1 2 SU8505966

ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E2-85-127

A.Galperin,[1] E.Ivanov, V.Ogievetsky, E.Sokatchev[2]

HARMONIC SUPERGRAPHS. GREEN FUNCTIONS

Submitted to "Classical and Quantum Gravity"

[1] Institute of Nuclear Physics, Tashkent, USSR
[2] Institute of Nuclear Research and Nuclear Energy, Sofia, Bulgaria

1985

I. Introduction

Recently a constructive approach to extended supersymmetry based on the new concept of harmonic superspace has been proposed [1,2]. It has led to the long-expected unconstrained off-shell manifestly supersymmetric formulations of N=2 matter, Yang-Mills and supergravity theories [1] as well as the N=3 Yang-Mills theory [2].

One of the main incentives to search for such formulations has been the desire to have a manifestly supersymmetric quantization scheme in terms of unconstrained superfields [3]. It has been supposed to allow a simple and convincing proof of the famous finiteness of N=4 supersymmetric Yang-Mills theory (SYM below) and other theories [4-9]. Partial success in this direction has been the unconstrained formulation of N=2 SYM and matter in terms of ordinary superfields proposed in [4]. It did indeed result in one of the first proofs of the finiteness theorems. However, it is hardly a scheme for practical quantum calculations (the propagators and vertices are too complicated involving a great number of spinor derivatives, the ghost structure is rather intricate).

The harmonic approach to extended supersymmetry provides a simple and efficient quantization technique. The Feynman rules and their application turn out to be not more difficult than those of N=1 supersymmetry (for a review of the N=1 supergraph technique see [3] and references therein). The present paper is devoted to the quantization rules for N=2 supersymmetric matter and Yang-Mills theories [x]. It consists of two parts, "Green functions" and "Feynman rules".

The first part contains the basic elements of quantization. In Sec.II we introduce some harmonic distributions similar to $\delta(x)$ and $\frac{1}{x^n}$ in the ordinary analysis. Detailed knowledge of their properties is essential for constructing the Green functions. This section is self-contained and includes the necessary information on harmonic functions and analysis. In Sec.III we introduce δ-functions for the analytic superspace and construct the Green functions for N=2 matter hypermultiplets. Section IV is devoted to the N=2 SYM theory.

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[x] Some very preliminary discussion of the Green functions for these theories has been given in [1] with several mistakes in formulae. It is to be replaced by the present complete and corrected version.

The main ideas of the formulation of this gauge theory in harmonic superspace are reviewed. In particular, the question of expressing the bridge e^{iv} between the analytic and u -independent representations of the gauge-group in terms of the prepotential V^{++} is revised. The gauge fixing and the introduction of Faddeev-Popov ghosts are explained. On this basis the Green functions for various gauges are constructed. Finally, the BRST transformations are discussed. It is remarkable that the quantisation of N=2 SYM has much in common with the case of N=0 (indeed, the potentials in both the cases have similar geometric meaning and transformation laws). No ghost-for-ghosts /3,4/ are needed.

The accompanying part of the paper contains the Feynman rules and a number of examples of supergraph calculations.

II. Harmonic distributions

The harmonic N=2 superspace introduced in /1/ contains the sphere S^2. We begin this section with briefly recalling some basic concepts of the harmonic analysis on S^2. Then we define and discuss the properties of some harmonic distributions, analogous to $\delta(x)$ and $\frac{1}{x^n}$ in the ordinary analysis. They are essential for the derivation of Green functions which will be carried out in Sec.III.

II.1. Preliminaries

In /1/ we have extended the standard superspace $\{x^m, \theta^\alpha_i, \bar\theta^{\dot\alpha i}\}$ by adding the sphere $S^2 \sim SU(2)/U(1)$. This was a crucial step since it allowed us to define the analytic subspace of the full N=2 harmonic superspace. As is shown in /1/, all the N=2 supersymmetric theories are formulated in terms of unconstrained analytic superfields.

The usual approach to the harmonic analysis on a coset manifold is to choose a particular parametrization of the coset and then consider a complete orthonormal set of harmonic functions of those parameters /10/. For our purposes we prefer to avoid using any parametrization of S^2. Instead, we describe the functions on S^2 as functions on $S^3 \sim SU(2)$ with the additional condition of conservation of the U(1) charge. This amounts to effectively gauging away one of the degrees of freedom on S^3 and reducing S^3 to S^2. The most important advantage of this approach is the simple integration rules on S^2, as will be shown below.

The sphere $S^3 \sim SU(2)$ will be parametrised by the "zweibeins" or basic harmonics $u^{\pm i}$:

$$\begin{pmatrix} u^{+i} \\ u^{-i} \end{pmatrix} \in SU(2) \Leftrightarrow u^{+i}u^{-}_{i} = 1 \,,\, u^{-}_{i} = \overline{(u^{+i})} \,,\quad i = 1,2 \qquad \text{(II.1)}$$
$$u^{\pm}_{i} = \varepsilon_{ij} u^{\pm j} \,,\, \varepsilon_{ij} = -\varepsilon_{ji} \,,\, \varepsilon_{12} = 1$$

The general functions on S^3 are expanded in terms of the irreducible products of $u^{\pm i}$:

$$f(u) = \sum_{\substack{n=0 \\ m=0}}^{\infty} f^{(i_1 \cdots i_n j_1 \cdots j_m)} u^{+}_{(i_1} \cdots u^{+}_{i_n} u^{-}_{j_1} \cdots u^{-}_{j_m)} \qquad \text{(II.2)}$$

where the parenthesis mean symmetrisation normalised with $\frac{1}{(n+m)!}$. The coefficients $f^{(i_1 \cdots j_m)}$ are irreducible SU(2) tensors with isospin $\frac{1}{2}(n+m)$. (Note that antisymmetric products of u^+, u^- do not have to be considered because of the SU(2) conditions (II.1)).

