

TIME-INDEPENDENT LIMIT OF A CREEP-RECOVERY
CONSTITUTIVE EQUATION*

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ABSTRACT

The effect of strain recovery is taken into consideration in ORNL efforts to establish unified constitutive equations for time-dependent plastic deformation for metals at elevated temperatures. Representation by internal state variables and Rice's flow potential are under consideration. Here the growth law for the internal state variables is discussed and interpreted in terms of a generalized form of the kinematic hardening condition of Prager. The yield condition is obtained from the flow potential representation of the inelastic strain rate. A consistency condition is derived from the yield condition and leads to a flow rule which assumes a slightly general form as compared with that of the classical plasticity due to the effect of strain recovery and the time-dependent property of the yield condition. Based on this representation, the time-independent limit is discussed. From a vanishing effect of recovery and a rate-independent limit for the yield condition at low temperature, this flow rule reduces to the well-known form of time-independent plasticity with a kinematic hardening condition. The duration of time (the characteristic time) required for the inelastic strain to reach its saturated value is defined for the inelastic loading condition. It provides the measure of a minimum duration of time which is required for a valid approximation made by the time-independent plasticity model.

INTRODUCTION

As discussed in Refs. 1-4, constitutive equations for creep and plasticity are currently recommended for use in high-temperature design of fast breeder reactors. Although those equations contain some provisions for interaction between creep and plastic strains, they are loosely

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related with respect to both their mathematical structures and physical interpretations. Efforts (e.g. Robinson^{1,5}) have been underway to develop a unified theory in the sense of the state variable representation of Onat⁶. To date, that effort has emphasized a constitutive model which is valid at temperatures in the creep range. It is desirable for a constitutive model which is valid at temperatures in the creep range to also be reducible to an appropriate form for time-independent plasticity at lower temperature by adjusting certain temperature sensitive parameters. The formulation considered by Robinson^{1,5} is motivated, in part, by the uniaxial creep model of Lagneborg⁷ which uses the dislocation density, or its equivalent, as an internal state variable and in which recovery effects are taken into consideration. In the present investigation, Robinson's formulation is recast into a slightly different form so that a flow rule is expressed as a generalized form of a classical plasticity theory and is reduced to a time-independent limit.

Time-independent plasticity is defined as a limiting case for low temperature phenomenon with slow loading rate compared with a characteristic time in which the associated inelastic strain reaches its saturated value. A quantitative definition for the characteristic time is established. Hence it provides a duration of time which is required for a valid approximation to be made by time-independent plasticity. Several observations are made in this work. The recovery term in the growth law (or the equation of evolution) for the state variable α is represented as a product of the state variable α and a scalar function $r(\alpha)$ for a J_2 yield condition. A loading function f is introduced in the flow

potential. The yield condition is obtained by solving the function f from the flow potential representation of the inelastic strain rate and the growth relation. The rate-independent limit for the yield condition at low temperature is, in particular, described by the large exponent in a power law representation for the flow potential. The growth law is viewed as a generalized form of the kinematic hardening condition in classical plasticity with a recovery term. The flow rule is obtained through use of a consistency relation which is derived from the rate-dependent yield condition. From a vanishing effect of recovery and a rate-independent limit for the yield condition at low temperature, this flow rule reduces to the well-known form of time-independent plasticity with a kinematic hardening rule as its limit for a duration of time, or the characteristic time, for the plastic strain saturation. As an alternative form, the time-dependent constitutive equations are inverted to state the stress rate in terms of the total strain rate in which the inelastic strain does not appear. The strain recovery and the rate-dependent yielding appear in this constitutive equation to result in a time-dependent structure.

In this context, emphasis here is on the time-independent limit for the creep recovery model, with the aim of providing a good approximation at room temperature. Lagneborg⁷ cautioned that his uniaxial creep-recovery model needed further study at lower temperatures due to other possible mechanisms being dominant. A comparison between the time-independent limit of the present model with that of Hahn⁸ is illustrated in this report. Both models use the effect of large exponent in a power law representation of creep rate to explain the room temperature yield condition. The effect of delayed time for the initiation of creep strain

is manifested in Hahn's model but not from the present model. Certain similarities between the present model and that of Ponter and Leckie⁹ and that of Hart¹⁰ are also mentioned.

FLOW POTENTIAL AND GROWTH RELATION

Biaxial and uniaxial tests for FBR structural alloys at room-temperature as well as at elevated temperatures suggest the kinematic nature of yield surfaces. The center of the yield surface $\underline{\alpha}$, or the back stress tensor, in general depends on the history of inelastic strain. In earlier developments, Robinson^{1,5} chose $\underline{\alpha}$ as a state variable and its appropriateness for a state variable is under investigation. A flow potential, Ω , similar to that employed by Rice is used and chosen to be a function of the stress tensor $\underline{\sigma}$ and the back stress tensor $\underline{\alpha}$. Hence, $\underline{\alpha}$ replaces the history of deformation as originally proposed by Rice. Note that the use of $\underline{\alpha}$, a second rank tensor, as a state variable is consistent with the result of Geary and Onat¹¹ that the state variables should consist of tensors of even ranks. It is consistent with the uniaxial creep model of Lagneborg, or the model of Bailey-Orowan¹² when α is interpreted as the density of dislocation. Rice¹³ proposed that the rate of inelastic strain could be derived from the flow potential $\Omega(\underline{\sigma}, \underline{\alpha})$ by

$$\dot{\epsilon}^i_p = \frac{\partial \Omega(\underline{\sigma}, \underline{\alpha})}{\partial \sigma^i} \quad (1)$$

