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HYDROLOGIC BEHAVIOR OF FRACTURE NETWORKS

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Abstract

This paper reviews recent research on the nature of flow and transport in discontinuous fracture networks. The hydrologic behavior of these networks has been examined using two- and three-dimensional numerical models. The numerical models represent random realizations of fracture networks based on statistical field measurements of fracture geometry and equivalent hydraulic aperture. We have compared the flux and mechanical transport behavior of these networks to the behavior of equivalent continua. In this way we are able to determine whether a given fracture network can be modeled as an equivalent porous media in both flux and advective transport studies. We have examined departures from porous media behavior both as a function of interconnectivity and heterogeneity. Parameter studies have revealed behavior patterns such as: given a fracture frequency that can be measured in the field, porous media like behavior and the magnitude of permeability are both enhanced if the fractures are longer and the standard deviation of fracture permeabilities is smaller. Transport studies have shown that the ratio between flux and velocity is not necessarily constant when the direction of flow is changed in systems which do behave like a porous media for flux. Thus the conditions under which porous media analysis can be used in transport studies are more restrictive than the condition for flux studies. We have examined systems which do not behave like porous media and have shown how the in situ behavior varies as a function of scale of observation. The behavior of well tests in fractured networks has been modeled and compared to a new analytical well test solution which accounts for the early time dominance of the fractures intersecting the well. Finally, a three-dimensional fracture flow model has been constructed which assumes fractures are randomly located discs. This model uses a semi-analytical solution for flow such that it is relatively easy to use the model as a tool for stochastic analysis.

INTRODUCTION

In recent years, attention has been focused on the possibility of storing nuclear waste in deep, underground facilities excavated in various rock types, many of which have low or negligible matrix permeability. In such rocks the transport of nuclides to the accessible environment is most likely through the fractures (joints, faults etc.) which form the major conduits for groundwater flow. All the major hydrologic techniques for determining how contaminants travel through groundwater use the assumption that the medium behaves like a continuum. As such, it has become a matter of some interest to determine whether or not the networks of fractures surrounding a potential repository behave like a continuum on scales appropriate for flux and transport analysis.

In this paper we give an overview of recent research on the hydrology of fracture networks. The initial work focused on the development of a technique for determining when a fracture network behaves like an equivalent continua. We developed computer models of fracture systems which were based on the occurence and properties of fractures as observed in the field and laboratory. We have assumed that the rock matrix is impermeable and that fractures are parallel plates. Through finite element analysis of flow through these fracture networks we are able to measure the directional permeability of the networks and determine whether or not an equivalent permeability tensor could be used to predict flux through the networks. We have also developed a model for mechanical transport which traces the flow tubes through the fracture network. With this model we are able to calculate the ratio of flux to mean velocity, i.e. the "hydraulic effective porosity", and compare this quantity to the total and connected porosity (or rock effective porosity). Thus, we can compare advective transport in fracture networks to that in ideal porous media. For both flux and transport we have examined the attributes of network geometry which cause the behavior to depart from that of an ideal porous medium.

Current research is focused on characterizing the insitu behavior of fracture networks, both under regional flow conditions and under well testing conditions. The network behavior depends primarily on the scale of observation, the degree of interconnectivity and the degree of heterogeneity. Given these three factors, we are able to examine the likelihood that the fracture network is connected (and thus conductive) on the scale of the regional flow problem. Then, if the network is connected, we wish to characterize the behavior of a volume of rock the size of an element in a groundwater flow model. This implies determining a relationship between gradient and flux as a function of the direction of the gradient when this relationship cannot necessarily be characterized by a permeability tensor. Future research will attempt to develop a groundwater flow model which incorporates such a characterization. That is, we would like to have a model which does not require the existence of a permeability tensor, and therefore Darcy's law, to hold over each element. Such a model "knows" about the vagaries of the fracture system and does not require that these vagaries average out to the mean behavior.

In our numerical models we can perform almost any experiment we like. However, in the real world our knowledge of fractured systems will rely to a very large extent on what we can learn from boreholes. Thus, it is extremely important to know how the behavior of a well test compares to flow under a regional gradient. This research has two major thrusts. First, what can we learn directly from well testing about the behavior and the average hydrologic parameters of the fracture network? Second, what can we learn about the network geometry in order to construct better network models. Well test results are probably the best way to learn about average parameters, but network models have the attraction of being much more flexible tools for learning about the nature of the hydraulic behavior.