On S^3 one can introduce covariant derivatives with respect to $u^{\pm}$ compatible with the defining conditions (II.1):

$$D^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} \quad , D^{--} = u^{-i}\frac{\partial}{\partial u^{+i}} \qquad \text{(II.3)}$$
$$D^{0} = [D^{++}, D^{--}] = u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}} \,.$$

They form the algebra of another SU(2) which acts on indices $\pm$ of $u^{\pm i}$

$$D^{++}u^{+i} = 0 \;,\quad D^{--}u^{+i} = u^{-i};\quad D^{0}u^{\pm}_{i} = \pm u^{\pm}_{i}$$
$$D^{++}u^{-i} = u^{+i};\; D^{--}u^{-i} = 0; \qquad \text{(II.4)}$$

and commutes with the initial one acting on indices i. In particular, the operator D^0 measures the U(I) charge (± 1) of the harmonics $u^{\pm}$. We can use this property and consider a subclass of functions (II.2) having a definite U(I) charge, i.e., being eigenfunctions of D^0

$$D^{0}F^{(q)}(u) = q\,F^{(q)}(u);\quad U(1):\; F^{(q)}(u') = e^{iq\alpha}F^{(q)}(u) \qquad \text{(II.5)}$$

Here q is the U(1) charge of $F^{(q)}(u)$. Such restricted functions have the following harmonic expansion (for $q \geq 0$; for $q < 0$ the change of definition is obvious)

$$F^{(q)}(u)=\sum_{n=0}^{\infty} f^{(i_1 \cdots i_{n+q} j_1 \cdots j_n)} u^+_{(i_1} \cdots u^+_{i_{n+q}} u^-_{j_1} \cdots u^-_{j_n)} . \qquad \text{(II.6)}$$

The difference from the general expansion (II.2) is in the conservation of the U(I) charge throughout (II.6). This means that one of the real degrees of freedom of $u^{\pm i}$ does in fact drops out from (II.6). Indeed, the SU(2)-matrix $\begin{pmatrix} u^{+i} \\ u^{-i} \end{pmatrix}$ can always be divided into the product of the SU(2)/U(I) and U(I)- parts and the latter can be absorbed in the overall U(I)-phase factor. As a result, the functions (II.6) effectively depend on the two real coordinates of $S^2 \sim SU(2)/U(I)$. Thus we see that the concept of U(I) charge and its conservation allow us to describe the sphere S^2 and the function on it without actual introducing a parametrization of S^2.

This approach simplifies considerably the integration rules on S^2. Instead of introducing an explicit parametrization of S^2 and then integrating over the manifold of parameters, we define the following (equivalent) integration rules

$$\int du \cdot 1 = 1$$
$$\int du \cdot (u^+)^{(m}(u^-)^{n)} = 0 \,, \quad m+n>0, \qquad \text{(II.7)}$$

where

$$(u^+)^{(m}(u^-)^{n)} \equiv u^{+(i_1} \cdots u^{+i_m} u^{-j_1} \cdots u^{-j_n)} . \qquad \text{(II.8)}$$

So, the integration on S^2 has become an algebraic operation (similar to the Grassmann integration). An important rule due to the conservation of the U(1) charge on S^2 is that the integral of a charged quantity is always zero

$$\int du \cdot F^{(q)}(u) = 0 \,, \quad q \neq 0. \qquad \text{(II.9)}$$

The rule for integration by parts is based on the property

$$\int du\, D^{++} F^{(-2)}(u) = \int du\, D^{--} F^{(+2)}(u) = 0$$

following from (II.4) and (II.6).

The set of symmetrized products of $u^{\pm}$ (II.8) is in fact the orthonormal set of harmonics on S^2:

$$\int du\, (u^+)^{(m} (u^-)^{n)} (u^+)_{(k} (u^-)_{\ell)} = \begin{cases} \frac{(-1)^n m!\, n!}{(m+n+1)!} \delta^{(i_1}_{(j_1} \cdots \delta^{i_{m+n})}_{j_{k+\ell})}, & \text{if } m=\ell,\ n=k \\ 0, & \text{otherwise} \end{cases}$$

This property allows to find the coefficients of the harmonic expansion (II.6) of a function $F^{(q)}(u)$ (just as one does in the ordinary Fourier analysis)

$$f^{(i_1 \cdots i_{n+q} j_1 \cdots j_n)} = \frac{(-1)^{n+q}(2n+q+1)!}{(n+q)!\ n!} \int du\, (u^+)^{(n} (u^-)^{n+q)} F^{(q)}(u) \qquad \text{(II.10)}$$

The last remark concerns the possibility to define real functions on the sphere S^2. Under the usual complex conjugation $\overline{F^{(q)}} = \bar{F}^{(-q)}$, so charged functions cannot be real in the usual sense. However, there exists another conjugation, a combination of the ordinary complex conjugation and the antipodal map on the sphere:

$$(u^{\pm i})^{*} = -u^{\pm}_{i}\ , \quad (u^{\pm}_{i})^{*} = u^{\pm i}\ , \quad (u)^{\pm *} = -u \qquad \text{(II.11)}$$

which is compatible with (II.1). Now one can impose the reality condition

$$\left(F^{(q)}(u)\right)^{*} = F^{(q)}(u)$$

for even charges q. In particular, the analytic superfields (see Sec.III) can be real too.

II.2. Harmonic δ-functions

Now we proceed to the main topic of this section. First we introduce harmonic δ-functions on S^2. Since the conservation of the U(1) charge is in the basis of our description of S^2, we have to consider separate δ-functions for each value of the U(1)-charge of the smearing functions:

$$\int du_2\, \delta^{(q,-q)}(u_1,u_2)\, F^{(p)}(u_2) = F^{(q)}(u_1)\, \delta^{pq} \qquad \text{(II.12)}$$

Note the balance of the two U(1) charges in this equation. Using (II.10) we can find the harmonic expansion of the δ-functions

$$\delta^{(q,-q)}(u_1,u_2)=\sum_{n=0}^{\infty}(-1)^{n+q}\frac{(2n+q+1)!}{n!\ (n+q)!} \qquad \text{(II.13)}$$

$$\cdot\ (u_1^+)_{(n+q}\ (u_1^-)_{n)}\ (u_2^+)^{(n}\ (u_2^-)^{n+q)}.$$

Of course, this is a formal expansion since the series diverges at $u_1 = u_2$ (modulo U(1) transformation). It is a generalisation of the Fourier expansion of the ordinary δ-function

$$\delta(x_1-x_2)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ip(x_1-x_2)}\,dp$$

that is divergent at $x_1 = x_2$

The δ-functions can be differentiated in a natural way:

$$\int du_2\, D_2^{++}\delta^{(q,-q)}(u_1,u_2)\cdot F^{(q-2)}(u_2) =$$

$$= -\int du_2\,\delta^{(q,-q)}(u_1,u_2)D_2^{++}F^{(q-2)}(u_2) = -D_1^{++}F^{(q-2)}(u_1);$$

$$D_2^{++}\delta^{(q,-q)}(u_1,u_2) = -D_1^{++}\delta^{(q-2,-q+2)}(u_1,u_2). \qquad \text{(II.14')}$$

A number of other interesting properties of the δ functions can be derived and we leave that to the reader. These are

(a) $\delta^{(q,-q)}(u_1,u_2)=\delta^{(-q,q)}(u_2,u_1)$;

(b) $F^{(p)}(u_2)\,\delta^{(q,-q)}(u_1,u_2)=F^{p}(u_1)\,\delta^{(q-p,p-q)}(u_1,u_2)$;

(c) $(u_1^+u_2^+)\delta^{(q,-q)}(u_1,u_2)=(u_1^-u_2^-)\delta^{(q,-q)}(u_1,u_2) = 0$; (II.14'')

