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THEORY PROBLEMS AT SMALL X

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MASTER

Summary

It is shown how the standard methods of perturbative QCD are valid to extremely small x. The methods are valid provided a quantity we call the 'packing fraction' of partons in a hadron is much less than one. One surprising consequence is that the cross-section for production of jets of a few GeV energy can be reliably calculated. Since this cross-section in tens of millibarns, the phenomenology of minimum bias events at the SSC will be different than at lower energy; this will have a significant effect on the backgrounds for new physics events.

Introduction

In this report is summarized work done by the subgroup on "Theory problems at small x" at the Snowmass workshop. The members of the subgroup were: S.J. Brodsky, J.C. Collins (leader), S.D. Ellis, R.J.Gonsalves, A.H. Mueller, and W.-K. Tung.

The issues addressed by the subgroup arise as follows: Many predictions for the rates of interesting processes and of background events at the SSC rely on the validity of the QCD-improved parton model, as is manifest in the whole of these proceedings. Calculations of cross-sections for the production of high-mass systems, like jets at large tranverse energy or new heavy particles, are made by first calculating the cross-section for the process at the parton level and by then convoluting with the distributions of partons in each of the incoming hadrons. The factorization theorems[1] of QCD state that when the produced system carries off a large fraction of the total center-of-mass energy, this method of calculation is valid. However, if the fractional energy (referred to as 'x' in the subgroup's title) is small, then the validity of the calculations is not manifest, even when the produced system has a large invariant mass.

The reason that the validity of the calculations is not manifest is that we are approaching a kinematic boundary, viz., x = 0. At kinematic boundaries perturbative calculations tend to break down, because higher-order terms in the perturbation series may dominate lower orders, and then a low-order calculation of the cross-section is inaccurate.

Fig. 1 illustrates the method of calculation just referred to. There is a hard collision of two partons, one out of each hadron. The produced system under consideration is part of the final state from the hard collision. The partons carry fractions x_A and x_B of their parent hadrons' momenta, so that the center-of-mass energy of the parton collision is given by

$$\hat{s} = x_A x_B s, \qquad (1)$$

where s is the square of the overall center-of-mass energy. We will usually denote a typical invariant mass in the subprocess by Q. Technically the basis for the calculations is the set of results called 'factorization theorems'[1]. Their predictive power is that the parton cross-sections can be expanded in powers of a small effective coupling $\alpha_s(Q)/\pi$, and that the parton distributions are the same for any process, with the distributions at different energies being related by the Altarelli-Parisi equation[2].

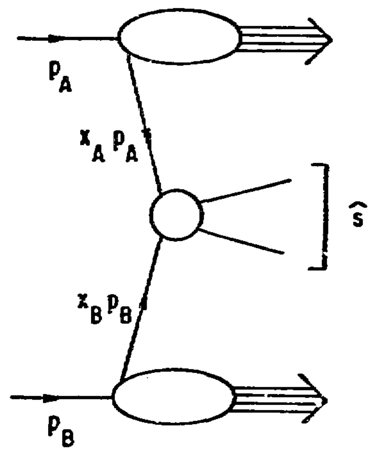

Fig. 1 QCD factorization for hard process

It is evident from, for example, the compilation of cross-sections given by Eichten et al[3] (EHLQ) that many interesting processes at the SSC involve small values of x. (We will henceforth use the symbol 'x' generically, to refer either to the fractional transverse energy or invariant mass of the system being produced, or to the typical fractional momentum carried by the partons entering the hard collision. Both numbers are usually comparable.) For example, consider production of single W's or Z's. The invariant mass is Q = 80 to 100 GeV, so that a typical x value for production by the Drell-Yan process is $Q/\sqrt{s}$ = 80GeV/40TeV = 2×10^{-3}. If one considers non-central production, then x's in the range 10^{-2} to 10^{-4} are important.

In the past, most theorists would have considered such values of x as being ridiculously small for applying perturbative methods in QCD. But if we are to make serious calculations for the SSC, we must be able to make predictions in these situations. As we will see, perturbative QCD does in fact apply, and it applies, moreover, at even smaller values of x.

