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QUANTUM FLUCTUATIONS AND SPONTANEOUS COMPACTIFICATION OF ELEVEN-DIMENSIONAL GRAVITY

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1. Introduction

The promising success of the $N = 8$ extended supergravity[1-4] derived from the $N = 1$ supergravity in eleven dimensions by means of the dimensional reduction in explaining many observed experimental data concerning the quark-lepton spectrum and its symmetry properties has revived the old idea of Kaluza[5] and Klein[6] that a unified theory of the gravitation and other matter fields in four dimensions might be deduced from a gravity (or supergravity) theory in higher dimensions. In many models of the Kaluza-Klein type the extra dimensions are spontaneously compactified[7,8]. However, in the existing eleven-dimensional supergravity theory[1-4] the spontaneous compactification takes place at the classical level if and only if the four-dimensional space-time is an anti-de Sitter one. To deduce a field theory in the four-dimensional Minkowski space-time from the eleven-dimensional supergravity (or gravity) by means of the spontaneous compactification of the extra dimensions it is necessary either to introduce the scalar fields (at the classical level) or to include the contribution of the quantum fluctuations.

The dynamical quantum effect in the Kaluza-Klein theories was studied in many works. In particular, Candelas and Weinberg[9] has proposed a method for calculating the contribution of the quantum fluctuations of the scalar fields to the curvatures of the four-dimensional space-time as well as of the spontaneously compactified space of the internal symmetry. Chodos and Myers[10] noted that **the quantum fluctuations of the multidimensional gravitation** symmetric second rank tensor field themselves must also contribute to the curvatures. Generalizing the reasonings of Candelas and Weinberg they have calculated the effective potential of this field in the one-loop approximation. The one-loop effective action of the gravitation field in higher dimensions was also studied by Randjbar-Daemi and Sarmadi [11,12]. However, the contribution of the quantum fluctuations of the multidimensional gravitation field to the curvatures was not considered. The study of this problem is carried out in the present work which would be the necessary beginning of the investigation of the spontaneous compactification of the eleven-dimensional quantum supergravity. We show that in the eleven-dimensional pure gravity theory with the spontaneous compactification to the direct product $M_4 \times S_7$ of the Minkowski space-time M_4 and the seven sphere S_7 the cosmological constant might be chosen in such a way that the generalized Einstein equations including the contribution of the quantum fluctuations are satisfied on the one hand, and there exist no tachyons on the other hand. For the simplicity we work in the light-cone gauge which has been used earlier by Randjbar-Daemi, Salam and Strathdee[13]. The problem of the gauge dependence of the effective action will be also considered in the sequel. We use the unit system with $\hbar = c = 1$.

2. Basic equations

We consider the second rank symmetric tensor field G_{AB} of the eleven-dimensional pure gravity theory, where A, B, ... label the coordinates in eleven dimensions, and assume the following action

$$S[G] = -\frac{1}{\varkappa^2}\int d^4x \sqrt{|\det G|}\,\left\{ R(G) + \lambda \right\} . \tag{1}$$

The appearance of the cosmological constant λ might be a consequence of the renormalization, as it was noted by Randjbar-Daemi, Salam and Strathdee /13/. Candelas and Weinberg /9/ showed that it is necessary to introduce this cosmological constant in order to satisfy the Einstein equation. In our notations

$$R(G) = G^{AB} R_{AB}(G) = G^{AB} R^{C}{}_{ABC}(G),$$

$$R^{D}{}_{ABC}(G) = \partial_B \Gamma^D_{AC} - \partial_C \Gamma^D_{AB} + \Gamma^E_{AC}\Gamma^D_{EB} - \Gamma^E_{AB}\Gamma^D_{EC},$$

$$\Gamma^C_{AB} = \frac{1}{2} G^{CD}\left(\partial_A G_{BD} + \partial_B G_{AD} - \partial_D G_{AB} \right),$$

$$\det G = \det G_{AB} .$$

For studying the quantum fluctuations we split G_{AB} into two parts: the background classical gravitation field g_{AB} and the fluctuating part $\varkappa h_{AB}$