To reflect the kinematic nature within the flow potential, a loading function $f(\underline{\sigma}' - \underline{\alpha})$ was chosen^{1,5} that depends on the difference of the deviatoric stress $\underline{\sigma}'$ and the back stress $\underline{\alpha}$ so that

$$\dot{\underline{\epsilon}}^p = \frac{\partial \Omega(f, \underline{\alpha})}{\partial \underline{\sigma}} \quad (1a)$$

Obviously, the rate of inelastic strain $\dot{\underline{\epsilon}}^p$ is perpendicular to the loading surface at $f = \text{constant}$. The introduction of f in Ω implies additional structures for Ω , though f may coincide with Ω as a special case. The choice of the loading function f provides a convenient way to interpret the yield¹¹ condition in time-independent plasticity, which will be discussed later. In reality the loading function f is more than a function of $\underline{\sigma}' - \underline{\alpha}$, but its form should be consistent with the loading surfaces $f = \text{constants}$ observed in multiaxial experiments.

In addition to the work by Robinson¹, the use of $\underline{\alpha}$ as a set of state variables has also been discussed by Ponter and Leckie⁹. A specific form of the flow potential was suggested so that the growth law assumes the form¹

$$\dot{\underline{\alpha}} = - \frac{c}{\sqrt{J_2(\underline{\alpha})}} \frac{\partial \Omega}{\partial \underline{\alpha}}, \quad (2)$$

where c is a constant. For purpose of discussion, the slightly generalized form of the growth law given in Ref. 1 is given here as

$$\dot{\underline{\alpha}} = h(\underline{\alpha}) \dot{\underline{\epsilon}}^p - r(\underline{\alpha}) \quad (3)$$

which is compatible to the Bailey-Orowan model in uniaxial case. In (3), $h(\underline{\alpha})$ and $r(\underline{\alpha})$ correspond to the effects of strain hardening and strain recovery, respectively, for temperatures in the creep range and a dot is used to denote the time derivative of the variable. It has been mentioned in the creep model of Hart¹⁰ that the functions h and n depend also on the applied stress $\underline{\sigma}'$. This can be compromised by substituting the flow equation (1) into the stress dependent h and r to eliminate $\underline{\sigma}'$ and taking a first order approximation of $\dot{\underline{\epsilon}}^P$ for small values of $\dot{\underline{\epsilon}}^P$. Equation (3) can be viewed as a generalized version of the kinematic hardening rule.

It is obvious that if the function f in the flow potential Ω is the second invariant $J_2(\underline{\sigma}' - \underline{\alpha})$, where $\underline{\sigma}'$ is the deviatoric stress, then from Eq. (1a), $\underline{\alpha}$ must be deviatoric and symmetric:

$$\alpha_{ij} = \alpha_{ji} \quad (4)$$

For an isotropic material, $h(\underline{\alpha})$ is a function of $J_2(\underline{\alpha})$ and $J_3(\underline{\alpha})$ where J_2 and J_3 are the second and third invariants of the back stress tensor $\underline{\alpha}$, respectively. The functions $r(\underline{\alpha})$ can be represented, in general, by

$$\underline{r}(\underline{\alpha}) = r(\underline{\alpha}) \underline{\alpha} + s(\underline{\alpha}) \underline{\alpha} \underline{\alpha}, \quad (5)$$

but $s(\underline{\alpha})$ must be zero since, from Eq. (3), \underline{r} is deviatoric and symmetric. In general, $r(\underline{\alpha})$ is a function of $J_2(\underline{\alpha})$ and $J_3(\underline{\alpha})$. $\underline{\alpha}$ can not be argued as deviatoric for other forms of f .

Ponter and Leckie⁹ used a potential representation for $\dot{\underline{\alpha}}$ with the form

$$\dot{\underline{\alpha}} = - h(\underline{\alpha}) \frac{\partial \Omega}{\partial \underline{\alpha}} . \quad (6)$$

It is seen that if

$$\Omega = F [J_2(\underline{\sigma} - \underline{\alpha})] + G [J_2(\underline{\alpha})] \quad (7)$$

and both $r(\underline{\alpha})$ and $\sigma(\underline{\alpha})$ are functions of $J_2(\underline{\alpha})$ only with

$$\frac{r(\underline{\alpha})}{h(\underline{\alpha})} = \frac{dG}{dJ_2(\underline{\alpha})} , \quad (8)$$

then (6) reduces to

$$\dot{\underline{\alpha}} = - h(\underline{\alpha}) \frac{\partial F}{\partial \underline{\alpha}} - h(\underline{\alpha}) \frac{\partial G}{\partial \underline{\alpha}} = h(\underline{\alpha}) \dot{\underline{\epsilon}}^P - r(\underline{\alpha}) \underline{\alpha} . \quad (9)$$

Thus the potential representation (6) assumes a form of (3). The potential representation of $\dot{\underline{\alpha}}$ provides a convenient formulation in numerical computation and the establishment of a variational principle. In the Bailey-Orowan model, tests for fast loading rate and steady-state loading rate have been used to determine $h(\underline{\alpha})$ and $r(\underline{\alpha})$.