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Let me summarize the questions addressed and answered by the small-x subgroup:

(1) Are standard perturbative QCD methods valid at very small x (while keeping $Q \gg 1$ GeV)? -- They are. To answer this question we had to gain an understanding of the Russian work recently reviewed by Gribov, Levin and Ryskin[4].
(2) Where do the methods break down? -- When the partons at the chosen values of x and Q fill the hadron.
(3) How accurate are numerical calculations of the solutions of the Altarelli-Parisi[2] equation, and to how low x can they be taken? -- Tung presented a fast and accurate program for this solution, and checked its accuracy. This work is described in his contribution[5], and will not be treated in this report.
(4) Granted that we trust standard perturbative calculations down to very small x, what are the cross-sections for production of jets of a few GeV transverse energy? -- These are in the mb region, and can have an important effect on background calculations.

We believe it is important for more theorists to explore in greater depth the field of perturbative methods at small x, and to build on the work reviewed in Ref. 4.

We had many fruitful interactions with other subgroups; this had impact on our work. We would particularly like to acknowledge discussions with the structure function, the small-p_T, and the jet fragmentation subgroups. The subject matter of these subgroups very much overlapped with our own. We would also like to thank Jim Rohlff for discussions on the jet data from the $Sp\bar{p}S$ collider.

Factorization at small x

In this section, we will summarize what the subgroup concluded were the appropriate basic results about factorization at small x.

Let us first remind ourselves of the steps by which perturbative calculations are made by the method summarized in Fig. 1:

(1) The cross-section σ at the parton level is calculated in low-order perturbation theory, being composed of the Born approximation plus higher-order corrections, which are a power series in the effective coupling $\alpha_s(Q)$.
(2) The parton distributions $f(x,\mu)$ with $\mu = O(Q)$ can be calculated by evolving the Altarelli-Parisi equation[2] from low values of Q.

Much of the predictive power of the calculations comes from the unversality of the parton distributions. That is, they can be measured in one process (say, deeply inelastic scattering) at one energy and the same values can then be used in other processes.

These methods are certainly valid[1,6] if we take the overall center-of-mass energy of the process to be large, while keeping Q^2/s fixed. The problem is to verify the validity of the methods in the situation that Q^2/s is very small but that the actual size of Q remains large, as in production of single W's or Z's

To see why we should not grant the methods automatic validity at small x, we must recall how the factorization theorem used in the calculations is proved. Feynman graphs for the cross-section are examined and then their leading-twist behavior is extracted. This behavior comes from regions of loop-momentum space that can be pictured roughly as in Fig. 1. It should be noted, however, that Fig. 1 somewhat misrepresents the structure of the momentum regions, since the whole scattering is bathed in a sea of soft partons. ('Soft' can be considered as meaning 'having low center-of-mass rapidity'.) These soft partons are in effect generated from gluon bremsstrahlung associated with the hard collision.

When x is small, the leading-twist behavior actually comes from regions with the same structure as at large x[4]. However, even if the factorization theorem is valid in the small-x region, it is not manifest that the cross-sections are perturbatively calculable. The problem is most easily seen in terms of Feynman graphs -- we must integrate over a range of momenta and typically we obtain logarithmically behaved integrals like

$$\ln(1/x) = \int_x^1 dy/y, \tag{2}$$

where $x = Q/\sqrt{s}$. The higher the order of a graph, the more independent integrals like this there are, and the greater, therefore, the number of powers of $\ln(x)$. Consequently higher orders of perturbation theory tend to have much larger coefficients than they do at normal values of x, so that simple low-order calculations are liable to become invalid.

We may summarize this section by listing some of the questions that it provokes:

(1) How low in x can one go before the basic factorization theorem fails?
(2) Assuming the validity of factorization, to what extent can one use it with only low-order calculations of the Altarelli-Parisi kernel and of the parton cross-sections σ?
(3) If low-order calculations are not sufficient, how can one find suitable resummation methods to take account of the large higher order corrections?