$$G_{AB} = g_{AB} + \varkappa h_{AB} . \tag{2}$$

The action $S_{qu}[g, h]$ and the Lagrangian $\mathcal{L}(g, h)$ of the fluctuating field h_{AB} in the background g_{AB} are defined in the following manner

$$S[G] = S[g] + S_{qu}[g, h], \tag{3}$$

$$S_{qu}[g, h] = \int d^4x \sqrt{|\det g|}\; \mathcal{L}(g, h) . \tag{4}$$

Chodos and Myers[10] noted that in general

$$\frac{\delta S[g]}{\delta g^{AB}} \neq 0 ,$$

i.e., the background field g_{AB} might not be the solution of the classical field equation. Therefore the Lagrangian $\mathcal{L}(g, h)$ must contain the terms of the first order in the fluctuating field h_{AB} subjected to the quantization. The terms of the second order in h_{AB} are known and may be found in Ref.[13] In the calculation of the effective potential the first order terms play no role, but in the study of the contribution of the quantum fluctuations to the curvatures they are essential and must be retained, as we shall see in the sequel.

In the one-loop approximation the effective potential of the background field g_{AB} equals

$$S_{eff}[g] = S[g] + \Sigma[g] , \tag{5}$$

$$\Sigma[g] = -i \ln \int [dh]\, e^{i S_{qu}[g, h]} , \tag{6}$$

where

$$\int [dh]$$

denotes the path integral over the quantum field h_{AB} in eleven dimensions. The generalized Einstein equation including the contribution of the quantum fluctuations is the field equation derived from the effective action $S_{eff}[g]$:

$$\frac{\delta S_{eff}[g]}{\delta g^{AB}} = 0 . \tag{7}$$

It may be written in the form

$$-\frac{1}{\varkappa^2}\left[R_{AB}-\frac{1}{2}g_{AB}\left(R+\lambda\right)\right]+\frac{\delta\Sigma[g]}{\delta g^{AB}}=0, \tag{8}$$

$$R_{AB}=R_{AB}(g), \quad R=R(g).$$

The last term in the r.h.s. of Eq.(8) is the vacuum expectation value of the functional derivative of the quantum action $S_{qu}[g,h]$

$$\frac{\delta\Sigma[g]}{\delta g^{AB}}=\left\langle \frac{\delta S_{qu}[g,h]}{\delta g^{AB}}\right\rangle, \tag{9}$$

$$\langle\{\cdots\}\rangle=\frac{\int[dh]\{\cdots\}e^{iS_{qu}[g,h]}}{\int[dh]\,e^{iS_{qu}[g,h]}}. \tag{10}$$

We consider the case when the eleven-dimensional space is spontaneously compactified into the direct product $M_4\times S_7$ of the Minkowski space-time M_4 and the seven sphere S_7, and denote the metric tensors in M_4 and S_7 by g^{ab} and $g^{\alpha\beta}$, resp. The indices a, b ... label the coordinates of the Minkowski space-time M_4 and the indices $\alpha,\beta\ldots$ label those of the seven sphere S_7. From the generalized Einstein equations (8) in S_7 and M_4, resp., we obtain two following relations

$$\frac{5R+7\lambda}{2\varkappa^2}+g^{\alpha\beta}\frac{\delta\Sigma[g]}{\delta g^{\alpha\beta}}=0, \tag{11}$$

$$\frac{2R+2\lambda}{\varkappa^2}+g^{ab}\frac{\delta\Sigma[g]}{\delta g^{ab}}=0. \tag{12}$$

Instead of the last terms in Eqs. (11) and (12) it is convenient for our purpose to introduce their integrals over the space $M_4 \times S_7$

$$\sigma^{(7)}[g] = \int d^{11}x \sqrt{|\det g|}\; g^{\alpha\beta} \frac{\delta \Sigma[g]}{\delta g^{\alpha\beta}} , \tag{13}$$

$$\sigma^{(4)}[g] = \int d^{11}x \sqrt{|\det g|}\; g^{ab} \frac{\delta \Sigma[g]}{\delta g^{ab}} . \tag{14}$$