(d) $\delta^{(q,-q)}(u_1,u_2)=(u_1^+u_2^-)\,\delta^{(q-1,-q+1)}(u_1,u_2)=$

$$\ldots\ldots = (u_1^+u_2^-)^q\,\delta^{(0,0)}(u_1,u_2);$$

(e) $$\left[\delta^{(q_1,-q)}(u_1,u_2)\right]^* = \delta^{(q_1,-q)}(u_1,u_2).$$

Here

$$u_1^{\pm} u_2^{\pm} = -u_2^{\pm} u_1^{\pm} \equiv u_1^{\pm i} u_{2\,i}^{\pm}.$$

Note also the useful relation

$$(u_1^+ u_2^+)(u_1^- u_2^-) = 1 + (u_1^+ u_2^-)(u_1^- u_2^+). \qquad \text{(II.15)}$$

One can conclude that the harmonic δ -functions have much in common with the ordinary δ -functions, the only peculiarity being the U(1) charge and its conservation. In fact, the chargeless harmonic δ -function $\delta^{(0,0)}(u_1,u_2)$ (which is the basic one, see (II.14d)) can be written down as an ordinary δ -function with a special argument:

$$\delta^{(0,0)}(u_1,u_2) = 2\,\delta\left[(u_1^+ u_2^+)(u_1^- u_2^-)\right].$$

II.3. Harmonic distributions

The second important class of distributions that we shall use consists of the ones given by their harmonic expansions

$$\frac{1}{(u_1^+ u_2^+)^n} = \frac{1}{n!} \sum_{k=0}^{\infty} (-1)^{n+k} \frac{(2k+n+1)!\,n}{(k+n)\,k!\,(k+1)!} \qquad \text{(II.16)}$$

$$\cdot (u_1^+)_{(k} (u_1^-)_{k+n)} (u_2^+)^{(k} (u_2^-)^{k+n)}; \qquad n \geq 0$$

The following properties justify the notation $1/(u_1^+ u_2^+)^n$ (which may otherwise seem peculiar since the r.h.s. of (II.16) is a function of u^- , not only u^+):

$$(u_1^+u_2^+)^k \frac{1}{(u_1^+u_2^+)^n} = \frac{1}{(u_1^+u_2^+)^{n-k}}, \quad k \leq n; \left(\frac{1}{(u_1^+u_2^+)^0} = 1\right)$$

$$\frac{1}{(u_1^+u_2^+)^n} = (-1)^n \frac{1}{(u_2^+u_1^+)^n} .$$

These properties can be checked directly by using the expansion (II.16) and the reduction identities

$$u_i^+ u^+_{(j_1} \cdots u^+_{j_n} u^-_{k_1} \cdots u^-_{k_m)} = u^+_{(i} u^+_{j_1} \cdots u^-_{k_m)} + \frac{m}{m+n+1} \varepsilon_{i(k_1} u^+_{j_1} \cdots u^+_{j_n} u^-_{k_2} \cdots u^-_{k_m)},$$

$$u_i^- u^+_{(j_1} \cdots u^+_{j_n} u^-_{k_1} \cdots u^-_{k_m)} = u^-_{(i} u^+_{j_1} \cdots u^-_{k_m)} - \frac{n}{m+n+1} \varepsilon_{i(j_1} u^+_{j_2} \cdots u^-_{k_m)} .$$

The distributions (II.16) are, in a way, analogues of the distributions $\frac{1}{x^n}$ in the ordinary analysis. However, there are also differences: $\frac{1}{x^n}$ is differentiated straightforwardly,

$$\frac{d}{dx}\left(\frac{1}{x^n}\right) = -n \frac{1}{x^{n+1}}$$

whereas only the harmonic derivative D^{--} of (II.16) produces a simple result:

$$D_1^{--} \frac{1}{(u_1^+u_2^+)^n} = -n \frac{u_1^- u_2^+}{(u_1^+u_2^+)^{n+1}} . \qquad \text{(II.17)}$$

The other derivative D^{++} gives (somewhat unexpectedly, since $D_1^{++}(u_1^+u_2^+) = 0$)

$$D_1^{++} \frac{1}{(u_1^+u_2^+)^n} = \frac{1}{(n-1)!} (D_1^{--})^{n-1} \delta^{(n,-n)}(u_1, u_2) . \qquad \text{(II.18)}$$

Both formulas (II.17-18) can be proved using the harmonic expansions (II.16) and (II.13).

It is well known that the distributions x^{-n} and x^{-m} cannot be multiplied because they have coinciding singularities. The same is true for $(u_1^+u_2^+)^{-n}$ which are singular at $u_1^+ = u_2^+$ (modulo U(1) phase). On the other hand, in perturbative QFT one is regularly forced to consider such meaningless expressions which cause the well-known problem of ultraviolet divergencies. The reader might suspect that a similar problem would occur in the perturbation theory in harmonic superspace, when propagators containing the above harmonic distributions with coinciding singularities are to be multiplied. In the second part of this work we shall show that this is not the case.

At the end of this section we offer the reader a few more exercises. First, using (II.3), (II.17), (II.18), (II.14a,d) show that

$$D_1^0 \frac{1}{(u_1^+ u_2^+)^n} = -n \frac{1}{(u_1^+ u_2^+)^n}$$

in accordance with (II.15). Second, show that

$$\left(\frac{1}{(u_1^+ u_2^+)^n}\right)^{\ast} = \frac{1}{(u_1^+ u_2^+)^n} \qquad \text{(II.19)}$$

III. Analytic δ-functions. Green functions for hypermultiplets

As is explained in /1/, the fundamental concept in our approach is the so-called analytic subspace $\{\zeta^M, u^\pm\}$ of the full harmonic superspace $\{Z^M, u^\pm\}$. The coordinates of the former are related to those of the latter as follows

$$\zeta^M \begin{cases} x_A^m = x^m - 2i\,\theta^{(i}\sigma^m\bar{\theta}^{j)} u_i^+ u_j^- \\ \theta_\alpha^+ = \theta_\alpha^i u_i^+ , \quad \bar{\theta}_{\dot\alpha}^+ = \bar{\theta}_{\dot\alpha}^i u_i^+ \end{cases} \qquad \text{(III.1)}$$

Note that the analytic subspace is closed under the conjugation (II.11).