Limits of validity of factorization

Gribov et al[4] (GLR) present a nice argument to find the lower limit on x beyond which factorization breaks down. They present their argument both as a very intuitive physical picture and as a corresponding statement about Feynman graphs. We will sketch the physical picture. The hard collision in Fig. 1 can be considered as a distinct physical process occurring on a time-scale $t \approx 1/Q$ that is much shorter than the ordinary hadronic processes in the rest of the event, provided only that the energy Q is large on a scale of 1 GeV. The (dressed) partons entering the hard collision should be the only ones to participate in the hard collision if factorization is to be valid, i.e., they should be independent of the other partons during the hard scattering. Note that in the wave function these partons must interact with other partons: their independence from other partons is only true and only matters in the vicinity of the hard scattering.

This independence implies a spatial separation from other partons with which there could be significant interaction, and from this GLR develop a useful quantitative criterion for the validity of factorization. The criterion is that the fraction of a hadron occupied by the relevant partons is much less than one. The relevant partons are those that, defined on a scale Q, are within about one unit of rapidity of the active parton under consideration. Interactions with partons of very different rapidities are already taken into account in the factorization theorem. Notice that it the scale Q of the hard scattering that is relevant here, not $\sqrt{s}$ or some other scale. The number of relevant partons in a hadron is then $xf(x,Q)$, where

$f(x,Q)$ is the total number density of partons summed over all parton species. In the circumstances with which we are concerned, the gluons are by far the most numerous, so we could just let $f(x,Q)$ be the gluon distribution.

We must treat the hadron as being Lorentz contracted, so that we have a two-dimensional problem. Validity of factorization requires then that a certain quantity $W(x,Q)$ be much less than one. This quantity, we may call the packing fraction of partons, and it is defined by

$$W(x,Q) = xf(x,Q)\; R^2_{\text{parton}}/R^2_{\text{hadron}}. \tag{3}$$

We need to know the radii of the partons and of the hadrons in order to use this formula. The hadron radius is ambiguous, because of the increase in σ_{tot} with energy. Up to SSC energies, it is a sufficient approximation to take R_{hadron} to be $1/m_\pi$. It is reasonably obvious that the parton radius is proportional to $1/Q$, but is not immediately obvious what the constant of proportionality should be. Should it just be the wavelength ($\propto 1/Q$), or should it be taken as related to the gluon-gluon cross-section

$$\sigma \approx N\, \alpha_s(Q)^2/Q^2? \tag{4}$$

Since the normalization N in eq. (4) is large (around 50), the consensus of the subgroup was that it is sufficient for our purposes to define

$$W = xf\, m_\pi^{2}/Q^2. \tag{5}$$

So we must require this to be much less than one in order for factorization to be valid.

A second criterion enters into GLR's considerations when it is recalled that $f(x,Q)$ has been evolved from lower values of Q. Consider Fig. 2, where we have drawn the (x,Q)-plane. The shaded region A is where the condition $W \ll 1$ is violated; in that region standard perturbative methods are invalid, and as a minimum some kind of resummation by a Reggeon-like technique is needed. Let us define the boundary of the forbidden region A as $x = x_{cr}(Q)$. The question to be addressed now is: When we evolve $f(x,Q)$ from lower values of Q, what effect has the existence of the forbidden region on our calculations of the evolution?

For the evolution we use the Altarelli-Parisi equation:

$$\partial f(x,Q)/\partial \ln Q = \int (dy/y)\; P(y/x,\alpha_s(Q))\; f(y,Q). \tag{6}$$

Suppose we wish to obtain $f(x_1,Q_1)$ at some small value of x_1 and at some large value of Q_1 from $f(x,Q_0)$ at some lower value of energy, Q_0. Then in eq. (6) we need to use the whole of the region $x_1 < x < 1$ and $Q_0 < Q < Q_1$. This apparently puts a severe restriction on the region in which we can do calculations, for we must keep x_1 above the minimum value $x = x_{cr}(Q)$ not at $Q = Q_1$, where we want to calculate f, but at $Q = Q_0$, where the evolution starts. In Fig. 2 this is the restriction that we stay above the line b.