YIELD CONDITION AND FLOW RULE

From the existence of the flow potential Ω and an assumed loading function f , the rate of inelastic strain is expressed by

$$\dot{\underline{\epsilon}}^P = \Lambda(\underline{f}, \underline{\alpha}) \frac{\partial \underline{f}}{\partial \underline{\sigma}}, \quad (10)$$

where

$$\Lambda(\underline{f}, \underline{\alpha}) = \frac{\partial \Omega}{\partial \underline{f}}. \quad (11)$$

The assumption of a loading function f implies the normality condition for the inelastic strain rate to the regular regimes of the loading surface $f = \text{constant}$. A convenient form of f which exhibits the property of kinematic hardening may be the J_2 form:

$$f = J_2(\underline{\sigma}' - \underline{\alpha}) = \frac{1}{2}(\underline{\sigma}' - \underline{\alpha})(\underline{\sigma}' - \underline{\alpha}), \quad (12)$$

where J_2 denotes the second invariant.

The yield condition is implicitly implied from the given flow potential and the growth law (3). Eliminating $\dot{\underline{\epsilon}}^P$ from (3) and (10), a differential equation of f is obtained:

$$\left(\frac{\partial \Omega}{\partial \underline{f}}\right)^2 \text{tr} \left(\frac{\partial \underline{f}}{\partial \underline{\sigma}} \frac{\partial \underline{f}}{\partial \underline{\sigma}}\right) = \frac{1}{h^2(\underline{\alpha})} \text{tr} \{[\dot{\underline{\alpha}} + \underline{r}(\underline{\alpha})][\dot{\underline{\alpha}} + \underline{r}(\underline{\alpha})]\} \quad (13)$$

or, in an abbreviated form,

$$\frac{\partial \Omega}{\partial \underline{f}} |\nabla f| = F(\underline{\alpha}, \dot{\underline{\alpha}}), \quad (14)$$

where ∇f is the gradient of f in the stress space and F denotes the square root of the right-hand side of (13). The loading function \underline{f} is

solved from the differential equation to obtain the instantaneous yield condition

$$f(\underline{\sigma}, \underline{\alpha}) = \kappa(\underline{\alpha}, \dot{\underline{\alpha}}) \quad (15)$$

Equation (15) represents the value of the function f in terms of $\underline{\alpha}$ and $\dot{\underline{\alpha}}$ at any instant, or, equivalently, in terms of the instantaneous rate of inelastic strain $\dot{\underline{\epsilon}}^P$. The value of $\underline{\alpha}$, or the instantaneous center of the yield surface, can be solved from (3) as a function of the inelastic strain history, but does not depend explicitly on the loading history. With an applied load of constant magnitude, for example, the inelastic rate $\dot{\underline{\epsilon}}^P$ is counter balanced by the recovery mechanism in the growth equation (3). The state variable $\underline{\alpha}$ reaches a limiting value $\underline{\alpha}_S$ while $\dot{\underline{\alpha}}_S = 0$. The quantity $\underline{\alpha}_S$ and therefore $\kappa(\underline{\alpha}_S)$ are functions of the history of inelastic strain $\underline{\epsilon}^P$. If the flow equation (10) is chosen to be a form of power law^{1,5} with exponent n as a generalized version of the Lagneborg model⁷, then, with more illustration shown later, the time-independent plasticity with kinematic hardening condition corresponds to the case of large exponent n and a vanishingly small recovery term $r(\alpha)$. In this case κ tends to be insensitive to $\dot{\underline{\alpha}}$ owing to the effect of large n as shown in Fig. 1. Furthermore, for an applied load of constant magnitude, the inelastic strain rate is initially very large owing to the large exponent n and rapidly tends to zero. With small value of $r(\alpha)$ in (3) $\underline{\alpha}$ increases to a value as dictated by $\dot{\underline{\epsilon}}^P$. The effect of cyclic hardening, i.e., the increasing κ with respect to ϵ^P , is considered as a secondary effect and is not to be included in the present model. It was also mentioned by Rice¹³ that in the case of time-independent

plasticity the surfaces (15) move rapidly at first then tend to be clustered together in a short period of time to define the commonly observed yield condition, whereas in the case of time-dependent plasticity the surfaces (15) are separated farther from each other. These phenomena are consistent with what we have just described.

The flow rule is derived by the usual procedure in plasticity. It poses a slightly general form owing to the recovery mechanism. The presence of the recovery mechanism destroys the linearity between the rate of stress and the rate of strain and, therefore, results in the time-dependent property of the solid. From the growth equation (3) and the flow equation (10), we obtain

$$\text{tr} \left(\frac{\partial f}{\partial \underline{\sigma}} \dot{\underline{\alpha}} \right) = h(\underline{\alpha}) \Lambda |\nabla f|^2 - \text{tr} \left(\underline{r} \frac{\partial f}{\partial \underline{\sigma}} \right) . \quad (16)$$