Recent work has focused on the development of a well test analysis which takes into account the early time dominance of the fractures intersecting the well. From this analysis, we gain information about the permeability of the fracture near the well, the average permeability of the network and about the distance from the well to the nearest fracture intersection. Parameter studies will be needed to study the applicablity of this analysis under various degrees of connectivity and heterogeneity.

Real fractures are three-dimensional, so we have been very interested in extending the results of our studies from two dimensions to three. We have developed a model in which fractures are discs, randomly located and oriented with random apertures (Long 1984b). The solution of flow through the network is accomplished with a mixed analytical-numerical scheme. Flow in each fracture is calculated analytically by assuming that each intersection acts like a line source or sink. The boundaries of the fracture are eliminated by the use of images for each intersection. Flow through the network is then calculated numerically through the construction of mass balance equations. The model will allow us to efficiently generate and solve for flow through three-dimensional, stochastically generated fracture systems.

This paper summarizes briefly the modeling techniques we have used, and some results from parameter studies performed to date. Some further discussion is devoted to current research and further research needs.

MODELING TECHNIQUES

A numerical code has been developed to generate sample fracture systems in two dimensions and determine the permeability of such systems (Long et al., 1982; Long, 1983). The two-dimensional mesh generator, FMG, produces random realizations of a fracture network in a square region called a generation region. Each set of fractures is generated independently and then the individual sets are superimposed (Figure 1). For each set, the density (number of fractures per unit area) and the orientation, length and aperture distributions must be supplied. The sets are then superimposed. This generator produces a fracture network model similar to that proposed by Baecher et al. (1977).

When all the sets have been generated, a flow region is selected for finite element analysis of flow through the network and measurement of K_g, permeability in the direction of the gradient. Boundary conditions, as shown in Figure 2 are applied to the flow region such that the material inside the region would experience a constant gradient throughout if in fact it were a homogeneous, anisotropic continuum (see Long, 1983 for details). Permeability can be measured in any direction, α, by rotating the boundaries of flow region α degrees and consequently rotating the direction of the gradient. For a homogeneous, anisotropic medium $1/\sqrt{K_g(\alpha)}$ versus α is an ellipse when plotted in polar coordinates. Thus, such a plot serves as a test of whether or not the given volume can be approximated as a homogeneous porous medium. If the data plot as an ellipse, then a unique symmetric conductivity tensor can be found to describe the relationship between gradient and flux. If the data do not plot as an ellipse, then we use a regression technique similar to that of Scheidegger (1954) to determine the best-fit permeability tensor (Figure 3) that in general will be anisotropic. For each direction of the gradient, the difference between the value of directional permeability calculated using the best-fit tensor and the value calculated by the finite element analysis is considered the error. The mean square error, normalized by dividing by the product of the principal permeabilities, is called NMSE. NMSE is a measure of the behavior of the fracture network because NMSE approaches zero as the behavior approaches that of an equivalent porous medium. These techniques are used to study the permeability, i.e., the relationship between flux and gradient, in fracture networks.

In order to study mechanical transport in fracture networks, we trace the flow tubes through the network. The procedure is illustrated in Figure 4 and is further explained in Endo (1984), Endo et al. (1984), and Endo and Witherspoon (1985). Figure 4a shows how we use the parabolic velocity distribution and conservation of mass to calculate the location of, and average velocity in each segment of each flow tube. This allows the construction of a breakthrough curve (Figure 4b) by plotting the percent flow carried by each tube against the travel time in that tube. Then the hydraulic effective porosity, ϕ_e, can be calculated for the direction of flux,

$$\phi_e = \frac{Q}{L/\bar{T}} \quad , \qquad (1)$$

where Q is the flux, L is the linear flow path length in the direction of the flux and $\bar{T}$ is the mean arrival time. In the general case of an anisotropic medium, the mean flux is not in the same direction as the mean gradient. The direction of mean flux must be estimated because in the model, we can only control the direction of the gradient (see Endo, 1984 for details). When ϕ_e is plotted as a function of the direction of flux (Figure 4c) it can be compared with the connected and total porosities ϕ_R and ϕ_T respectively. If the medium behaves like a classical porous medium, then ϕ_e should be invariant with direction and equal to the connected porosity, ϕ_R.