However, GLR make the observation that it is not the whole of the mathematically necessary region that is actually important in the evolution of the distributions. If the target value of x is small, then the predominant contribution to the solution of (6) comes from a relatively small range of x for each value of Q. Therefore the evolution can be regarded as following trajectories in the (x,Q)-plane. The value of $f(x_1,Q_1)$ is determined mostly by the behavior of $f(x,Q)$ in the neighborhood of the trajectory that goes through (x_1,Q_1). In the evolution, when we need the value of $f(x,Q)$ in the forbidden region, it is a reasonable approximation to replace $f(x,Q)$ by its maximum value, which corresponds to $W = 1$.

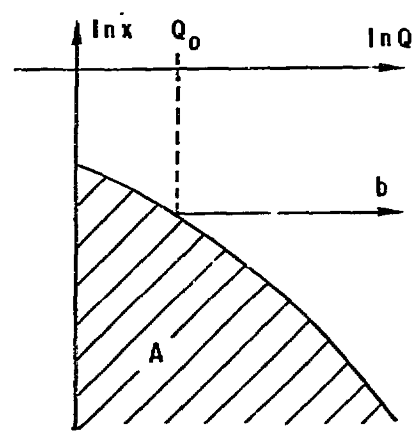

Fig. 2 Regions of validity of the Altarelli-Parisi equation.

The range of x and Q in which we can legitimately evolve $f(x,Q)$ from its value at $Q = Q_0$ is thereby extended to more-or-less the whole of the (x,Q)-plane that lies outside the forbidden region. However, the results of calculations using the usual unmodified Altarelli-Parisi are incorrect if the trajectories pass through the forbidden region. The fix-up of replacing f by its limiting value with $W=1$ remedies this: the replacement equation is non-linear near the forbidden region. A more complicated modified evolution equation is given by GLR[4]; it is supposed to be valid over the whole of the exterior of the forbidden region, including the neighborhood of the boundary. It includes non-linear terms that take account of what one may call 'parton overcrowding' when the density W gets close to unity. These terms arise diagrammatically in a manner similar to the perturbative description of Reggeon theory.

We have checked that the curves of parton distributions given by EHLQ have a parton packing fraction of at most a per cent or so, and they are therefore safe from breakdown of the formalism used for the calculations.

GLR do in fact give estimates which could be used to estimate the useful range of x for perturbative calculations. However, these are based on saddle-point approximations, and it is not clear how trustworthy these are at the energies we are considering. In particular, it is not manifest how good the normalizations are. Anyone working through their review[4] should also be aware that estimates given early in the paper are based on crude approximations that are greatly improved later in the paper. GLR do not appear to base results on numerical solutions of the Altarelli-Parisi equation, and the subgroup's feeling was that it is necessary to use just such numerical data in order to make sound estimates of where the forbidden region is.

Two other points are worth noting. The first is that EHLQ[2] give curves for parton distributions down to $x = 10^{-4}$ and out to $Q = 10^4$ GeV. Now, the energy μ that should be used in the parton distributions in the factorization formula is of order the parton subenergy, viz.,

$$\mu \approx (x_A x_B s)^{1/2}. \qquad (7)$$

Thus central production of a system with $x \approx 10^{-4}$ corresponds to a scale for the parton distributions of only $\mu \approx 4$ GeV. Even if one goes to the kinematic limit of forward production, where $x_A = 10^{-4}$ and $x_B = 1$, we still only get $Q = 400$ GeV, which is a long way from $Q = 10^4$GeV. So the most extreme values on EHLQ's curves are not applicable to the SSC. The second point is that evolution of the Altarelli-Parisi can be carried out more accurately and orders of magnitude faster than EHLQ's calculation -- see Tung's contribution[5].