Substituting (16) into (10), the flow rule is obtained:

$$\dot{\underline{\sigma}}^P = \frac{\partial f / \partial \underline{\sigma}}{h(\underline{\alpha}) |\nabla f|^2} \left[\text{tr} \left(\frac{\partial f}{\partial \underline{\sigma}} \dot{\underline{\alpha}} \right) + \text{tr} \left(\underline{r} \frac{\partial f}{\partial \underline{\sigma}} \right) \right] . \quad (17)$$

the yield condition (15) implies

$$\text{tr} \left[\frac{\partial f}{\partial \underline{\sigma}} (\dot{\underline{\sigma}} - \dot{\underline{\alpha}}) \right] = \frac{dk}{dt} . \quad (18)$$

Equation (18) can be substituted into the flow rule (17) to obtain a familiar form

The flow rule as described above may be inverted to obtain a form of the stress rate as a function of the rate of total strain. In this form, inelastic strain will not appear. With the deviatoric elastic strain defined by

$$\dot{\underline{\underline{\epsilon}}}'^e = \dot{\underline{\underline{\sigma}}}' / 2G, \quad (20)$$

it is obvious that the total strain rate is the sum of (19) and (20),

$$\dot{\underline{\underline{\epsilon}}}' = \frac{\dot{\underline{\underline{\sigma}}}'}{2G} + \frac{1}{h(\alpha) |\nabla f|^2} \frac{\partial f}{\partial \underline{\underline{\sigma}}} \left\{ \text{tr} \left[\frac{\partial f}{\partial \underline{\underline{\sigma}}} (\dot{\underline{\underline{\sigma}}}' + \underline{\underline{r}}) \right] - \frac{dk}{dt} \right\}, \quad (21)$$

where $\dot{\underline{\underline{\epsilon}}}'$ is the rate of total deviatoric strain, referring to the sum of the elastic and the inelastic strains, and $\underline{\underline{r}} = \underline{\underline{r}}(\alpha)\underline{\underline{\alpha}}$ if $f = J_2$. Multiplying by $\partial f / \partial \underline{\underline{\sigma}}$ and taking the trace of the product, we obtain

$$\text{tr} \left(\frac{\partial f}{\partial \underline{\underline{\sigma}}} \dot{\underline{\underline{\epsilon}}}' \right) = \frac{1}{2G} \text{tr} \left(\frac{\partial f}{\partial \underline{\underline{\sigma}}} \dot{\underline{\underline{\sigma}}}' \right) + \frac{1}{h(\alpha)} \left\{ \text{tr} \left[\frac{\partial f}{\partial \underline{\underline{\sigma}}} (\dot{\underline{\underline{\sigma}}}' + \underline{\underline{r}}) \right] - \frac{dk}{dt} \right\}. \quad (22)$$

The substitution of this equation into (21) results in

$$\begin{aligned} \frac{\dot{\underline{\underline{\sigma}}}'}{2G} = \dot{\underline{\underline{\epsilon}}}' - \frac{\partial f}{\partial \underline{\underline{\sigma}}} \frac{2G}{(2G + h) |\nabla f|^2} \text{tr} \left(\frac{\partial f}{\partial \underline{\underline{\sigma}}} \dot{\underline{\underline{\epsilon}}}' \right) \\ - \frac{\partial f}{\partial \underline{\underline{\sigma}}} \frac{1}{(2G + h) |\nabla f|^2} \left[\text{tr} \left(\frac{\partial f}{\partial \underline{\underline{\sigma}}} \underline{\underline{r}} \right) - \frac{dk}{dt} \right]. \end{aligned} \quad (23)$$

Equation (23) is of the form

$$\underline{\dot{\sigma}} = \underline{L} \underline{\dot{\epsilon}} - \underline{M}, \quad (24)$$

where \underline{L} is independent of the stress rate and the strain rate and provides the linear relation between them. In (24), \underline{M} represents the effects of strain recovery and rate-dependent yield which results in a time-dependent structure for the constitutive equation. \underline{L} and \underline{M} can be represented in index forms as

$$L_{ijkl} = 2G (\delta_{ik} \delta_{jl} - \frac{2G}{h + 2G} \frac{1}{|\nabla f|^2} \frac{\partial f}{\partial \sigma_{ij}} \frac{\partial f}{\partial \sigma_{kl}}) \quad (25)$$

$$M_{ij} = \frac{2G}{h + 2G} \frac{1}{|\nabla f|^2} \frac{\partial f}{\partial \sigma_{ij}} \left(\frac{\partial f}{\partial \sigma_{kl}} r_{kc} - \frac{dk}{dt} \right). \quad (26)$$

From $\underline{M} = 0$, (24) reduces to a form similar to that of the time-independent case with an isotropic yield condition^{14,15}.

UNIAXIAL MODEL

Qualitative discussions on the effect of temperature variation on the multiaxial model shown above are based on several temperature sensitive parameters. We have tacitly assumed that (a) the function $\kappa(\underline{\alpha}, \underline{\dot{\alpha}})$ in the yield condition (15) varies with respect to the temperature and tends to be insensitive to $\underline{\alpha}$ and $\underline{\dot{\alpha}}$ as the temperature decreases to the room-temperature range. (b) the effect of the recovery mechanism $M(\underline{\alpha})$ decreases with decreasing temperature and becomes negligible as the temperature reaches the room temperature range, and (c) the characteristic time which defines an average rate for the transient inelastic strain to saturate at room

temperature is assumed to be short compared with the time for each loading step and the steady-state creep rate in this temperature range is negligibly small. With these assumptions, we have shown how the flow law and the growth law can be reduced to time-independent limits. In this section we present known creep models in the uniaxial case and illustrate how the rate-independent yield is reached and how to define the characteristic time for the inelastic strain saturation.