From the definition (4) it follows that the functional derivative of the quantum action may be written in the form

$$\frac{\delta S_{qu}[g,h]}{\delta g^{AB}} = -\frac{1}{2} g_{AB} \mathcal{L}(g,h) + M_{AB}(g,h). \tag{15}$$

Therefore we have

$$\sigma^{(7)}[g] = -\frac{7}{2} \langle S_{qu}[g,h] \rangle + \int d^{11}x \sqrt{|\det g|} \cdot \langle g^{\alpha\beta} M_{\alpha\beta}(g,h) \rangle , \tag{16}$$

$$\sigma^{(4)}[g] = -2 \langle S_{qu}[g,h] \rangle + \int d^{11}x \sqrt{|\det g|} \cdot \langle g^{ab} M_{ab}(g,h) \rangle . \tag{17}$$

The expressions of the last terms in the r. h.s. of Eqs. (16) and (17) will be given in the next section. We note that if $\mathcal{L}(g,h)$ contains only the terms of the second order of the quantized field h_{AB} then the expectation value $\langle S_{qu}[g,h] \rangle$ would equal zero. However due to the presence of the terms of the first order of the field h_{AB} in $\mathcal{L}(g,h)$ this expectation value does not vanish. It will be also calculated in the next section.

3. Light-cone gauge

For the simplicity we work in the light-cone gauge, since in this gauge the unphysical modes and the Fadeev-Popov ghosts do not arise. The second order terms of the Lagrangian $\mathcal{L}(g,h)$ have been calculated in this gauge by Randjbar-Daemi, Salam and Strathdee[13]. Remember that in the seven sphere S_7 of the radius a

$$R_{\alpha\beta\gamma\delta} = -\frac{1}{a^2}\left(g_{\alpha\delta}\, g_{\beta\gamma} - g_{\alpha\gamma}\, g_{\beta\delta}\right).$$

Consider the four-dimensional Lagrangian

$$L(h) = \int_{S_7} d^7x \sqrt{|\det g_{\alpha\beta}|}\; \mathcal{L}(g,h). \tag{18}$$

In the notations of Ref.[13] we have

$$L(h) = \sqrt{\Omega_7}\,\frac{6}{\varkappa a^2}\, h^{(0)}_{ii} + L^{(2)}(h), \tag{19}$$

$$\begin{aligned}
L^{(2)}(h) = \sum_{n=0}^{\infty} \Big\{ & \frac{1}{4} h^{(n)}_{ij}\left[\partial^2 - \frac{n(n+6)+42-\alpha}{a^2}\right] h^{(n)}_{ij} \\
& + \frac{9}{56} h^{(n)}_{ii}\left[\partial^2 - \frac{n(n+6)+110/3-\alpha}{a^2}\right] h^{(n)}_{ii} \\
+ \sum_{n=1}^{\infty} \Big\{ & \frac{1}{2} h^{t(n)}_{i}\left[\partial^2 - \frac{n(n+6)+35-\alpha}{a^2}\right] h^{t(n)}_{i} \\
& + \frac{1}{2} H^{(n)}_{i}\left[\partial^2 - \frac{n(n+6)+30-\alpha}{a^2}\right] H^{(n)}_{i} \Big\} \\
+ \sum_{n=2}^{\infty} \Big\{ & \frac{1}{4} h^{tt(n)}\left[\partial^2 - \frac{n(n+6)+30-\alpha}{a^2}\right] h^{tt(n)}
\end{aligned} \tag{20}$$

$$+\frac{1}{4}H^{t(n)}\left[\gamma^2-\frac{n(n+6)+23-\alpha}{a^2}\right]H^{t(n)} \tag{20}$$

$$+\frac{1}{4}H^{(n)}\left[\gamma^2-\frac{n(n+6)+18-\alpha}{a^2}\right]H^{(n)}\Big\},$$

where the fields $h^{(n)}_{ij}$, $h^{(n)}_{ii}$, ... $H^{(n)}$ in the Minkowski space-time are the coefficients of the Fourier expansions of the corresponding fields h_{ij}, h_{ii}, ... H in the eleven-dimensional space in terms of the spherical harmonic in seven dimensions, Ω_7 is the volume of the compact seven sphere, and

$$\alpha = \lambda a^2 .$$

From the expression (20) it follows that there will be no tachyons if $\alpha \leqslant 34$.