It should be noted that only mechanical transport is calculated. We do not calculate the diffusional mixing that takes place across stream tube boundaries. However, because there is no diffusion, this technique can also be used to calculate the longitudinal dispersivity due to advection alone as a

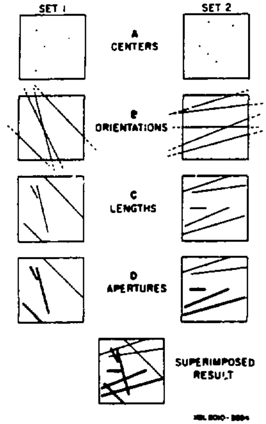

Fig. 1. Generation of a fracture network.

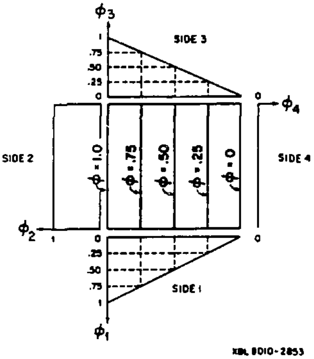

Fig. 2. Boundary conditions applied to fracture models for permeability measurement.

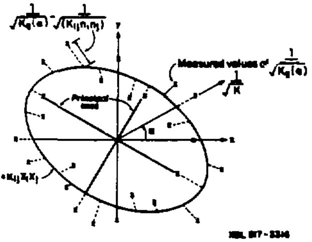

Fig. 3. Plot of the measured values of $1/\sqrt{Kg}$, and the corresponding "best fit" ellipse.

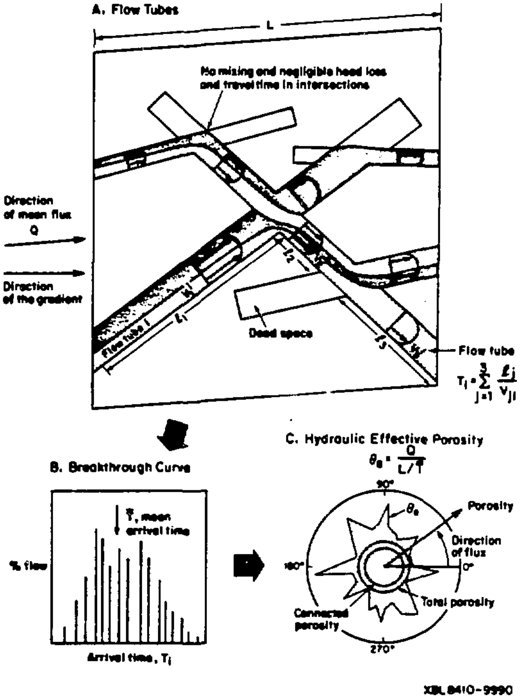

Fig. 4. Calculation of hydraulic effective porosity.

function of direction of flux and the standard deviation of the breakthrough curve. With these techniques we have studied transport in fracture networks with varying degrees of interconnection and heterogeneity.

SOME PARAMETER STUDY RESULTS

We present here a summary of some of the studies that have been performed with the techniques described above. Only a brief discussion of the transport studies is given because another paper in these Proceedings, Endo and Witherspoon (1985), is devoted to this topic.

The behavior of flow in fracture networks departs from that of a porous medium for two main reasons: lack of connection between fractures and heterogeneity of aperture (i.e., spread in the aperture distribution). Furthermore, the behavior of any given system depends on the scale of observation. We have performed a study on interconnection effects at various scales of observation for systems of constant aperture (Long, 1984a). The effect of the proximity of the boundary conditions has been addressed in order to understand the expected insitu behavior of such systems at different scales of observation. We have begun to look at heterogeneity effects, but much more needs to be done in this area. This approach forms a good framework for understanding the theoretical behavior of flow in fracture networks.

In the study of interconnection effects, we began by looking at systems of fractures with constant apertures that would all appear to be the same from borehole observation. That is, the line sample through any of the networks in the series would intersect the same number of fractures with the same orientation distribution. Such systems can actually be very different because the number of fractures intersected by a line sample per unit length, λ_ℓ, is proportional to both the mean length of the fractures, $\bar{\ell}$, and the number of fractures per unit area, λ_A, that is:

$$\lambda_P = \bar{\ell}\lambda_A \overline{\cos\theta} \quad , \qquad (2)$$

where θ are the angles between the poles of the fractures and the borehole. Thus, given the same orientation distribution, systems where the $\bar{\ell}$ and λ_A are different would all appear the same from a borehole as long as the product of $\bar{\ell}$ and λ_A is constant. We call this product LD.