The Lagneborg model⁷, or Bailey-Orowan model¹², is represented by the flow law:

$$\dot{\epsilon}^P = a \exp\left(-\frac{H}{kT}\right) (\sigma - \alpha)^{b/kT} \quad (27)$$

and the growth law

$$\dot{\alpha} = h(\alpha) \dot{\epsilon}^P - r(\alpha)\alpha, \quad (28)$$

where T is the absolute temperature and the parameters $h(\alpha)$ and $r(\alpha)$ are temperature dependent. It has been mentioned by Lagneborg that $r(\alpha)$ decreases with decreasing temperature but the coefficients a , b , k , and H are insensitive to temperature. After simple calculations, the flow law (27) can be written in a slightly different form:

$$\dot{\epsilon}^P = a \left(\frac{\sigma - \alpha}{\sigma_y}\right)^n, \quad (29)$$

where

$$\sigma_y = \exp(H/b) \quad (30)$$

and

$$n = b/kT . \quad (31)$$

Therefore, only n is temperature sensitive and σ_y is a constant. It has been observed by Blum¹⁶ that at high temperature n tends to 4 and at low temperature n can be as large as 50 from his measurement of several metals.

Equation (29) can be written as

$$\frac{\sigma - \alpha}{\sigma_y} = \left(\frac{\dot{\epsilon}}{a} \right)^{1/n} . \quad (32)$$

For large n , the right-hand side of (32) tends to be independent of the strain rate and reaches one in the limit. With a vanishing recovery term the growth law (28) assumes

$$\dot{\alpha} = h \dot{\epsilon}^p , \quad (33)$$

which is the kinematic hardening condition of classical plasticity theory.

The variation of the yield condition (32) with respect to n is shown schematically in Fig. 1. First, we assume that n is large. The condition

$$\frac{\sigma - \alpha}{\sigma_y} < 1 \quad (34)$$

leads to $\dot{\epsilon}^p \ll 1$ and reduces approximately to elastic range. The condition

$$\frac{\sigma - \alpha}{\sigma_y} > 1 \quad (35)$$

leads to plastic flow. At moderate value of n , the tests¹ shown in Fig. 2 indicate during periods of unloading a fast reduction of the inelastic strain rate which is followed by a constant $\dot{\epsilon}^P$ provided that the holding time is sufficiently long. This phenomenon can be explained by the inequality (34) which implies a rapid decrease of $\dot{\epsilon}^P$ even with a moderate exponent n . The yield condition (32) can be reduced to

$$\left(\frac{\sigma - \alpha}{\sigma_y}\right)^n = \left(\frac{\dot{\epsilon}^P}{\dot{\epsilon}^0}\right) = \frac{1}{ah(\alpha)} [\dot{\alpha} + r(\alpha)\alpha] \quad (36)$$

This form is consistent with the multiaxial case shown in (15).

Owing to the uncertainty of using Lagneborg's model to deal with the room temperature case, we shall present a flow equation as proposed by Hahn⁸. It is encouraging to note that it only differs slightly from that of Lagneborg. The flow law of Hahn was based on the idea of Johnston and Gilman¹⁷ for their treatment of LIF by the theory of dislocation dynamics. Hahn applied this idea to mild steel at room temperature to obtain

$$\dot{\epsilon}^P = A [\rho_0 + c (\epsilon^P)^a] (2\tau_0^{-n})(\sigma - q\epsilon^P)^n, \quad (37)$$

where A , c , a , τ_0 , and n are constants. ρ_0 is the initial dislocation density and q is a constant work-hardening coefficient. The difference between (37) and the Lagneborg model only appears in the constant a and the terms in the square brackets which account for the phenomenon of the

delay time for the initiation of the plastic strain. As mentioned by Hahn, a rearrangement of (37) leads to

$$\sigma - q\epsilon^p = 2\tau_0 \left\{ \frac{\dot{\epsilon}^p}{A [\rho_0 + c(\epsilon^p)^a]} \right\}^{1/n} \quad (38)$$

The exponent n was measured as 35 for the iron at 298°K^{17} . While we are not in a position to assess which equation is more appropriate to describe the plastic flow of FBR structural alloys, it is interesting to note that Hahn has also made use of the effect of n to explain the phenomenon of yield.

It has been mentioned by Rice that the time-independent limit corresponds to a slow loading rate at room-temperature range. Again slow loading rate is to be compared with the average strain rate for the inelastic strain to saturate. A way to define such a characteristic time, which is required for the inelastic strain to reach its saturated value, is illustrated here. We chose the cases of a constant h and a variable h in the following discussion.