The analytic superfields $\Phi^{(t)}(\zeta, u)$ defined in this subspace satisfy the analyticity conditions

$$D^+_\alpha \varphi^{(q)} = \bar{D}^+_{\dot\alpha} \varphi^{(q)} = 0,$$
$$D^+_\alpha = D^i_\alpha u^+_i \ , \ \bar{D}^+_{\dot\alpha} = \bar{D}^i_{\dot\alpha} u^+_i \tag{III.2}$$

which just mean that they depend only on $\theta^+_\alpha, \bar\theta^+_{\dot\alpha}$ but not on $\theta^-_\alpha = \theta^i_\alpha u^-_i, \bar\theta^-_{\dot\alpha} = \bar\theta^i_{\dot\alpha} u^-_i$ (if written down in the analytic basis (III.1)). Due to commutation relations

$$[D^{++}, D^+_\alpha] = [D^{++}, \bar{D}^+_{\dot\alpha}] = 0 \tag{III.3}$$

harmonic derivative D^{++} preserves analyticity: $D^{++}\Phi$ is an analytic superfield if Φ does. This is not true for D^{--} since

$$[D^{--}, D^+_{\alpha(\dot\alpha)}] = D^-_{\alpha(\dot\alpha)} . \tag{III.4}$$

Finally, D^0 measures the U(1) charge of the analytic superfields (cf. (II.5)

$$D^0 \varphi^{(q)}(\zeta, u) = q \ \varphi^{(q)}(\zeta, u) .$$

In the analytic subspace one can define integration with a charged analytic measure $d\zeta^{(-4)} du$. Note the important relation

$$d^{12} z \, du = d\zeta^{(-4)} du \ (D^+)^4,$$
$$(D^+)^4 \equiv \frac{1}{16} \ (D^{+\alpha} D^+_\alpha)(\bar{D}^+_{\dot\alpha} \bar{D}^{+\dot\alpha}) . \tag{III.5}$$

which allows us to restore the full superspace measure in analytic integrals. It is analogous to the N=1 relation between the full and chiral measures, e.g.

$$d^8 z = d^6 \zeta \ \bar{D}^2 .$$

Finally, note that analytic superfields with even U(1) charges q can be made real, i.e.,

$$(\varphi^{(q)}(\zeta, u))^{*} = \varphi^{(q)}(\zeta, u) \ , \ q = 2n .$$

III.1. Analytic δ-functions

The δ-functions for the analytic subspace are defined by the equation

$$\int d\zeta_2^{(-4)} du_2\, \delta_A^{(q,4-q)}(\zeta_1,u_1|\zeta_2,u_2)\, \Phi^{(p)}(\zeta_2,u_2) = \\ = \delta^{qp}\, \Phi^{(p)}(\zeta_1,u_1). \tag{III.6}$$

Note the exact balance of the U(1) charges with respect to both arguments 1 and 2. Now we proceed to construction of such δ-functions starting from the δ-function for the full harmonic superspace. It is defined as follows

$$\int d^{12}z_2\, du_2\, \delta^{12}(z_1-z_2)\, \delta^{(q,-q)}(u_1,u_2) f^{(p)}(z_2,u_2) = \\ = \delta^{qp} f^{(p)}(z_1,u_1).$$

where

$$\delta^{12}(z_1-z_2) \equiv \delta^4(x_1-x_2)\, \delta^8(\theta_1-\theta_2).$$

Let us now consider an analytic superfield as a function of the coordinates (z^M, u) of the full superspace, i.e.,

$$\Phi^{(p)} = \Phi^{(p)}(\zeta^M(z,u),u)$$

(see (III.1)). Then one can write down

$$\int d^{12}z_2\, du_2\, \delta^{12}(z_1-z_2)\, \delta^{(q,-q)}(u_1,u_2)\, \Phi^{(p)}(\zeta(z_2,u_2),u_2) = \\ = \delta^{qp}\, \Phi^{(p)}(\zeta(z_1,u_1),u_1).$$

Using (III.5) and treating ζ^M as independent variables one finds

$$\int d\zeta_2^{(-4)} du_2 \left[(D_2^+)^4 \delta^{12}(z_1 - z_2) \right] \delta^{(q,-q)}(u_1,u_2)\, \phi^{(p)}(\zeta_2, u_2) =$$
$$= \delta^{qp}\, \phi^{(p)}(\zeta_1, u_1).$$

Comparing this equation with the definition (III.6) one obtains

$$\delta_A^{(q,4-q)}(\zeta_1, u_1 | \zeta_2, u_2) = (D_2^+)^4 \delta^{12}(z_1 - z_2)\, \delta^{(q,-q)}(u_1,u_2) =$$
$$= (D_1^+)^4 \delta^{12}(z_1 - z_2)\, \delta^{(q-4,4-q)}(u_1,u_2). \qquad \text{(III.7)}$$

The second form is derived with the help of the property

$$D_1^i\, \delta^{12}(z_1 - z_2) = - D_2^i\, \delta^{12}(z_1 - z_2)$$

and the identity (II.14b). Note that in the expressions (III.7) the analyticity of δ_A is manifest.

In fact, the analytic δ-functions are analogues of the chiral ones in N=1 supersymmetry just as analyticity is analogous to chirality. For the left-handed chiral superspace one has

$$\delta_L^6(\zeta_1^L - \zeta_2^L) = (\bar{D}_2)^2 \delta^8(z_1 - z_2) = (\bar{D}_1)^2 \delta^8(z_1 - z_2).$$

Let us emphasize, however, that δ_L^6 factorises into $\delta(x)$ and $\delta(\theta)$

$$\delta_L^6(\zeta_1^L - \zeta_2^L) = \delta^4(x_1^L - x_2^L)\, \delta^2(\theta_1 - \theta_2),$$

whereas δ_A (III.7) does not (the reason is that one cannot write down $\delta^4(\theta_1^+ - \theta_2^+)$ because θ_1^+ and θ_2^+ transform under different U(1) groups).

III.2. Green functions for hypermultiplets

At last we are prepared to start constructing Green functions for various N=2 theories. The first case is the hypermultiplet matter in its Fayet–Sohnius form. As explained in /1/, the FS–hypermultiplet is described by an analytic superfield $q^+(\zeta, u)$ which satisfies the following equation of motion

$$D^{++} q^+(\zeta, u) = J^{(3)}(\zeta, u), \qquad \text{(III.8)}$$

where $J^{(3)}$ is an analytic source of U(1) charge +3. The corresponding Green function

$$G^{(1,1)}(\zeta_1, u_1|\zeta_2, u_2) \equiv \langle \overset{*}{q}{}^+(1)\, q^+(2) \rangle$$

obeys the equation

$$D_1^{++} G^{(1,1)}(1|2) = \delta_A^{(3,1)}(1|2) \qquad \text{(III.9)}$$

and gives the solution of (III.8)

$$q^+(\zeta_1, u_1) = \int d\zeta_2^{(-4)} du_2\, G^{(1,1)}(1|2)\, J^{(3)}(\zeta_2, u_2).$$

