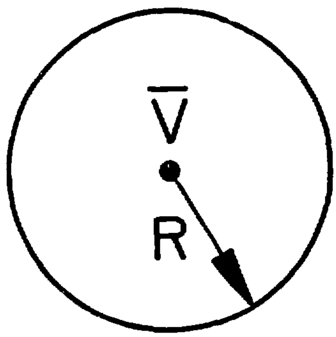

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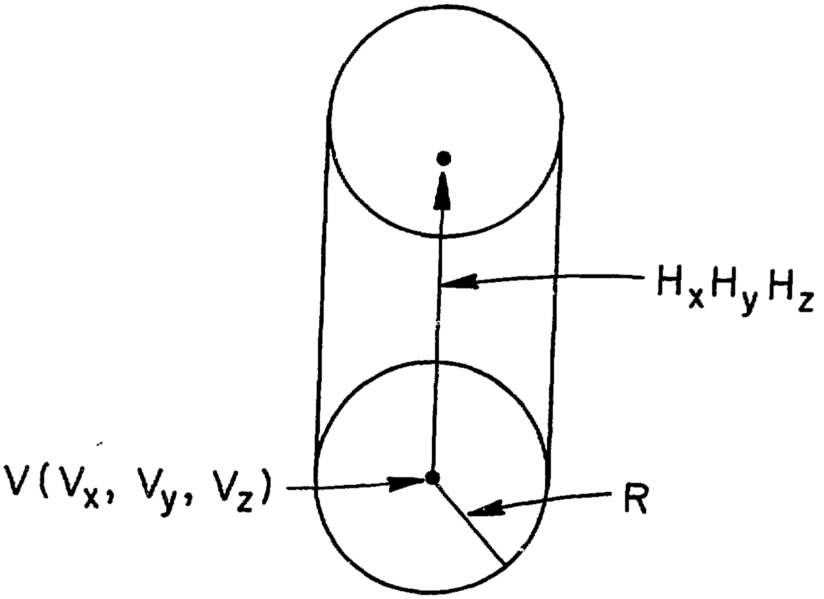

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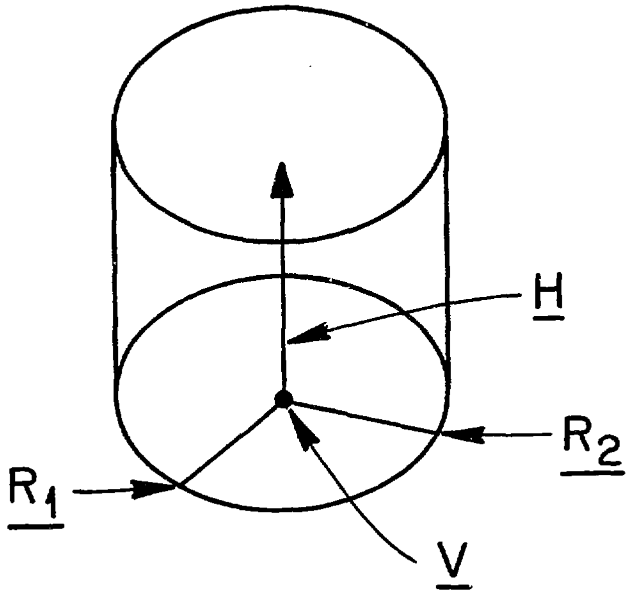

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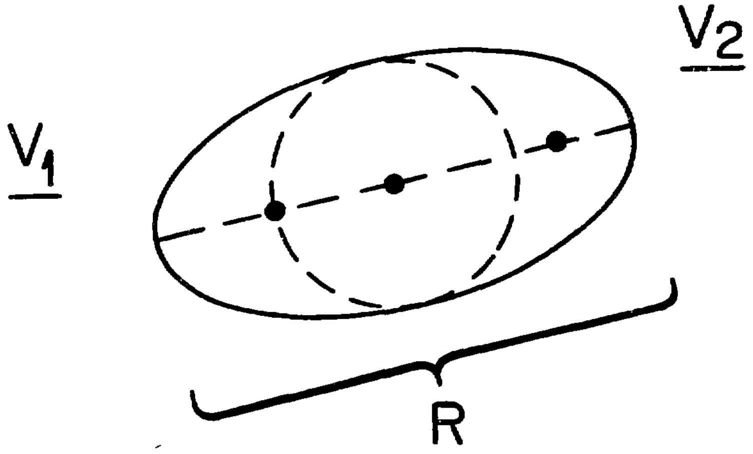