Figure 5 shows some of the fracture networks and permeability plots that were generated with a constant value of LD = 0.288. In each case, $L/\bar{\ell}$ is kept constant, where L is the dimension of the flow region tested. One can see that as $\bar{\ell}$ increases and λ_A decreases the degree of interconnection increases and the systems behave more like porous media. Figure 6 shows how NMSE and average permeability vary as a function of length. It appears that the system becomes connected for lengths greater than about 8 cm. Once connection has occurred, further increases in fracture length cause the system to behave more like a porous medium. Also, the permeability increases by about an order of magnitude and approaches that which would be predicted using Snow's (1965) technique for extensive fractures. For constant aperture and length systems like this, it seems that one needs to know that the fracture length is longer than some minimum, say 20 cm in this case, to know that the medium behaves like a continuum for flux. However, the magnitude of the average permeability will only vary by about an order of magnitude due to interconnection effects. Thus once interconnection is achieved, the magnitude of permeability will be controlled by the aperture distribution because of the cubic relationship between flux in a fracture and aperture.

The above discussion does not include the effect of scale of observation. Percolation theory provides a good framework for understanding this effect. Among others, Englman et al. (1983) have pointed out that interconnection of a fracture network is proportional to a factor ζ where $\zeta = \pi\lambda_A\bar{\ell}^2$. In our study we kept $\lambda_A\bar{\ell}$ constant and plotted the results against $\bar{\ell}$. Thus, the $\bar{\ell}$ axis is proportional

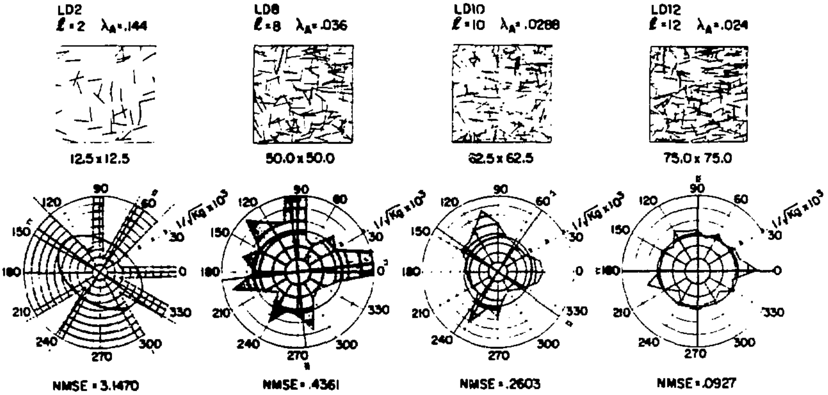

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Fig. 5. Permeability results for fracture lengths 2, 8, 10 and 12.

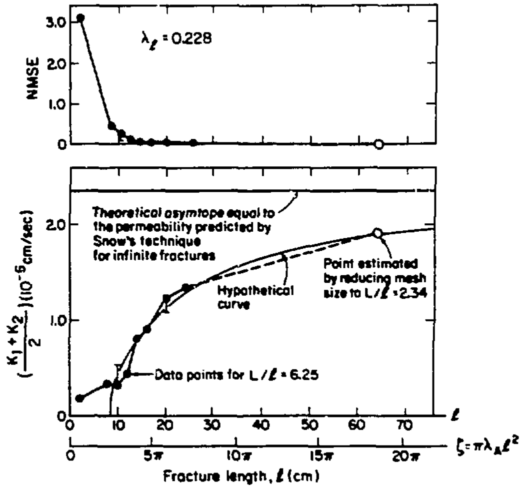

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Fig. 6. Permeability and NMSE versus fracture length.

to a ζ axis (see Figure 6). As ζ increases, the size and number of fracture clusters increases. Englman et al. point out that there is a critical value of ζ, ζ^*. On the infinite scale of observation, systems with $\zeta > \zeta^*$ are always connected and the percolation frequency, ν is said to be one. Thus, the size of the clusters is infinite. On the other hand, observations on the infinite scale of systems with $\zeta < \zeta^*$ are always unconnected and ν is zero. This means the size of the clusters is finite. If one observes any system on a small enough scale, some realizations will be connected and some won't. Thus, given ζ and the scale of observation the value of ν will be between zero and one. If $\zeta > \zeta^*$ then increases in scale of observation will increase ν because increases in scale will decrease the probability that the particular sample will be between percolating (or connected) clusters. If $\zeta < \zeta^*$ then increases in scale will decrease ν because larger scales of measurement are more likely to exceed the finite size of the fracture clusters.