It is not difficult to see that the following manifestly analytic expression

$$G^{(1,1)}(1|2) = -\frac{1}{\Box_1}(D_1^+)^4 (D_2^+)^4 \delta^{12}(z_1 - z_2) \frac{1}{(u_1^+ u_2^+)^3} \qquad \text{(III.10)}$$

is the solution of equation (III.9). Indeed, with the help of (II.18) one finds

$$D_1^{++} G^{(1,1)}(1|2) = -\frac{1}{\Box_1}(D_1^+)^4 (D_2^+)^4 \delta^{12}(z_1 - z_2) \frac{1}{2}(D_1^-)^2 \delta^{(3,-3)}(1|2). \qquad \text{(III.11)}$$

Now, using the algebra of the covariant derivaties /1/ one can prove a very useful identity:

$$-\frac{1}{2}(D^+)^4 (D^{--})^2 \Phi(\zeta, u) = \Box\, \Phi(\zeta, u) \qquad \text{(III.12)}$$

for any analytic superfield Φ. In (III.11) such a superfield is $(D_2^+)^4 \delta^{12}(z_1 - z_2)\, \delta^{(3,-3)}(u_1, u_2) \equiv \delta_A^{(3,1)}(1|2)$ so, one arrives at (III.9).

From (III.10) one clearly sees that the Green function is antisymmetric,

$$G^{(1,1)}(1|2) = -G^{(1,1)}(2|1)$$

and real with respect to the conjugation ($\overset{*}{-}$).

The second type of hypermultiplet, that of Howe, Stelle and Townsend, is described by a chargeless real analytic superfield

$\omega(\zeta,u)$, $\omega^{*}=\omega$ /1/. Its equation of motion is

$$(D^{++})^2\omega(\zeta,u)=J^{(4)}(\zeta,u). \qquad \text{(III.13)}$$

The Green function

$$G^{(0,0)}(1|2)\equiv\langle\omega(1)\,\omega(2)\rangle$$

satisfies the equation

$$(D_1^{++})^2 G^{(0,0)}(1|2)=\delta_A^{(4,0)}(1|2)$$

and provides the solution for (III.13):

$$\omega(\zeta_1,u_1)=\int d\zeta_2^{(-4)}du_2\, G^{(0,0)}(1|2)\, J^{(4)}(2).$$

The suitable expression for $G^{(0,0)}$ is

$$G^{(0,0)}(1|2)=-\frac{1}{\Box_1}(D_1^+)^4(D_2^+)^4\delta^{12}(z_1-z_2)\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}. \qquad \text{(III.14)}$$

Indeed,

$$(D_1^{++})^2\frac{u_1^-u_2^-}{(u_1^+u_2^+)^3}=D_1^{++}\left[\frac{u_1^+u_2^-}{(u_1^+u_2^+)^3}+\frac{1}{2}(u_1^-u_2^-)(D_1^{--})^2\delta^{(3,-3)}(1|2)\right]=$$

$$=(u_1^+u_2^-)\frac{1}{2}(D_1^{--})^2\delta^{(3,-3)}(1|2)=\frac{1}{2}(D_1^{--})^2\left[(u_1^+u_2^-)\,\delta^{(3,-3)}(1|2)\right]=$$

$$=\frac{1}{2}(D_1^{--})^2\,\delta^{(4,-4)}(1|2). \qquad \text{(III.15)}$$

Here we used (II.4), (II.18), (II.14c,d). Once again (III.12) yields the result.

It is clear that $G^{(0,0)}$ (III.14) is real and symmetric,

$$G^{(0,0)}(1|2)=G^{(0,0)}(2|1)$$

The Green functions for the hypermultiplets derived above are the basis for the perturbation theory for N=2 quantum matter. Now we proceed to the quantization of N=2 gauge theories.

IV. Quantisation of N=2 supersymmetric Yang-Mills theory

IV.I. Formulation of N=2 SYM in harmonic superspace

We begin with a review of the formulation of N=2 SYM in harmonic superspace first presented in /1/.

The N=2 SYM unconstrained prepotential is a Hermitean analytic Lie algebra valued superfield with U(1)-charge 2

$$V^{++}(\zeta, u) \equiv V^{++a} T_a \, , \ (V^{++a})^{*} = V^{++a} \, .$$

Here T_a are generators of the adjoint representation of the YM group:

$$[T_a, T_b] = i f_{abc} T_c \, , \ tr(T^a T^b) = \delta^{ab} . \qquad \text{(IV.1)}$$

This prepotential plays the role of the connection for the covariant harmonic derivative,

$$(\mathcal{D}^{++})_{\lambda} = D^{++} + i V^{++} \qquad \text{(IV.2)}$$

in close analogy with the N=0 case. Correspondingly, V^{++} transforms as follows

$$V^{++\prime} = e^{i\lambda} (V^{++} - i D^{++}) e^{-i\lambda} , \qquad \text{(IV.3)}$$

where $\lambda = \lambda(\zeta, u)$ are the Hermitean $(\overset{*}{\lambda} = \lambda)$ analytic gauge group parameters. The subscript λ in (IV.2) means that $\mathcal{D}^{++}$ is considered in the analytic or λ-representation of the gauge group. In this representation

$$(\mathcal{D}^{+}_{\alpha(\dot\alpha)}) = D^{+}_{\alpha(\dot\alpha)} \qquad \text{(IV.4)}$$

which means that analyticity is a covariant notion.

At the same time one can consider another, τ representation of the gauge group where the parameters $\tau(z)$ are Hermitean u-independent superfunctions. One may introduce a bridge between the two representations, i.e., the quantity e^{iv} with the following transformation law

$$e^{iv\prime} = e^{i\lambda} e^{iv} e^{-i\tau} . \qquad \text{(IV.5)}$$

Here $v(z,u)$ is a Hermitean ($\overset{*}{v} = v$) Lie algebra valued, but non-analytic superfield. In the τ representation the covariant harmonic derivative becomes simple

$$(\mathcal{D}^{++})_{\tau} = e^{-iv}(\mathcal{D}^{++})_{\lambda} e^{iv} = D^{++} \qquad \text{(IV.6)}$$

(so the statement of u-independence becomes covariant) but the spinor ones $\mathcal{D}^{+}_{\alpha}, \bar{\mathcal{D}}^{+}_{\dot{\alpha}}$ acquire connections

$$(\mathcal{D}^{+}_{\alpha(\dot{\alpha})})_{\tau} = e^{-iv} D^{+}_{\alpha(\dot{\alpha})} e^{iv}.$$

So, one can say that the λ representation is most natural for describing analytic superfields, and the τ one – for ordinary, u-independent superfields.