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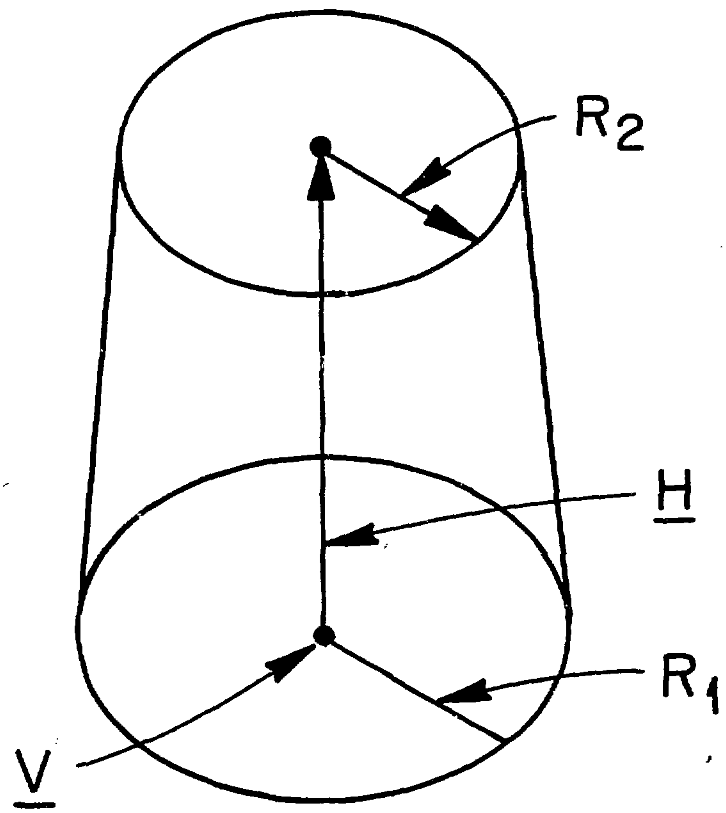

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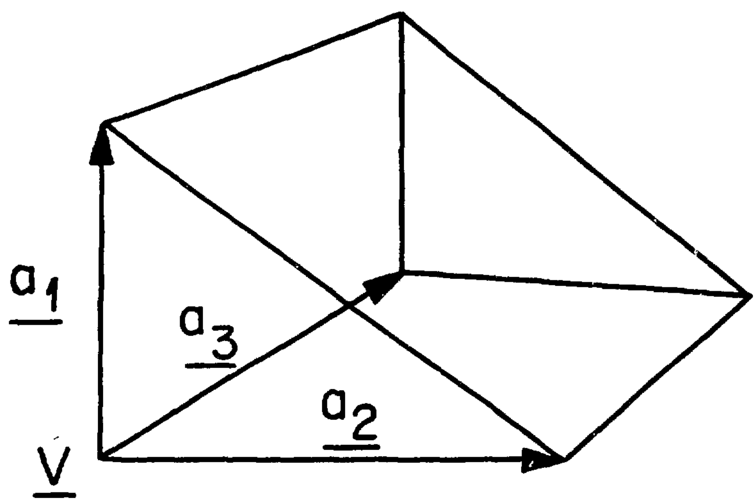

