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A Comparison of Finite Element J-Integral Evaluations for the Blunt Crack Model and the Sharp Crack Model

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Introduction

In assessing the safety of a liquid metal fast breeder reactor (LMFBR), a major concern is that of hot sodium coming into contact with either unprotected concrete or steel-lined concrete equipment cells and containment structures. An aspect of this is the potential of concrete cracking which would significantly influence the safety assessment.

Concrete cracking in finite element analysis can be modeled as a blunt crack in which the crack is assumed to be uniformly distributed throughout the area of the element. A blunt crack model based on the energy release rate and the effective strength concepts which was insensitive to the element size was presented by Bazant and Cedolin [1]. Some difficulties were encountered in incorporating their approach into a general purpose finite element code [2]. An approach based on the J-integral to circumvent some of the difficulties was proposed by Pan, Marchertas and Kennedy [3].

Alternatively, cracking can also be modeled as a sharp crack where the crack surface is treated as the boundary of the finite element mesh. The sharp crack model is adopted by most researchers and its J-integral has been well established [4,5]. It is desirable to establish the correlation between the J-integrals, or the energy release rates, for the blunt crack model and the sharp crack model so that data obtained from one model can be used on the other.

J-Integral for the Sharp Crack Model

If a crack of length a is assumed to advance in the x-direction, the rate of energy change can be shown to be the well known J-integral [4]

$$J = \int \left(W dy - T \cdot \frac{\partial U}{\partial x} d\ell\right) \qquad (1)$$

where x and y are Cartesian coordinates with y perpendicular to the crack surface, W is the strain energy, T is the surface traction, U is the displacement, $d\ell$ is a line segment in an arbitrary integration loop surrounding the crack tip. If the integration is reduced to an infinitesimally small loop surrounding the crack tip, then

$$J = \int_{tip} W dy \qquad (2)$$

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J-Integral for the Blunt Crack Model

If the same crack described in the above section is modeled as a blunt crack, then the material in the crack still retains its load carrying capacity in the x-direction. In this case, the rate of energy change associated with the crack advancement was shown in Ref. [3] as

$$G = \int_{tip} \left(W - \frac{1}{2} \sigma_x \varepsilon_x\right) dy \tag{3}$$

where σ_x is the stress and ε_x is the strain in the x direction before crack advances. It is noted that the first term in Eq. (3) is equal to the well known J-integral and the second term accounts for the remaining load carrying capacity of the cracked element. Hence, G and J differ by a constant

$$G = k \cdot J = k \cdot \int \left(W dy - T \cdot \frac{\partial U}{\partial x} d\ell\right) \tag{4}$$

where the integration is performed in an arbitrary path surrounding the crack tip.

Since the tip of a blunt crack is not the same as that of a sharp crack, J values evaluated by Eqs. (1) and (4) may not be the same. However, if the distribution of the elastic field of a blunt crack is similar to that of the sharp crack, then a correlation can be established between the two J values.

Numerical Results and Conclusions

Numerical results for the J-integrals of the sharp crack model and the blunt crack model were obtained for the static problem of a center-cracked plate subjected to uniaxial loading. The finite element mesh for the sharp crack model is shown in Fig. 1(a). Three different meshes for the blunt crack model are shown in Figs. 1(b) and (c) where the solid line shows the coarse mesh and the dotted line shows the refined mesh. The shaded area indicate the blunt crack. The zig-zag crack band in the slanted mesh shown in Fig. 1(c) was predicted by the procedure reported in Ref. [3]. All computations were carried out for $E = 2.193$ MN/m^2, $\nu = 0.2$, $b = 0.24$ m, $h = 0.26$ m and $J_c = .0236$ MN/m. The applied stress σ was kept at 5000 N/m^2 for all runs. The integration loop for the J-integral is shown as the bold line in Figs. 1(a)-(c). The calculated J-integral values and the stress multiplier α, where $\alpha\sigma$ gives the critical stress for crack extension, are plotted in Fig. 1(d). It is noted that the values for the sharp crack model and the blunt crack model correlate with each other very well. Hence, the following conclusions can be made:

(i) The J-integral approach indeed can be used with the blunt crack model. Also, it can be seen that the zig-zag crack band can be used to represent the straight crack as long as the length of the crack is taken to be the projected length on the crack direction.