Based on (29) and (33), we can estimate the amount of $\dot{\epsilon}^p$ drop from an increase of back stress α or, equivalently, ϵ^p if we choose a constant h . Differentiating (29) with respect to α , with the assumption of a large n , we obtain approximately

$$\frac{d\dot{\epsilon}^p}{d\alpha} = na \left(\frac{\sigma - \alpha}{\sigma_y} \right)^{n-1} \frac{1}{\sigma_y} = \frac{n\dot{\epsilon}^p}{\sigma_y} \quad (39)$$

which, after integration, gives

$$\dot{\epsilon}^p = \dot{\epsilon}_0^p \exp \left[- \frac{n}{\sigma_y} (\alpha - \alpha_0) \right] \quad (40)$$

where $\dot{\epsilon}_0^P$ is the strain rate for $\alpha = \alpha_0$.

For the case of $\alpha_0 = 0$, we shall use the above equation to define the characteristic strain ϵ_r^P at which $\dot{\epsilon}_0^P$ reduces to $\dot{\epsilon}_0^P/e$, or roughly 1/3 of the original rate. Obviously, for this case the terms in the square brackets of (40) is minus one which, after substituting it into (33), gives

$$\epsilon_r^P = \frac{\alpha}{h} = \frac{\sigma_y}{nh} \quad (41)$$

It is noted that ϵ_r^P depends only on mechanical properties of the material. The relaxation of the inelastic strain rate $\dot{\epsilon}^P$ follows the solid curve in Fig. 3. Integrating (39) we obtain the characteristic time t_r required to reduce $\dot{\epsilon}_0^P$ to $\dot{\epsilon}_0^P/e$,

$$\begin{aligned} t_r &= \frac{1}{\dot{\epsilon}_0^P} \frac{\sigma_y}{nh} \int_0^1 \exp\left(\frac{nh}{\sigma_y} \epsilon^P\right) d\left(\frac{nh}{\sigma_y} \epsilon^P\right) \\ &= \frac{\epsilon_r^P}{\dot{\epsilon}_0^P} (e - 1) = \frac{e - 1}{\dot{\epsilon}_0^P} \frac{\sigma_y}{nh} = \frac{1}{\sigma^n} \end{aligned} \quad (42)$$

which varies inversely with respect to σ^n .

Suppose that we use a variable $h(\alpha) = c/\alpha$ as suggested in a previous report¹ then the growth relation, instead of (33), is

$$\dot{\alpha} = c\dot{\epsilon}^P \quad (43)$$

Equation (43) can be integrated to obtain, instead of (41),

$$\dot{\epsilon}_r^P = \frac{1}{2C} \alpha^2 = \frac{1}{2C} \left(\frac{\sigma_y}{n} \right)^2, \quad (44)$$

whereas (40) and (43) imply the equation

$$\frac{1}{C} \alpha \frac{d\alpha}{dt} = \dot{\epsilon}_o^P \exp \left(- \frac{n}{\sigma_y} \alpha \right). \quad (45)$$

It can be integrated to obtain

$$\dot{\epsilon}_o^P t = \frac{1}{C} \left(\frac{\sigma_y}{n} \right)^2 [ye^y - (e^y - 1)], \quad (46)$$

where, from (43),

$$y = \frac{n}{\sigma_y} \alpha = \frac{n \sqrt{2C}}{\sigma_y} \sqrt{\dot{\epsilon}^P}. \quad (47)$$

The time required to reduce $\dot{\epsilon}_o^P$ to $\dot{\epsilon}_o^P/e$, $y = 1$ in (47), is

$$t_r = \frac{1}{\dot{\epsilon}_o^P} \frac{1}{C} \left(\frac{\sigma_y}{n} \right)^2, \quad (48)$$

where t_r is proportional to $1/\sigma^n$ as well. It is noted that α at the value of $y = 1$ is

$$\alpha_r = \frac{\sigma_y}{n}. \quad (49)$$

The two values of t_r 's from (42) and (48) will be identical if in (42) we choose

$$t_1 = \frac{C(e-1)}{\dot{\epsilon}_r} \quad (50)$$

It is interpreted as an "average" value of h instead of a variable α in $h = c/\alpha$.

FURTHER MULTIAXIAL CONSIDERATIONS

For multiaxial conditions, we may define a quantity I_2 by

$$I_2 = \frac{1}{2} \underline{\underline{\dot{\epsilon}}}^P \cdot \underline{\underline{\dot{\epsilon}}}^P \quad (51)$$

which is related to F in (7) by

$$I_2 = \left(\frac{dF}{dJ_2} \right) J_2 \quad (52)$$

where J_2 is defined in (12). The right-hand side of (52) is a function of $J_2(\underline{\underline{\sigma}}' - \underline{\underline{\alpha}})$ only and denoted by a function H ,

$$I_2 = H [J_2(\underline{\underline{\sigma}}' - \underline{\underline{\alpha}})] \quad (53)$$

