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OPTIMAL CONTROL THEORY APPLIED TO FUSION PLASMA THERMAL STABILIZATION*

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Many authors have investigated stability characteristics and performance of various burn control schemes (e.g., Ref. 1). The work presented here represents the first application of optimal control theory to the problem of fusion plasma thermal stabilization. The objectives of this initial investigation were to develop analysis methods, demonstrate tractability, and present some preliminary results of optimal control theory in burn control research.

The analysis was conducted with a zero dimensional, one-group plasma model. The model consists of the coupled, nonlinear state equations representing particle and power balance of a fusioning DT plasma,

$$\frac{dn}{dt} = r(ext) - r(burnup) - r(trans) \tag{1}$$

$$\frac{d(nT)}{dt} = P(ext) + P(fusion) - P(brem) - P(trans) \tag{2}$$

The particle balance equation, (1), includes terms representing:fueling rate from an externally controlled source, *r(ext)*; particle loss rate due to fusion events, *r(burnup)*; and particle loss rate due to transport processes, *r(trans)*. The power balance equation, (2), includes terms representing: power input from an externally controlled source, *P(ext)*; power input due to the thermalization of the 3.5 MeV fusion product alpha particle, *P(fusion)*; power loss due to electron bremsstrahlumg radiation, *P(brem)*; and power loss due to transport processes, *P(trans)*.

The above plasma dynamics equations were recaste in a form suitable for implementing linear optimal control. This was accomplished by linearizing about the equilibrium point corresponding to maximum plasma power density. Using vector notation, and denoting the state and control variables by **x** and **u**, i.e.,

$$\mathbf{x} = [n, nT]^{\mathrm{T}}, \qquad \mathbf{u} = [r(ext),\, P(ext)]^{\mathrm{T}},$$

the plasma dynamics equations may now be stated in linear canonical form

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + B\mathbf{u},$$

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where A and B are 2 × 2 matrices.

In optimal control theory, "optimal" controls are obtained from the complete family of admissable controls by defining and minimizing a performance index which represents a "cost" of the control action. In this work, quadratic integral cost functionals of the form,

$$J = \int (\Delta \mathbf{x}^{\mathrm{T}} Q \Delta \mathbf{x} + \Delta \mathbf{u}^{\mathrm{T}} R \Delta u) dt$$

were formulated to represent operational considerations, e.g., control power consumption, net power loss due to deviations from the optimum operation point, settling time, etc. Quadratic Linear Problem (QLP), (e.g. Ref. 2), methods were then applied to minimize the integral cost for a perturbation in equilibrium conditions and the ensuing optimal trajectory was calculated. The effects of competing operational considerations were investigated. Sensitivity to control perturbations was also investigated.

In order to assess the validity of the linearized dynamical equations, costs computed from the linear and nonlinear models were compared. Using the control, $\mathbf{u}(\mathbf{x})$, prescribed by the linear optimal control theory, the state trajectory, $\mathbf{x}(t)$, was computed from the nonlinear state equations. The resulting cost, J(nonlinear), was computed and compared with the cost in the linear model, J(linear). It was found that the two were in good agreement, indicating that the suppressed nonlinearities don't contribute significantly.

An evaluation of the merits of the optimal control approach was made. Trajectories were found to display good damping along with inherent stability. Computational organization and techniques allow manageable analysis. Such characteristics indicate that optimal control theory is well suited for burn control applications, and in particular, to cope with more realistic extensions involving multi-input/output control and state descriptions. Extensions of the presented work were suggested.

[1] I. Maya, "Control for Fusion Thermal Stability," University of Florida Doctoral Dissertation, 1983.

[2] M. Athans and P. Falb, *Optimal Control*, New York: McGraw-Hill, 1966.