In order to carry out the calculation of the path integrals it is necessary to avoid the first order terms in the r.h.s. of Eq. (19) by setting

$$\tilde{h}^{(0)}_{ii} = h^{(0)}_{ii} - \sqrt{\Omega_7}\,\frac{56}{\varkappa(110-3\alpha)} . \tag{21}$$

We have then

$$L(h) = \Omega_7 \frac{1}{a^2\varkappa^2}\frac{168}{110-3\alpha} + \tilde{L}^{(2)}(h), \tag{22}$$

where $\tilde{L}^{(2)}(h)$ is obtained from the r.h.s. of Eq.(20) after the substitution

$$h^{(0)}_{ii} \rightarrow \tilde{h}^{(0)}_{ii} .$$

In general we can write

$$\left\langle \int d^4x\, \tilde{L}^{(2)}(h) \right\rangle = Z \int d^{11}x \sqrt{|\det g|}\ .$$

The expression of Z will be given in the next section. Therefore we have

$$\langle S_{qu}[g,h] \rangle = \left\langle \int d^4x\, L(h) \right\rangle =$$

$$= \left\{ \frac{1}{a^2 x^2} \cdot \frac{168}{110-3\alpha} + Z \right\} \int d^{11}x \sqrt{|\det g|}\ . \tag{23}$$

Note that if we omit the first order term in the r.h.s. of Eq. (19) then we shall loss the first term in the r.h.s. of Eq. (23). This means that in our study it is essential to retain the first order term in the Lagrangian.

In the second order approximation with respect to the field h_{AB} the expression of the operator $M_{AB}(g,h)$ in the r.h.s. of Eq. (15) contains both the first and second order terms. It is straightforward to derive this expression and then to calculate the last terms in the r.h.s. of Eqs. (16) and (17) in the light-cone gauge. We obtain following results:

$$\int d^{11}x \sqrt{|\det g|} \left\langle g^{\alpha\beta} M_{\alpha\beta}(g,h) \right\rangle = \tag{24}$$

$$= \int d^4x \left\{ -\sqrt{\Omega_7}\, \frac{18-\alpha}{2x^2 a} \langle h_{ii}^{(0)} \rangle + \left\langle \frac{\partial R(h,\xi)}{\partial \xi} \right\rangle_{\xi=1} \right\},$$

$$\int d^{11}x \sqrt{|\det g|} \left\langle g^{ab} M_{ab}(g,h) \right\rangle =$$

$$= \int d^4x \left\{ \sqrt{\Omega_7}\, \frac{42-\alpha}{2x^2 a} \langle h_{ii}^{(0)} \rangle + \left\langle \frac{\partial Q(h,\xi)}{\partial \xi} \right\rangle_{\xi=1} \right\}, \tag{25}$$

$$R(h,\xi) = \sqrt{\Omega_7}\,\frac{6}{\varkappa a^2}\, h_{ii}^{(0)} + R^{(2)}(h,\xi), \tag{26}$$

$$Q(h,\xi) = \sqrt{\Omega_7}\,\frac{6}{\varkappa a^2}\, h_{ii}^{(0)} + Q^{(2)}(h,\xi), \tag{27}$$