Clearly, there should be a relationship between scale of measurement, ζ, ν, permeability and NMSE. For systems with $\zeta > \zeta^*$ we might expect to underestimate permeability on the infinite scale when we use finite scales of measurement. However, the technique we use to measure permeability forces flow through fractures which are truncated by the boundaries of the flow region. In this way permeability is consistently overestimated and we call this truncation error. Also for this reason, in systems, where the fractures are long compared to flow region dimension we expect NMSE to be artificially low. So as scale is increased, we expect to approach the infinite permeability from above. For $\zeta < \zeta^*$, we expect to overestimate the infinite permeability due to both random and truncation effects because the infinite permeability is zero. Most importantly, as ζ is increased, the number of percolating clusters and therefore the number of paths for fluid flow, increase. So as ζ increases, we expect the inifnite permeability to increase. The results we observe, as illustrated on Figure 7 are consistent with the above discussion. All the results in Figure 7 are based on different realizations except the "REV" study. The REV study examined the permeability of the same realization on different scales. In this study, it seems ζ^* corresponds to $\overline{l} \lesssim 10$ cm.

In order to understand how insitu behavior varies with scale, the effect of truncation error must be eliminated from the scale effect study. To do this we are in the process of examining the behavior of various size regions as the boundary conditions are applied increasingly far away. An example of the results for the case of $\overline{l} = 10$ is shown in Figure 8. In this figure, Δ is the difference between the dimensions of the flow region and the dimensions of the region being measured (the study region). The inset shows all the different size regions that were studied. Clearly, we have overestimated the permeability and underestimated NMSE by applying the boundary conditions directly to the region of interest. On the other hand, if the study region is at least 60 cm and the Δ is at least 40 cm it seems that we are able to get a good estimate of the behvaior of the system on the inifinite scale. When the study is complete we will have a better idea of the size of the REV and the insitu behavior as a function of scale of observations and the degree of interconnection. We expect to see that, for systems with $\zeta > \zeta^*$, measuring permeability on a small scale will on the average underestimate permeability on a larger scale and a reverse for systems with $\zeta < \zeta^*$.

In the above discussion, only the effect of interconnection has been examined. Significant changes in behavior are observed when we allow aperture to vary. A study has been performed on a well connected system where we have allowed, ν_b the coefficient of variation of aperture (i.e. the standard deviation over the mean) to vary while the mean of aperture cubed, $E(b^3)$, was held constant. Thus, except for the effect of heterogeneity, the permeabilty should be the same in each case, because the average permeability of the individual fractures is the same in each case. However, the heterogeneity itself causes a decrease in the flow because the fractures with small apertures control the

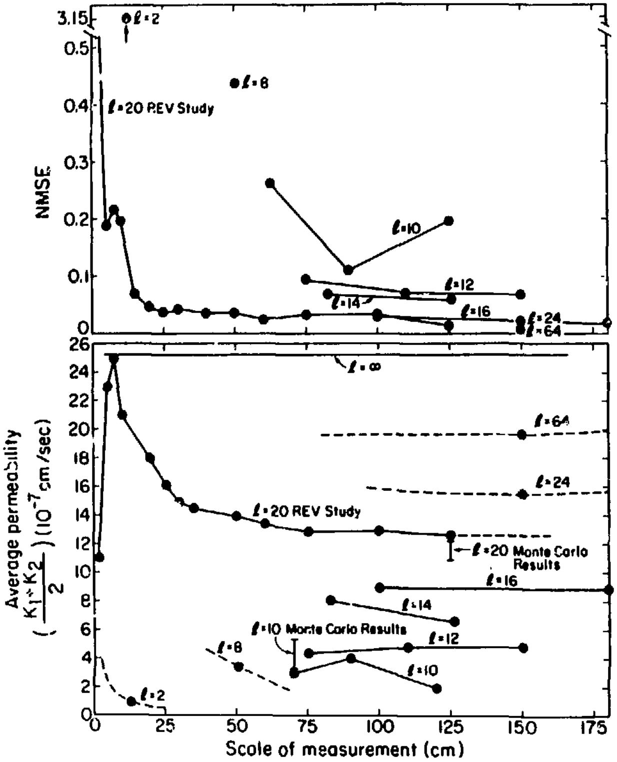

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Fig. 7. Summary of all permeability results plotted as a function of scale of measurement.