The bridge e^{iv} is not an independent quantity. It can be found by solving the equation

$$e^{iv}(D^{++}e^{-iv}) = iV^{++} \qquad \text{(IV.7)}$$

which is obtained by comparing (IV.2) and (IV.6). Here the analytic superfield V^{++} is given and the non-analytic e^{iv} is an unknown. This equation can be solved iteratively [x)]. Namely, the bridge e^{iv} can be written down as a Taylor series in V^{++}

$$e^{iv(z,u)} = \sum_{n=0}^{\infty} \frac{1}{n!} \int du_1 \dots du_n \cdot \left[\frac{\delta^n e^{iv(z,u)}}{\delta V^{++}_{a_1}(z,u_1) \dots \delta V^{++}_{a_n}(z,u_n)} \right]_{V^{++}=0} \cdot V^{++}_{a_1}(z,u_1) \dots V^{++}_{a_n}(z,u_n) \qquad \text{(IV.8)}$$

Here all the V's depend on the same Z (via $\zeta = \zeta(z,u)$, see (III.1)) but on different u's. The reason is that the operator D^{++} in (IV.7) does not involve $\partial/\partial Z$ (in the central basis).

x) The solution presented here is different from the one in /1/. The latter is not correct, which has also been noted in /11/.

Now we have to determine the functional derivatives of e^{iv} so that (IV.8) will be the general solution of (IV.7). It will turn out that the first-order derivative is the basic one, the others are simply obtained from it. So, let us consider the first-order variation of e^{iv}. From (IV.7) we get

$$i\,\delta V^{++} = \delta\left(e^{iv} D^{++} e^{-iv}\right) = -e^{iv} D^{++}\left(e^{-iv}\delta e^{iv}\right)e^{-iv}$$

or

$$D^{++}\left(e^{-iv}\delta e^{iv}\right) = -i\left(\delta V^{++}\right)_{\tau} . \tag{IV.9}$$

Here

$$\left(\delta V^{++}\right)_{\tau} \equiv e^{-iv}\,\delta V^{++} e^{iv} \tag{IV.10}$$

is the τ-representation form of the λ tensor δV^{++} (V^{++} itself is a gauge superfield, but its variation δV^{++} transforms as a λ-tensor, see (IV.3)). Equation (IV.9) can easily be solved with the help of (II.18) and (II.14d):

$$e^{-iv}\delta e^{iv} = -i\int du_1 \frac{u^+u_1^-}{u^+u_1^+}\left(\delta V^{++}\right)_{\tau}(z,u_1). \tag{IV.11}$$

Note that the r.h.s. of (IV.11) is anti-Hermitean just as the l.h.s. is (see (II.11) and (II.19)). The careful reader might ask why some arbitrary u-independent superfield is not added to the r.h.s. of (IV.11), as eq. (IV.9) allows to do. The reason is that such a superfield would result in a τ gauge transformation of the bridge e^{iv}

From (IV.11) we can obtain the first-order derivative of e^{iv} (see (IV.10)):

$$\frac{\delta\, e^{iv(z,u)}}{\delta V^{++}_{a_1}(z,u_1)} = -i\,\frac{u^+u_1^-}{u^+u_1^+}\, e^{iv(z,u)} e^{-iv(z,u_1)} T_{a_1} e^{iv(z,u_1)}$$

In (IV.8) we need the value of this derivative for $V^{++}=0$.

Turning once again to (IV.7) we see that for $V^{++}=0$ $D^{++}e^{iv}=0$, i.e., $e^{iv(z,u)}\Big|_{V^{++}=0}=e^{-i\tau(z)}$ where $\tau(z)$ is a Hermitean u-independent superfunction. Then we find

$$\left[\frac{\delta e^{iv}(z,u)}{\delta V_{a_1}^{++}(z,u_1)}\right]_{V^{++}=0}=-i\,\frac{u^+u_1^-}{u^+u_1^+}T_{a_1}e^{-i\tau(z)}. \tag{IV.12}$$

Obviously, the arbitrary factor $e^{-i\tau(z)}$ in (IV.12) corresponds to the τ-gauge freedom in e^{iv} (see (IV.5)).

The second-order variation of e^{iv} can be found from the first-order one (IV.11), and (IV.10):

$$\delta_2\delta_1 e^{iv}=e^{iv}\int du_1du_2\frac{u^+u_1^-}{u^+u_1^+}\Big\{\frac{u_1^+u_2^-}{u_1^+u_2^+}\left[(\delta_2V^{++})_\tau(\delta_1V^{++})_\tau\right]- \tag{IV.13}$$

$$-\frac{u^+u_2^-}{u^+u_2^+}(\delta_2V^{++})_\tau(\delta_1V^{++})_\tau\Big\}.$$

Once again, computing the second-order derivative at $V^{++}=0$ we obtain the gauge freedom $e^{-i\tau(z)}$ as in (IV.12). Note that the distributions in (IV.13) can be multiplied since their singularities do not coincide.

The process of finding subsequent variations of e^{iv} and thus reconstructing the complete solution (IV.8) is straightforward. In the second part of this work we shall use a similar procedure to expand the N=2 SYM action and find the vertices.

This N=2 SYM action is given as an integral over the chiral subspace of N=2 superspace

$$S=\frac{1}{2}\int d^4x_R\,d^4\bar\theta\,\mathrm{tr}\,\overline{W}^2.$$

Here

$$\overline{W}=-\frac{i}{4}e^{-iv}\left[(D^+)^2(e^{iv}D^{--}e^{-iv})\right]e^{iv}$$

is the u-independent chiral ($D_{\alpha i}\overline{W}=0$) field-strength tensor (see [1]). Having expressed e^{iv} in terms of V^{++}, the action becomes a functional of the unconstrained prepotentials. In what follows we shall use the variation of S with respect to V^{++}. It can be written down as an analytic superspace integral /1/

$$\delta S = -i\int d\zeta^{(-4)} du\, tr\left[\delta V^{++}(D^+)^4\left(e^{iv} D^{--} e^{-iv}\right)\right] \qquad \text{(IV.14)}$$

or, with the help of (III.5), as a full superspace integral

$$\delta S = -i\int d^{12}z\, du\, tr\left(\delta V^{++} e^{iv} D^{--} e^{-iv}\right) \qquad \text{(IV.15)}$$

IV.2. Gauge fixing and ghosts

Let us consider the linearised N=2 SYM action. It can be derived from (IV.14) by taking the second-order variation and then replacing δV^{++} by V^{++}:

$$S_{lin} = \frac{1}{2}\int d\zeta_1^{(-4)} du_1\, tr\left[V^{++}(1)\,(D_1^+)^4 D_1^{--}\int du_2 \frac{u_1^+ u_2^-}{u_1^+ u_2^+} V^{++}(2)\right] = \qquad \text{(IV.16)}$$

$$= \frac{1}{2}\int d\zeta_1^{(-4)} du_1 du_2\, tr\left[V^{++}(1)\,(D_1^+)^4 \frac{1}{(u_1^+ u_2^+)^2} V^{++}(2)\right].$$

Here $\zeta_{1,2} = \zeta_{1,2}(z, u_{1,2})$; eqs. (IV.11), (II.17), (II.15) have been used.