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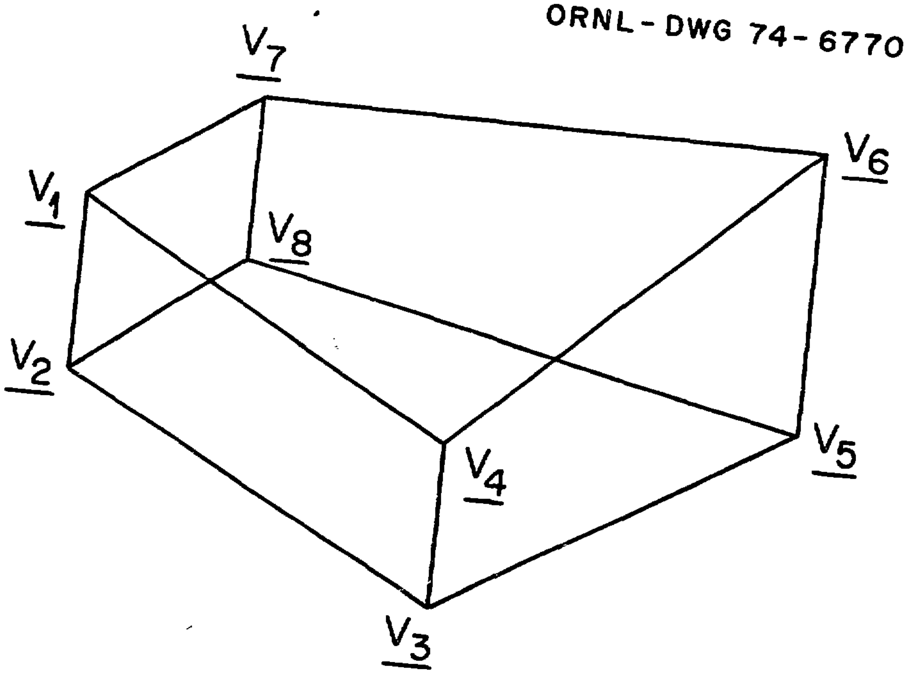

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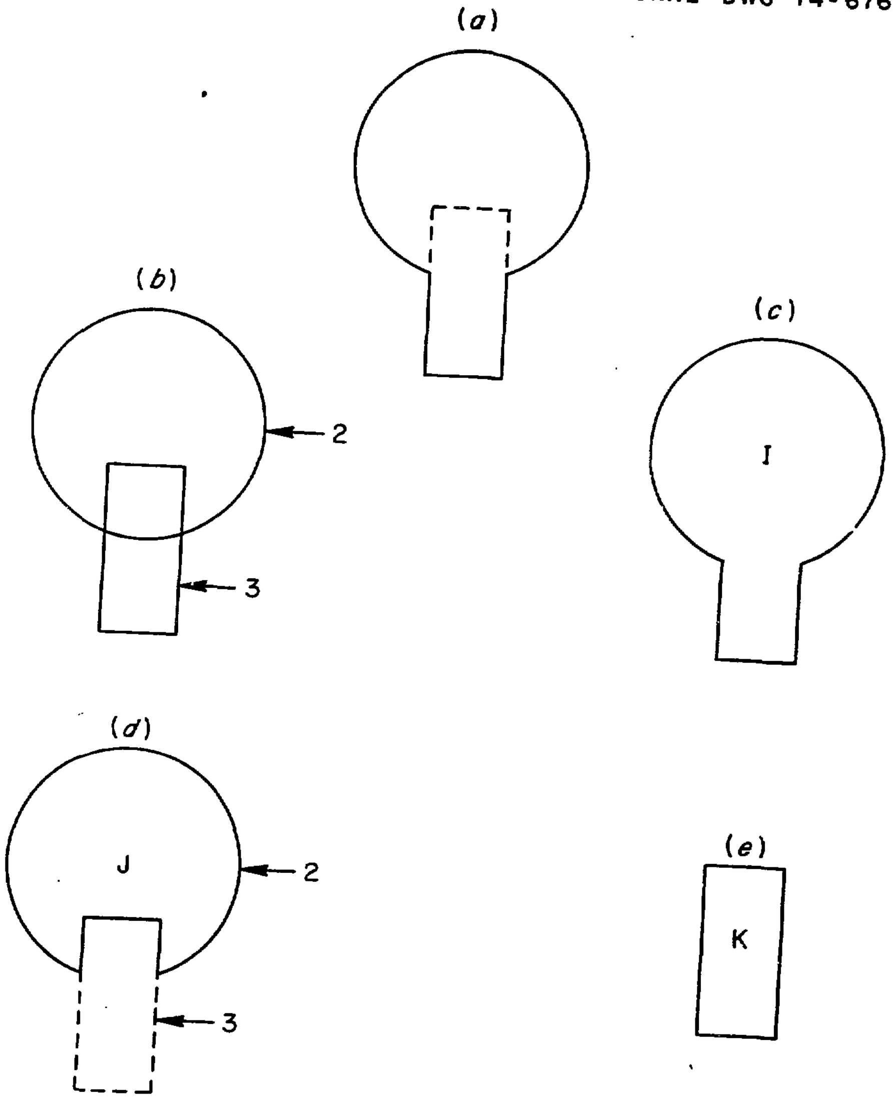