Calculability

Suppose that we are in a region of x and Q where we can apply factorization. One issue implicitly addressed in the previous section was whether the validity of low-order perturbation theory at small x is affected by the pile-up of logarithms in higher order. A typical way in which these logarithms arise is from the integral over longitudinal momentum of a particle inside a Feynman graph. This was schematically shown in eq. (2), where the 1/y factor on the right-hand side could reprsent the value of a parton distribution.

Now, the canonical expectation[7] (from Regge theory), is that parton distributions behave like 1/x at small x. Moreover, the 1/x behavior, modified by logarithms, is typical in fixed-order perturbation theory. However, when we perform the Altarelli-Parisi evolution, these logarithms modify the small-x behavior to make it steeper than 1/x at those values of x with which we are concerned. Indeed, if one tries fitting a power law (at small x) to the curves for f(x,Q) given in Sec. 2 of EHLQ, one finds an effective power that varies but that is typically in the range of $x^{-1.3}$ to $x^{-1.6}$. (Note that EHLQ plot x times f rather than f itself.) Hence the integrals over the momentum fractions x_A and x_B in Fig. 1 are dominated by the central values of the x's rather than having contributions equally from all ranges of x, which is what would result from a 1/x distribution (cf (2)). Similar remarks apply to the Altarelli-Parisi equation (6). It is essentially this deviation from 1/x behavior that produces the trajectories in the (x,Q)-plane that give the dominant contributions to the Altarelli-Parisi evolution. These we discussed in the provious section.

The true asymptotic formulae for the parton distributions at small x and large Q are rather complicated[4]. So it suffices for our purposes to be more qualitative, and a toy example will give the general idea of the way in which the logarithms are avoided. Suppose that we define $\Delta f = \partial f/\partial \ln Q$, and that the evolution of f satisfies:

$$\Delta f = \int_x^1 dy/y \; C(x/y)\; f(y,Q), \qquad (8)$$

with

$$C(x,y) = (y/x)\;[1 + \alpha \ln(y/x)]. \qquad (9)$$

First, we will set $\alpha = 0$ and assume that f(x) is $x^{-1-\epsilon}$ at the value of Q where we start the evolution. Then

$$\Delta f = \int_x^1 dy\; y^{-1-\epsilon}\; x^{-1} = x^{-1}(x^{-\epsilon}-1)/\epsilon. \qquad (10)$$

If ϵ has its canonical value of zero, then

$$\Delta f = x^{-1}\ln(1/x).$$

The logarithm acts to steepen the distribution when Q is higher than the starting value. The distribution is therefore no longer proportional to 1/x at the higher values of Q.

Let us now add in the one-loop term in eq. (10). The order α part of Δf is

$$\alpha\;[\epsilon^{-2}\;(x^{-1-\epsilon}-x^{-1}) - \epsilon^{-1}x^{-1}\ln(1/x)]. \qquad (11)$$

If we again put in the canonical value $\epsilon = 0$, (11) becomes

$$\alpha\;(2x)^{-1}\ln^2 x, \qquad (12)$$

which has one more logarithm than the lowest-order contribution. This is an example of the pile-up of logarithms in higher order calculations that was refer-red to earlier.

Suppose now that the initial distribution is steeper than in the canonical case, i.e., that $\epsilon > 0$. Then the leading power behavior of the one-loop part of Δf, as $x \to 0$, is

$$\alpha\; x^{-1-\epsilon}\;\epsilon^{-2},$$

which has no logarithms compared to the lowest-order term. This result arises because the main contribution to the integral comes from the neighborhood of its lower limit, whereas the logarithms only arise if all ranges of ln(y) in eq. (8) contribute equally.

The conclusion, then, is that the standard methods -- using low finite-order calculations of the Altarelli-Parisi kernel and of the parton cross-sections -- are valid provided only that Q is far enough above 1 GeV to have $\alpha_s(Q)$ small and that the parton density W(x,Q) is well below unity.