The above equation shows that the second invariant of the inelastic strain rate is constant along surfaces of constant $J_2(\underline{\underline{\sigma}}' - \underline{\underline{\alpha}})$ values. From the potential representation of $\underline{\underline{\dot{\epsilon}}}^P$, the direction of $\underline{\underline{\dot{\epsilon}}}^P$ vector is perpendicular to the surfaces $J_2(\underline{\underline{\sigma}}' - \underline{\underline{\alpha}}) = \text{constants}$. The material property of the solid being time-independent or time-dependent will be reflected in the equation $I_2 = H$. For a time-independent solid with a given $\underline{\underline{\sigma}}'$, I_2 should eventually approach zero after the transient period is passed under constant σ . We may, therefore, use this equation to define the characteristic time of the solid similar to the one-dimensional case. For a sufficiently large work-hardening coefficient n , the following relation is approximately true:

$$\frac{dI_2}{dJ_2(\underline{\sigma}' - \underline{\alpha})} = H' [J_2(\underline{\sigma}' - \underline{\alpha})] \approx AI_2, \quad (54)$$

where F is assumed to be an exponential form and A is a constant. Simple integration of (54) leads to

$$I_2 = (I_2)_0 \exp \{A[J_2(\underline{\sigma}' - \underline{\alpha}) - J_2(\underline{\sigma}')] \}, \quad (55)$$

where $(I_2)_0$ is the initial value of I_2 at $J_2(\underline{\sigma}')$. Hence, a characteristic time of the plastic strain saturation is defined as the period of time required for $(I_2)_0$ to be reduced to $(I_2)_0/e$. It corresponds to a value of $\underline{\alpha}$ to be determined from the equation

$$J_2(\underline{\sigma}' - \underline{\alpha}) = J_2(\underline{\sigma}') - \frac{1}{A}. \quad (56)$$

A modification of the growth relation has the form

$$(\underline{\sigma}' - \underline{\alpha}) \frac{d(\underline{\sigma}' - \underline{\alpha})}{dt} = -h \frac{\partial F}{(\partial \underline{\sigma}' - \underline{\alpha})} (\underline{\sigma}' - \underline{\alpha}) \quad (57)$$

or

$$\frac{dJ_2(\underline{\sigma}' - \underline{\alpha})}{dt} = -2h \frac{dF}{dJ_2} J_2. \quad (58)$$

Hence, the characteristic time for the time-independent solid is

$$\tau_r = - \int_{J_2(\underline{\sigma}')}^{J_2(\underline{\sigma}' - \underline{\alpha})} \frac{dJ_2}{2hF' J_2}. \quad (59)$$

In the above equation, h is assumed to be a constant and F' is a function of $J_2(\underline{\sigma}' - \underline{\alpha})$. The integration can be evaluated readily. If h is a function of the second invariant of $\underline{\alpha}$, then we may change the integration variable to $\underline{\alpha}$ and evaluate the integration.

SUMMARY

A time-independent limit of a creep-recovery constitutive equation is established and shown to be the classical theory of plasticity model with kinematic hardening condition if certain temperature sensitive parameters in this creep-recovery model are appropriately adjusted. Thus, the creep-recovery model which has been initially developed to describe the inelastic deformation for reactor alloys at high temperature can be used to represent the time-independent plasticity model at low temperature with the same set of temperature sensitive parameters. The method of numerical computations which ultimately would use this creep-recovery model can, therefore, include both creep and plastic deformations while no longer treating them separately. Since this model is based, in part, on models of physical mechanisms of metal deformation, it reflects naturally a unified description of both time-dependent and time-independent plastic deformations. The time-independent theory is taken to be an asymptotic approximation of the creep process in the room-temperature range. It is consistent with experimental observations. In the process of reaching the asymptotic approximation, we established a quantitative measure of the time required for the saturation of the inelastic strain. It, therefore provides a measure of the "minimum

time" which is required so that the classical plasticity can be used as a valid approximation to describe a plastic deformation.