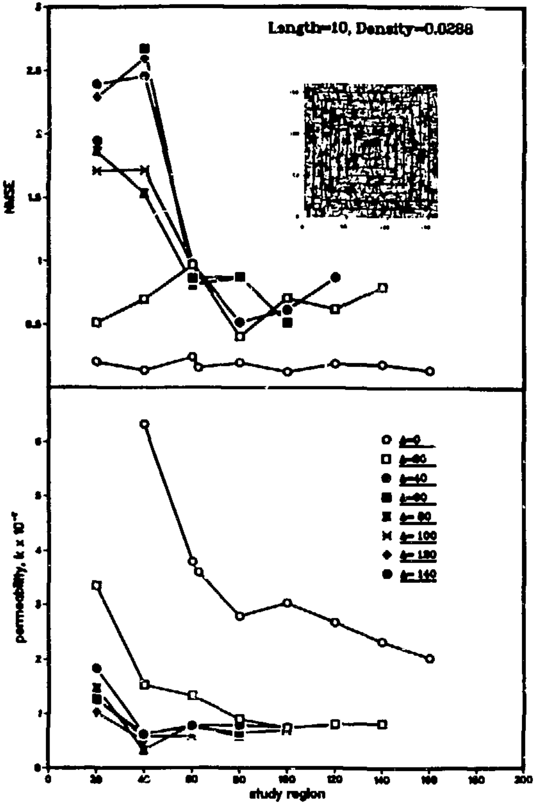

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Fig. 8. Permeability and NMSE results as a function of scale of measurement for various distances to the assigned boundaries.

flow. In the case we examined, the permeability of a system with ν_b equal to one had a permeability about one half the permeability of the same system with constant apertures. As expected, the value of NMSE increases with ν_b.

In the cases described above, the apertures were randomly assigned to the fractures. If we now reassign the same aperture distribution in such a way that the longer fractures are assigned the larger apertures, i.e., length is correlated to aperture., the permeability dramatically increases as shown in Figure 9. In fact, when aperture is roughly proportional to length, we have found that the mean permeability of such systems can be about twice the mean permeability of the same networks with constant apertures and is roughly equal to the mean permeability of an infinite fracture system with the same λ_ℓ. Furthermore, correlated systems have entirely different behavior than uncorrelated systems. In both cases increases in the coefficient of variation of aperture cause the medium to behave less like a porous medium. However, when the system is uncorrelated the increases in NMSE are due to both erratic decreases and increases in permeability. This occurs in the correlated case too, but long, high aperture fractures are also regularly produced. These "super conductors" cause a dramatic increase in permeability, especially when the gradient is in the same direction as the fracture. This can be seen on the permeability plot for the correlated case on Figure 9. Whether or not such correlations exist is problematic, but super conductors are a well documented occurrence in many underground explorations.

When these same networks are analyzed for transport behavior, we find that increases in ν_b cause the ϕ_e to vary erratically with the direction of the flux. Even in cases where the permeability is well behaved, the transport behavior may not resemble that of a continuum. The conditions under which one may use an equivalent porous medium analysis are more restrictive for transport than for flux.

It is significant that values of $\nu_b > 0.0$ can yield values of hydraulic effective porosity ϕ_e which are directionally dependent and significantly larger than both the connected porosity, Φ_R and the total porosity, ϕ_T. These heterogeneous systems have break through curves with long tails since some particles travel through much slower, longer paths and this causes the mean travel time to increase significantly. In such cases mean groundwater velocities will be much slower than that predicted using porosity. Figure 10 illustrates this effect for a system statistically identical to the correlated case of Figure 9.

For heterogeneous systems, we have yet to understand the scale dependence of permeability and transport parameters and we have not yet studied the effect of the proximity of the boundary conditions. Such studies may provide insight into the concept of "self similarity". That is, can we describe discrete fracture systems which behave in a similar manner on difference scales? Mandelbrot (1983) addresses these concepts in the study of fractals. The behavior of fracture networks might be self similar because as one examines larger and larger volumes of rock, one observes larger and larger fractures.

WELL TESTING

In a porous medium the size of each flow conduit is microscopic and a large number of conduits are connected to the well and thus the medium appears to behave like a continuum on the scale of the well test. In a fractured medium, only a small number of fractures may intersect the pumping well. These particular fractures will be stressed by a large gradient under well test conditions, so in early time the behavior will be dominated by these fractures which may be quite unrepresentative of the system.