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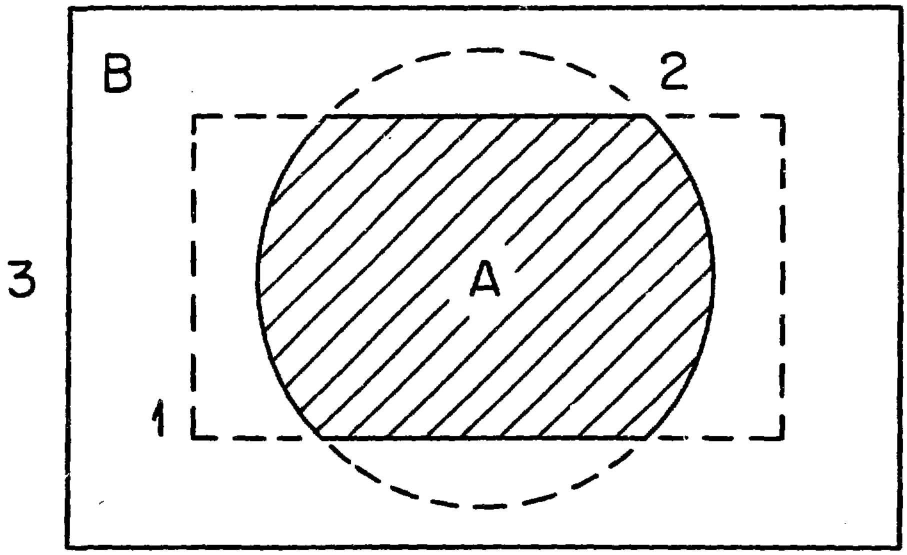

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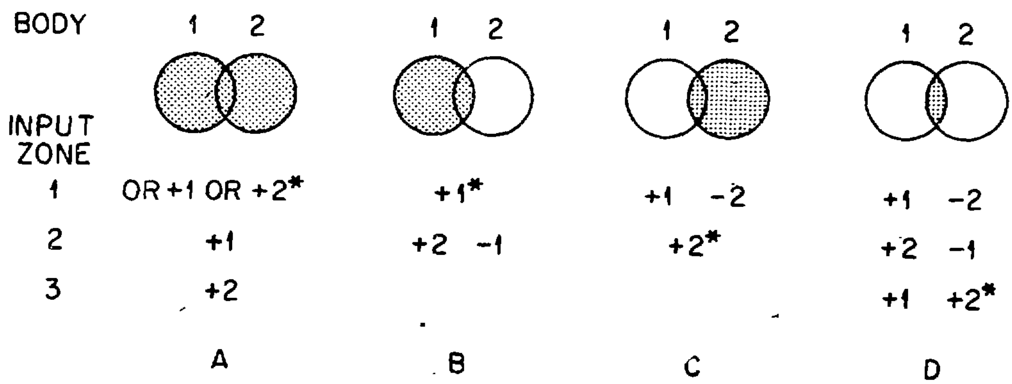

*INDICATES THE SHADED ZONE

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