Being invariant with respect to the linearised gauge transformations (IV.3), the operator in (IV.16) is degenerate. Indeed,

$$D_1^{++}\left[\int du_2\, (D_1^+)^4 \frac{1}{(u_1^+ u_2^+)^2} V^{++}(2)\right] = \qquad \text{(IV.17)}$$

$$= \int du_2\, (D_1^+)^4 D_1^{--} \delta^{(2,-2)}(u_1, u_2) V^{++}(2) = (D_1^+)^4 D_1^{--} V^{++}(1) \equiv 0$$

since V^{++} is analytic (see (II.18) and (III.4)). So, one has to fix the gauge and introduce Faddeev-Popov ghosts.

As a convenient gauge fixing one may take, e.g.,

$$D^{++} V^{++} = 0 \qquad \text{(IV.18)}$$

which is a generalisation of the Lorentz gauge $\partial^m A_m(x) = 0$ of the case N=0 (indeed, $D^{++} V^{++}$ contains $\partial^m A_m$ as a component field). To see that (IV.18) fixes the gauge completely, one makes a linearized gauge transformation $\delta V^{++} = -D^{++}\lambda$ (IV.3) and obtains from (IV.18)

$$(D^{++})^2\lambda = D^{++}V^{++}.$$

According to eqs. (III.13,14) this equation has a unique solution for λ (provided appropriate boundary conditions are imposed).

In order to make a gauge fixing in the functional integral

$$Z = N\int DV^{++} e^{iS_{SYM}} \qquad \text{(IV.19)}$$

we shall follow the standard Faddeev-Popov and 't Hooft procedure. Consider the Faddeev-Popov determinant

$$\Delta_{FP}(V) = \int D\lambda\, \delta\{D^{++}[e^{i\lambda}(V^{++} - iD^{++})e^{-i\lambda}] - f^{(4)}\},$$

where $f^{(4)}$ is a Hermitean analytic superfunction. Next, introduce $\Delta^{-1}_{FP}\Delta_{FP}$ into (IV.19), make a gauge transformation and a change of the integration variables, and then drop the λ integral. The result is

$$Z = N\int DV^{++}\Delta^{-1}_{FP}\,\delta[D^{++}V^{++} - f^{(4)}]\,e^{iS_{SYM}}. \qquad \text{(IV.20)}$$

The next step is to average over $f^{(4)}$ with an appropriate weight factor, so that Feynman-like and other familiar gauges can be obtained. This factor turns out to be

$$\exp\left[\frac{i}{2\alpha}\operatorname{tr}\int d\zeta_1^{(-4)} du_1 du_2 f^{(4)}(1)(D_1^+)^4\frac{u_1^- u_2^-}{(u_1^+u_2^+)^3}f^{(4)}(2)\right]. \qquad \text{(IV.21)}$$

Here everything depends on the same x and θ but on different u's which is indicated by the subscripts 1,2. It is essential that the operator in (IV.21) is not degenerate (otherwise the integral over $f^{(4)}$ is not well defined). The reader may verify this by checking the identity

$$(D_1^{++})^2\left[\int du_2 (D_1^+)^4\frac{u_1^- u_2^-}{(u_1^+u_2^+)^3}f^{(4)}(2)\right] = -\Box_1 f^{(4)}(1)$$

(use (III.15) and (III.12)).

The integration over $f^{(4)}$ means replacing $f^{(4)}$ in (IV.21) by $D^{++}V^{++}$ because of the δ-function in (IV.20). The evaluation is similar to that in (III.15). The resulting gauge fixing term in the action is

$$S_{gf} = \frac{1}{2\alpha} tr \int d\zeta_1^{(-4)} du_1 du_2 V^{++}(1)(D_1^+)^4 \cdot \left[\frac{1}{(u_1^+ u_2^+)^2} - \frac{1}{2}(D_1^{--})^2 \delta^{(2,-2)}(u_1,u_2) \right] V^{++}(2) =$$

$$= \frac{1}{2\alpha} tr \int d\zeta_1^{(-4)} du_1 du_2 V^{++}(1) (D_1^+)^4 \frac{1}{(u_1^+ u_2^+)^2} V^{++}(2) + + \frac{1}{2\alpha} tr \int d\zeta^{(-4)} du\, V^{++} \Box V^{++}. \tag{IV.22}$$

Here (III.12) has been used.

The Faddeev-Popov determinant is computed in complete analogy with the N=0 case:

$$\Delta_{FP}^{-1} = \int DFDP \exp i\, tr \int d\zeta^{(-4)} du\, i F D^{++}(D^{++} + iV^{++}) P \tag{IV.23}$$

(cf. the N=0 expression $tr \int d^4x\, i c \partial^m (\partial_m + i A_m) c'$). The two ghost superfields F,P are similar to the ω hypermultiplet (III.13) except for their statistics. It is remarkable that the kinetic operator in (IV.23) is not degenerate thus avoiding the ghost-for-ghost problem /3,4/ .

Finally, putting together (IV.16), (IV.22) and (IV.23) one obtains

$$S_{SYM} + S_{gf} + S_{FP} = \frac{1}{2\alpha} tr \int d\zeta^{(-4)} du\, V^{++} \Box V^{++} + + \frac{1}{2}\left(1 + \frac{1}{\alpha}\right) tr \int d\zeta^{(-4)} du_1 du_2 V^{++}(1) (D_1^+)^4 \frac{1}{(u_1^+ u_2^+)^2} V^{++}(2) + \tag{IV.24}$$

$$+ tr \int d\zeta^{(-4)} du\, i F D^{++} (D^{++} + i V^{++}) P + \ldots$$

where the dots denote self-interaction terms for V^{++} .

IV.3. Green functions

After the gauge has been fixed and the degeneration lifted one can derive the Green function for the gauge superfield V^{++}

$$G^{(2,2)}(1|2) = \langle V^{++}(1)\,V^{++}(2)\rangle .$$

The corresponding equation can be read off from (IV.24):

$$\frac{1}{\alpha}\Box_1 G^{(2,2)}(1|3) + \left(1+\frac{1}{\alpha}\right)(D_1^+)^4 \int du \frac{1}{2(u_1^+u_2^+)^2} G(z_1,u_2|3) = \delta_A^{(2,2)}(1|3).$$

It can be rewritten as follows (see (III.7))

$$\int d\zeta_2^{(-4)} du_2 \left[-\Box_1 \Pi^{(2,2)}(1|2) + \frac{1}{\alpha}\Box_1\left(\delta_A^{(2,2)}(1|2) - \Pi^{(2,2)}(1|2)\right)\right] G^{(2,2)}(2|3) = \delta_A^{(2,2)}(1|3). \qquad \text{(IV.25)}$$