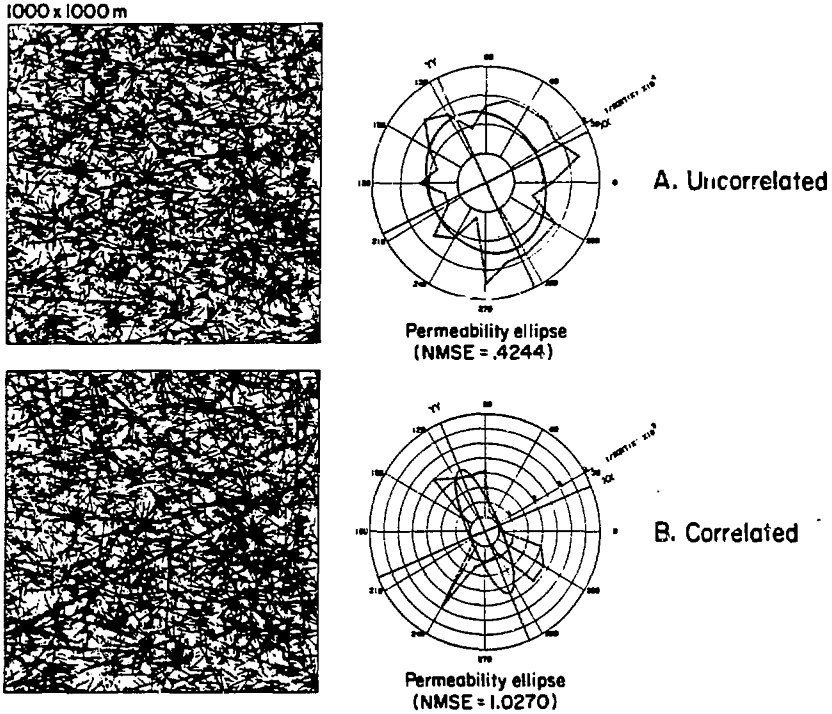

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Fig. 9. Permeability of a heterogeneous system, $\nu_b = 1$ for (a) aperture uncorrelated to length and (b) correlated.

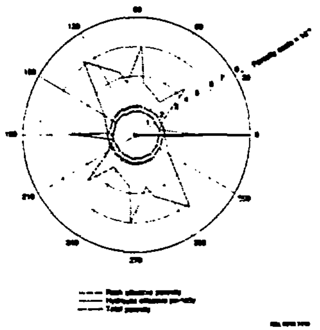

Fig. 10. Porosity for a correlated system, $\nu_b = 1$.

We have developed a new analytical model for well test analyses in a fractured medium where the matrix is impermeable. The model accounts for the difference in flow regime near the active well from that of the system as a whole. In the two-dimensional solution we have assumed that the fractures are all vertical and extend from the top to the bottom of the formation of uniform thickness. In the outer region the usual equivalent porous medium approximation is assumed to hold i.e., the flow properties of the fractures are volumetrically averaged and a single continuum replaces the fractures. The hydraulic conductivity and the storage coefficient for the region are K_2 and S_{s_2}, respectively. The well is located in the center of the inner region and communicates with the outer region through n fractures in the inner region. The radius of the well is r_w and the radius of the boundary between the outer and inner region is r_f. All the fractures in the inner region have the same aperture, b, and the hydraulic parameters K_1 and S_{s_1}. It is assumed that there is an infinitesimally thin ring of infinite conductivity between the two regions so that otherwise incompatible boundaries can be matched. The following dimensionless parameters are defined:

$$h_D = \frac{2\pi K_2 h}{Q}, \quad t_D = \frac{\alpha_2 t}{r_w^2} \cdot \frac{r_w^2}{r_f^2} = \frac{\alpha_2 t}{r_f^2}, \quad r_D = \frac{r}{r_f}, \qquad (3)$$

$$r_c = \frac{r_w}{r_f}, \quad \alpha_c = \frac{\alpha_1}{\alpha_2}, \quad \beta = \frac{nbK_1}{2\pi r_f K_2}.$$

The solutions are given in Karasaki et al. (1985) and these can be presented in a series of type curves for ranges of dimensionless parameters, α_c, β and r_c.

In a real network, r_f is a measure of distance to the nearest fracture intersection. Figure 11 is an example of a numerical well test simulation in the fracture network shown in the inset. The calculated r_f was 4.9 m in this case. In the actual network the distance to the nearest intersection was 7.3 m in one direction and 3.8 m in the other. When a fit can be found, this analytical model can also be used to accurately determine the storage coefficient of a fracture system from a one-well test. More parameter studies are needed to examine the uniqueness of the solution, on the applicability of this technique under varying states of interconnection and heterogeneity.