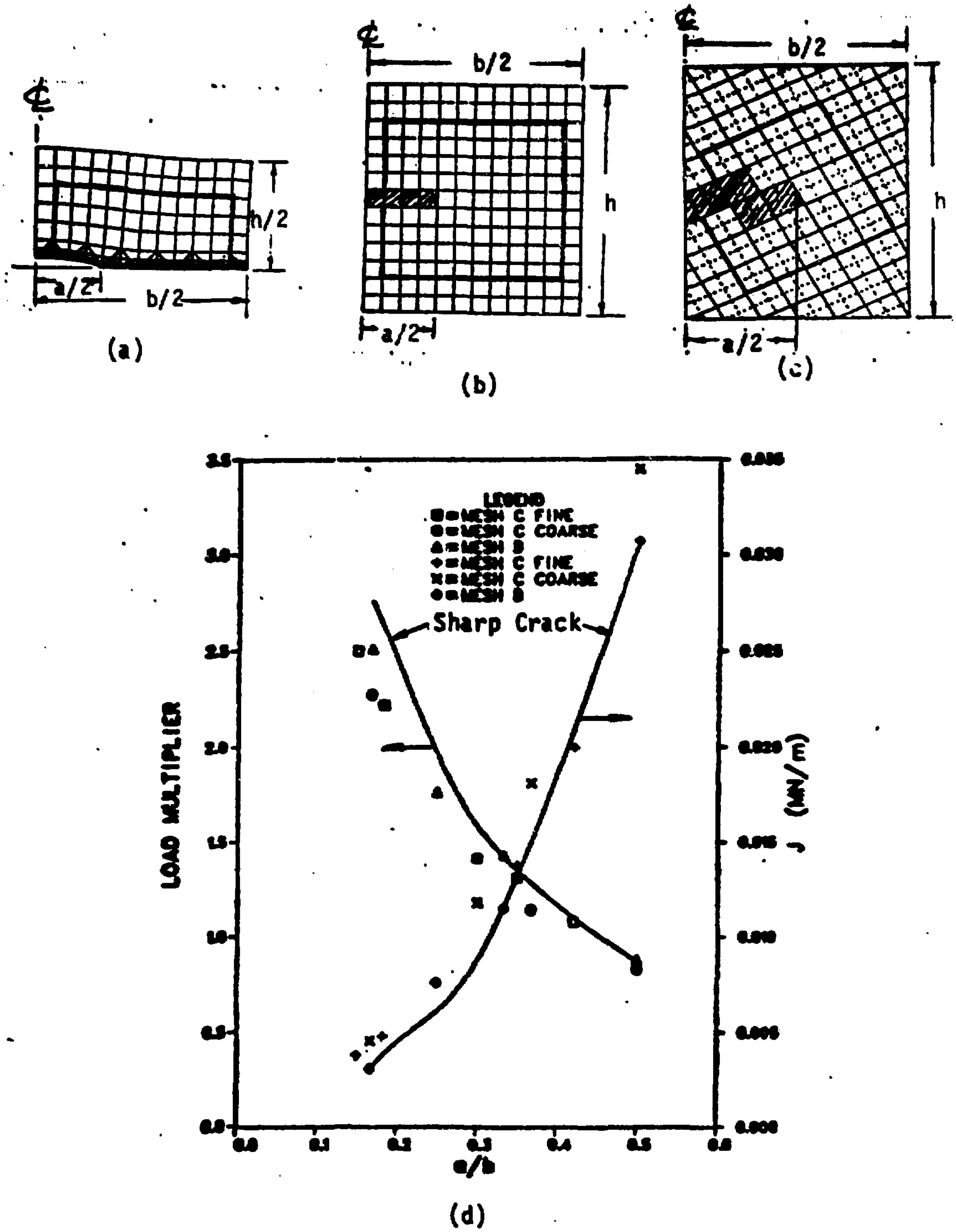

Fig. 1. (a) Sharp Crack Model
(b) Blunt Crack Model in Rectangular Mesh
(c) Blunt Crack Model in Slanted Mesh
(d) Variation of the Load Multiplier and the J-Integral vs. the Crack Length to Plate Width Ratio.

(2) Since the J-integrals for the blunt crack model and the sharp crack model are the same, they can be used interchangeably.

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