Here

$$\Pi^{(2,2)}(1|2) = -\frac{(D_1^+)^4 (D_2^+)^4}{\Box_1} \frac{1}{(u_1^+u_2^+)^2} \delta^{12}(z_1 - z_2) \qquad \text{(IV.26)}$$

is the projection operator for superisospin 0 for the analytic superfields $\Phi^{(2)}(\zeta,u)$ (the analytic superfields $\Phi^{(q)}$ contain superisospins $I = |n - \frac{q}{2}|$, $n = 1, 2, \dots$, see /1/). Indeed, the supplementary condition for this superisospin is $D^{++}\Phi^{(2)} = 0$ /1/ , and it is easy to check that (see (IV.17))

$$D_1^{++}\Pi^{(2,2)}(1|2) = D_2^{++}\Pi^{(2,2)}(1|2) = 0 .$$

Further, $\Pi^{(2,2)}$ has the projection property

$$\int d\zeta_2^{(-4)} du_2\, \Pi^{(2,2)}(1|2)\, \Pi^{(2,2)}(2|3) = \Pi^{(2,2)}(1|3). \qquad \text{(IV.27)}$$

To see this, one may use $(D_2^+)^4$ from, e.g., $\Pi^{(2,2)}(2|3)$ to restore the full measure $d^{12}z_2$ (see (III.5)) and then integrate over z_2 with the help of $\delta^{12}(z_2 - z_3)$. Thus the l.h.s. of (IV.27) becomes

$$l.h.s. = \int du_2 \frac{(D_1^+)^4 (D_2^+)^4 (D_3^+)^4}{\Box_1^2} \delta^{12}(z_1 - z_3) \frac{1}{(u_1^+u_2^+)^2 (u_2^+u_3^+)^2} .$$

To evaluate the harmonic integral, one may use the identity $(D_2^+)^4 f(z_2) \equiv \frac{1}{4} D_2^{++} D_2^{--} (D_2^+)^4 f$ (see (II.4)) and integrate by parts taking into account (II.18):

$$\text{l.h.s.} = -\frac{(D_1^+)^4 (D_3^+)^4}{4\,\Box_1^2} \int du_2 \Big[\frac{D_2^{--} \delta^{(2,-2)}(u_2,u_1)}{(u_2^+ u_3^+)^2} + $$

$$+ \frac{D_2^{--} \delta^{(2,-2)}(u_2,u_3)}{(u_1^+ u_2^+)^2} \Big] D_2^{--} (D_2^+)^4 \delta^{12}(z_1 - z_3).$$

Now one can take D off the harmonic δ-functions and use the latter to do the u_2-integral, thus identifying $(D_2^+)^4$ with $(D_1^+)^4$ or $(D_3^+)^4$. The only nonvanishing contribution is obtained when D_2^{--} from the harmonic δ-functions hits $D_2^{--}(D_2^+)^4 \delta^{12}$ and (III.12) may be used. The result is just the r.h.s. of (IV.27).

Coming back to the Green function equation (IV.25) we note that it is analogous to the one for the case N=0:

$$-\Box_1 \Pi_{mn} G_{nk}(1|2) + \frac{1}{\alpha} \Box_1 (\delta_{mn} - \Pi_{mn}) G_{nk}(1|2) = \delta_{mk} \delta^4(1|2),$$

where $\Pi_{mn} = \delta_{mn} - \partial_m \partial_n / \Box$ is the spin 1 projector for a vector field $A_n(x)$. The situation in N=1 SYM is similar. Now, using the projection property of $\Pi^{(2,2)}$ we can solve (IV.25):

$$G^{(2,2)}(1|2) = \frac{\alpha}{\Box_1} \delta_A^{(2,2)}(1|2) - \frac{1+\alpha}{\Box_1} \Pi^{(2,2)}(1|2). \qquad \text{(IV.28)}$$

In (IV.28) one can recognize the familiar α-gauges. For instance, at α =0 one obtains the Landau-Lorentz gauge, at $\alpha = -1$ – the Fermi-Feynman gauge. The latter is usually preferred in the literature due to a better infrared behaviour of the propagator. Moreover, it is best suited for the background field method.

The Green functions for the Faddeev-Popov ghost superfields in (IV.24) are the same as for the ω-hypermultiplet (III.14).

Finally, we mention that the well-known Becchi-Rouet-Stora-Tyutin transformations are easily generalised to the case N=2:

$$\delta V^{++} = i\xi (D^{++} + iV^{++}) P$$
$$\delta F = -\frac{1}{\alpha}\xi (D_1^+)^4 \int du_2 \frac{u_1^- u_2^-}{(u_1^+ u_2^+)^3} D_2^{++} V^{++}(2)$$
$$\delta P = \xi P^2 \qquad \text{(IV.29)}$$

Here ξ is a nilpotent constant parameter. These transformations leave invariant $S_{gf} + S_{FP}$.

Concluding the first part of our work we may say that the introduction of harmonic distributions in Sec.II has allowed us to find the Green functions for N=2 matter (Sec.III) and N=2 SYM (Sec.IV). The complete set of Feynman rules and examples of supergraph calculations are given in the second part of our paper.

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Received by Publishing Department
on January 29, 1985.

Гальперин А. и др. E2-85-127

Гармонические суперграфики. Функции Грина

Разрабатывается процедура квантования в гармоническом пространстве. Вводятся гармонические обобщенные функции. Они нужны для построения δ-функций в аналитическом подпространстве и функций Грина суперполей N = 2 гипермультиплета и мультиплета Янга-Миллса. Описана процедура закрепления калибровки и определены соответствующие духовые суперполя Фаддеева-Попова. Найдены БРСТ преобразования. Квантование N = 2 калибровочной теории в гармоническом суперпространстве оказывается довольно простым и имеет много общего с таковым для обычной (N = 0) теорией Янга-Миллса. В частности, не возникает нужды в так называемых духах для духов.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна 1985

Galperin A. et al. E2-85-127

Harmonic Supergraphs. Green Functions

The quantization procedure in the harmonic superspace approach is worked out. Harmonic distributions are introduced and are used to construct the analytic superspace δ-functions and the Green functions for the hypermultiplet and the N = 2 Yang-Mills superfields. The gauge fixing is described and the relevant Faddeev-Popov ghosts are defined. The corresponding BRST transformations are found. The harmonic superspace quantization of the N = 2 gauge theory turns out to be rather simple and has many parallels with that for the standard (N = 0) Yang-Mills theory. In particular, no ghosts-for-ghosts are needed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna 1985

26 коп.

Редактор Э.В.Ивашкевич. Макет Р.Д.Фоминой.

Подписано в печать 28.02.85.
Формат 60х90/16. Офсетная печать. Уч.-изд.листов 1,69.
Тираж 530. Заказ 35936.
Издательский отдел Объединенного института ядерных исследований.
Дубна Московской области.