FUTURE RESEARCH

These numerical models have proved to be a powerful tool in understanding the hydology of fracture networks. Other than providing insight, however, we have examined how such an approach can play a role in performance assessment. Figure 12 illustrates a research plan which has been developed at LBL to improve such technology. All the research described so far in this paper falls under the box labeled "Network Properties". Clearly the biggest weakness in this approach is providing appropriate input data to the analysis. We need to know more about "Network Geometry". There are two approaches to this problem. One is to improve our ability to "see" fractures and measure their properties in situ, mainly through geophysics and well testing. The other is to study the genesis of fractures in order to better predict their characteristics and occurrence. Also we need to learn to analyze the statistical data in a manner which is appropriate to the existing fracture pattern.

In order to find fractures with geophysics, we need to better understand how the "Individual Fracture Properties" such as fracture roughness, contact area, volume and permeability affect the geophysical responses. We need to study how transport occurs in real rough fractures. This research also provides the basic data necessary to understand how network behavior changes under changes in stress state, temperature etc.

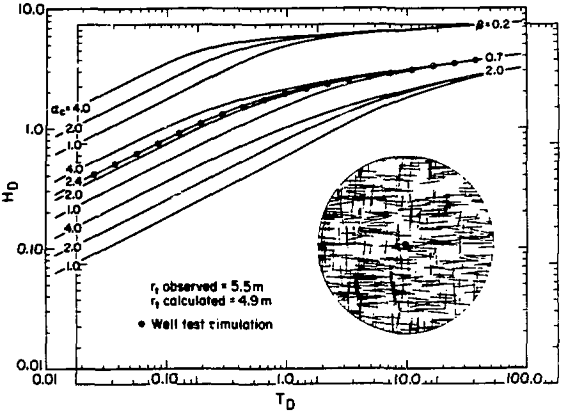

Fig. 11. Injection results.

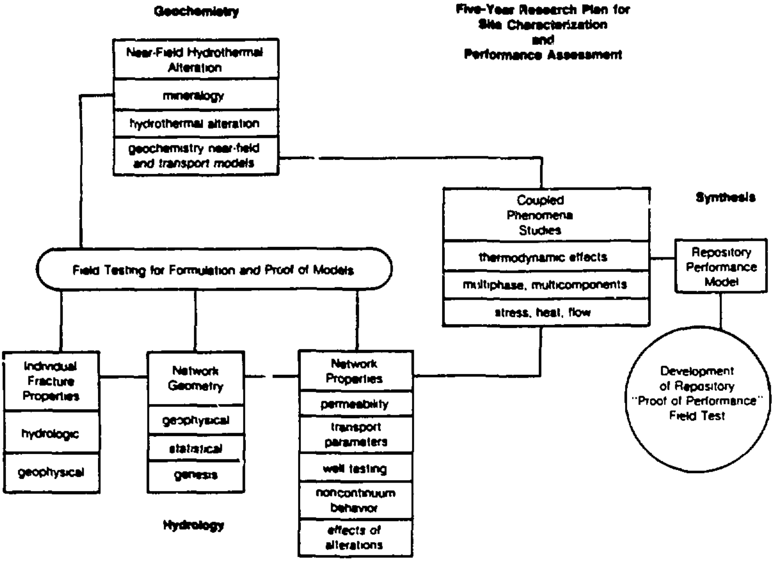

Fig. 12. Five-year research plan.

Parallel efforts in "Near Field Hydrothermal Alterations" will provide the means to calculate geochemical retardation factors. All such research requires "Field Testing for Fomulation and Proof of Models". When these measurements and analyses are conducted for the same site, then it will be possible to relate and check results. This work provides input to "Coupled Phenomena Studies" which must be performed to predict the changes in performance due to repository construction and operation. Finally, all such analysis must be combined into a "Repository Performance Model" and checked through "Proof of Performance Field Tests".

In conclusion, through network analysis, the basic tools are available to decide whether or not equivalent continua analysis is appropriate. Progress is being made on determining how to treat flow systems which do not behave like ideal porous media. Such analysis must be extended to three dimensions for quantitative accuracy. Well test technology is being extended to provide data for fracture network models. However, the data requirements of network modelling remain an obstacle to their use in a predictive mode. A comprehensive research program has been proposed for extending our ability to obtain such data and assess nuclear waste repository performance in fracture rock.

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