Large cross-sections at low $E_T/\sqrt{s}$

Now that we believe that we can trust perturbative methods down to very small x, we must take them seriously as a calculation of cross-sections for jet production at transverse energies of a few GeV. The cross-sections, as we will now show, are at a level of tens of millibarns, which implies that most events at the SSC will contain a hard scattering. This, clearly, has a great impact on estimates of the backgrounds to new physics at the SSC, particularly because an important background, much discussed at this workshop, arises from ordinary 'minimum bias' events superimposed on a hard scattering event. At the high luminosities typical of those planned for the SSC, there may be several events per beam crossing, on average. Because the jet cross-sections are now at the same level, the character of minimum-bias events will change. Steve Ellis[8] called this result the 'death of low p_T'.

These large cross-sections for jet production have already been noticed by Paige (private communication) and doubtless by other people. Our point is that they should be taken as valid predictions of QCD, and be used both for their own sake, as interesting physics, and in realistic background calculations. Precise numerical calculations can be made by the standard programs already available, so the point here is only to emphasize the order of magnitude of the cross-sections and their significance.

Consider the cross-section for jet production per unit rapidity at a transverse energy Q. This is

$$d\sigma/dy d\ln Q \approx N\, Q^{-2}\, \alpha_s(Q)^2\, [xf(x,Q)]^2, \qquad (13)$$

where N is a normalization. The $1/Q^2$ factor arises on purely dimensional grounds, and x is $Q/\sqrt{s}$. Now, if the parton distributions have the canonical 1/x behavior, then (for fixed Q) the jet cross-section is independent of the total energy and is fairly small. However, the parton distributions are substantially steeper, as we saw in the previous section. Thus, when $\sqrt{s}$ increases, x decreases in proportion, the jet cross-section increases, and this increase has, roughly speaking, a power-law behavior.

We may estimate the number of jets of the type considered per hadron-hadron collision by

$$R = N\, \alpha_s(Q)^2\, (xf)^2\, (m_\pi/Q)^2. \qquad (14)$$

The normalization factor N can be extracted from, say, the formulae in Sec. 3 of EHLQ, and it is large, around 10. We have estimated the total hadron-hadron cross-section by πm_π^{-2}; its logarithmic increase with energy is minor compared to the effect we are investigating.

Evidently at high enough energy the jet cross-section is higher than the total cross-section -- on average there is more than one jet per collision. The character of 'minimum bias' events, i.e., run-of-the-mill events, must change then. The energy range of the SSC is just where this change occurs. This is easily verified by substituting some numbers into eq. (12). If we set $\alpha_s = 1/5$ and take xf = 10 from EHLQ, then we find that R = 3% (per unit rapidity). When we remember the large available range of rapidity, this implies that a large fraction of events at the SSC will contain jets of several GeV energy. Normally one expects minimum bias events only to contain particles of low transverse momentum; this will no longer be true at the SSC.

These large cross-sections are important for the background calculations that were so prominent at this workshop. But they also provide an important piece of QCD physics which should be investigated: They give a window onto perturbative physics at very low x, a region with very little experimental data at present.

It is important to check the corresponding results for current hadron colliders, that is, at $\sqrt{s}$ around 1/2 to 1 TeV. Let us estimate the ratio R in eq. (12), for the same value of Q. Since we now have $x \approx 10^{-2}$, we should set xf ≈ 3. This gives R ≈ 1/4% per unit of rapidity. In other words, the cross-section is of a size that is easily measurable. Indeed, many events of this type should already have been seen. They are characterized by having what we can term 'mini-jets'. We suspect that the 'hot-spots' seen by UA5 are in fact caused by these mini-jets. The hot-spots do not at first sight appear to be jet-like: Although confined to a small range of rapidity, the particles in the hot-spots have their directions uniformly distributed in azimuthal angle. However, the experiment does not measure particle energies or momenta, and we know from e^+e^--annihilation that two-parton final-states at this energy do not provide manifestly jetty events of the sort we are now accustomed to at 100 GeV transverse energies. The situation obviously needs investigation. Mini-jets will become rarer at higher values of Q, but should also become clearer. They should form part of a continuous distribution, all of which should fit with the predictions of perturbative QCD.

Acknowledgements

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