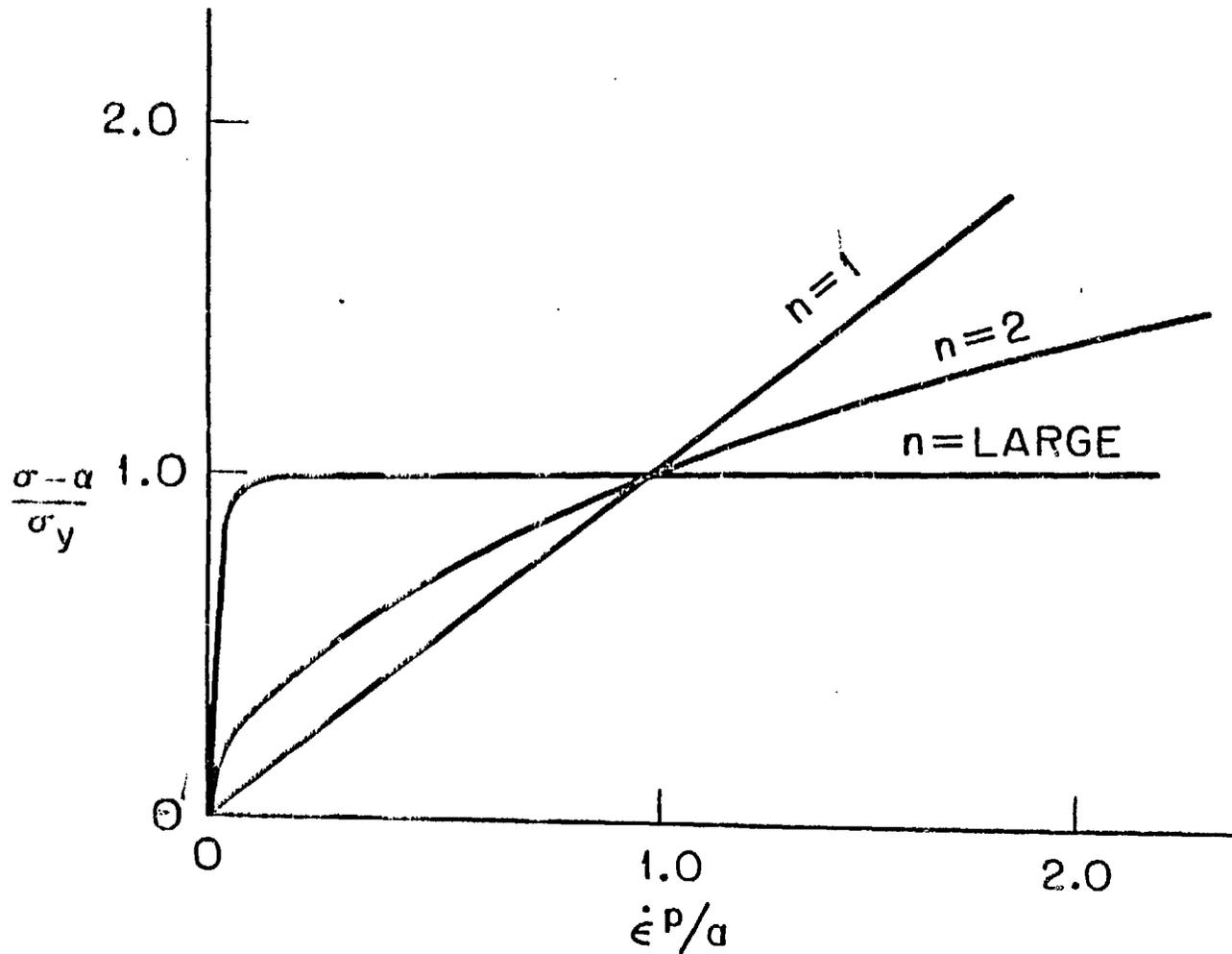


Fig. 1 Variation of the yield condition

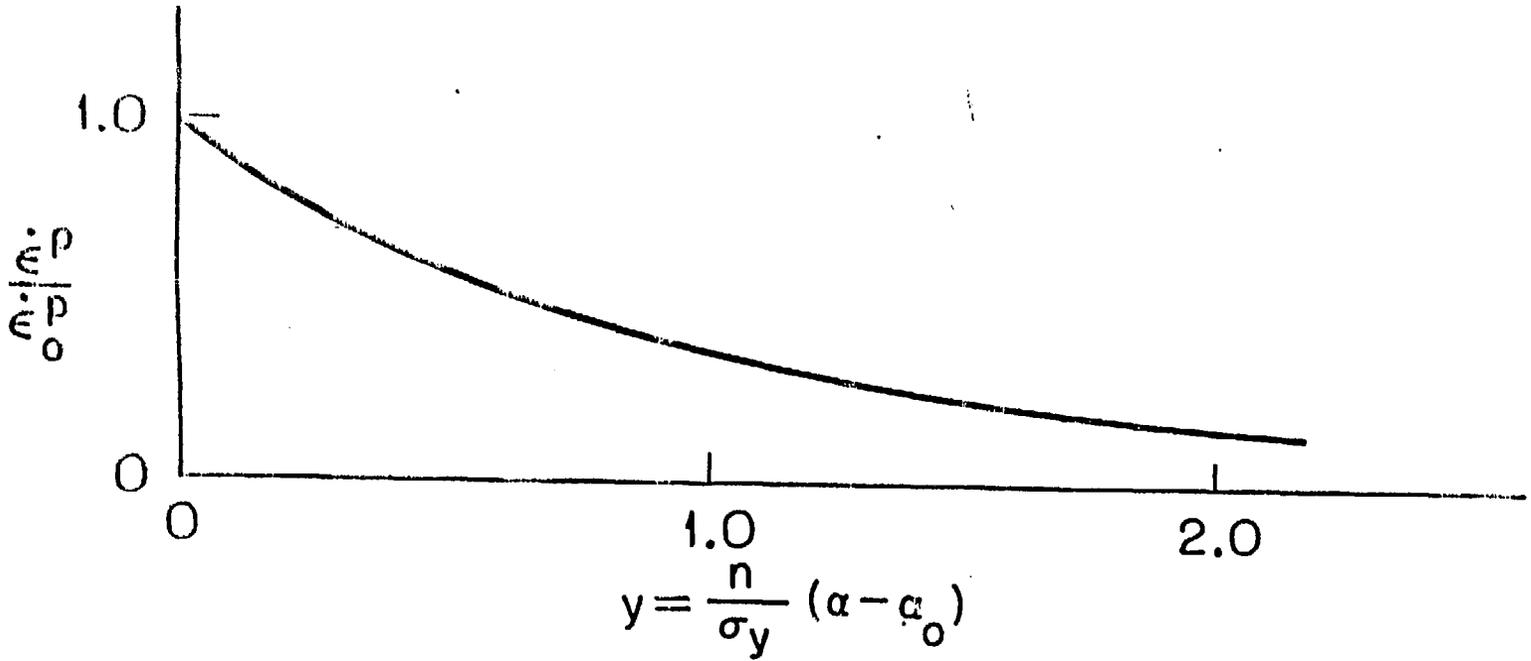


Fig. 3 Decrease of the rate of inelastic strain

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