$$\begin{aligned}
R^{(2)}(h,\xi) = \sum_{n=0}^{\infty} \Big\{ & \frac{1}{4} h_{ij}^{(n)} \Big[\partial^2 - \frac{\xi\,[n(n+6)+42]-\alpha}{a^2} \Big] h_{ij}^{(n)} \\
& + \frac{7+2\xi^2}{56} h_{ii}^{(n)} \Big[\partial^2 - \frac{\xi\,[n(n+6)+30 f(\xi)]-\alpha}{a^2} \Big] h_{ii}^{(n)} \Big\} \\
+ \sum_{n=1}^{\infty} \Big\{ & \frac{\xi}{2} h_i^{t(n)} \Big[\partial^2 - \frac{\xi\,[n(n+6)+35]-\alpha}{a^2} \Big] h_i^{t(n)} \\
& + \frac{\xi}{2} H_i^{(n)} \Big[\partial^2 - \frac{\xi\,[n(n+6)+30]-\alpha}{a^2} \Big] H_i^{(n)} \Big\} \\
+ \sum_{n=2}^{\infty} \Big\{ & \frac{\xi^2}{4} h^{tt(n)} \Big[\partial^2 - \frac{\xi\,[n(n+6)+30]-\alpha}{a^2} \Big] h^{tt(n)} \\
& + \frac{\xi^2}{4} H^{t(n)} \Big[\partial^2 - \frac{\xi\,[n(n+6)+23]-\alpha}{a^2} \Big] H^{t(n)} \\
& + \frac{\xi^2}{4} H^{(n)} \Big[\partial^2 - \frac{\xi\,[n(n+6)+18]-\alpha}{a^2} \Big] H^{(n)} \Big\},
\end{aligned} \tag{28}$$

$$f(\xi) = \frac{4\xi^2+7}{2\xi^2+7} \,, \tag{29}$$

$$\begin{aligned} Q^{(2)}(h,\xi) = \sum_{n=0}^{\infty} \Big\{ & \frac{\xi^3}{4} h_{ij}^{(n)} \Big[\partial^2 - \frac{n(n+6)+42-\alpha}{\xi a^2} \Big] h_{ij}^{(n)} \\ & + \frac{7\xi^3+2\xi}{56} h_{ii}^{(n)} \Big[\partial^2 - \frac{n(n+6)+30\varphi(\xi)-\alpha}{\xi a^2} \Big] h_{ii}^{(n)} \Big\} \\ + \sum_{n=1}^{\infty} \Big\{ & \frac{\xi^2}{2} h_i^{t(n)} \Big[\partial^2 - \frac{n(n+6)+35-\alpha}{\xi a^2} \Big] h_i^{t(n)} \\ & + \frac{\xi^2}{2} H_i^{(n)} \Big[\partial^2 - \frac{n(n+6)+30-\alpha}{\xi a^2} \Big] H_i^{(n)} \Big\} \\ + \sum_{n=2}^{\infty} \Big\{ & \frac{\xi}{4} h^{tt(n)} \Big[\partial^2 - \frac{n(n+6)+30-\alpha}{\xi a^2} \Big] h^{tt(n)} \\ & + \frac{\xi}{4} H^{t(n)} \Big[\partial^2 - \frac{n(n+6)+23-\alpha}{\xi a^2} \Big] H^{t(n)} \\ & + \frac{\xi}{4} H^{(n)} \Big[\partial^2 - \frac{n(n+6)+18-\alpha}{\xi a^2} \Big] H^{(n)} \Big\} \,, \end{aligned} \tag{30}$$

$$\varphi(\xi) = \frac{4+7\xi^2}{2+7\xi^2} \,. \tag{31}$$

From the expressions (19)-(22) determining the Lagrangian $L(h)$ it follows that

$$\langle h_{ii}^{(0)} \rangle = \sqrt{\Omega_7}\, \frac{56}{\varkappa(110-3\alpha)} \,. \tag{32}$$

Therefore Eqs. (24) and (25) give

$$\int d^{11}x \sqrt{|\det g|} \left\langle g^{\alpha\beta} M_{\alpha\beta}(g, h) \right\rangle =$$

$$= -\frac{1}{\varkappa^2 a^2} \frac{28(18-\alpha)}{110-3\alpha} \int d^{11}x \sqrt{|\det g|} + \left\langle \int d^4x \frac{\partial R(h,\xi)}{\partial \xi} \right\rangle_{\xi=1}, \quad (33)$$

$$\int d^{11}x \sqrt{|\det g|} \left\langle g^{ab} M_{ab}(g, h) \right\rangle =$$

$$= \frac{1}{\varkappa^2 a^2} \frac{28(42-\alpha)}{110-3\alpha} \int d^{11}x \sqrt{|\det g|} + \left\langle \int d^4x \frac{\partial Q(h,\xi)}{\partial \xi} \right\rangle_{\xi=1}. \quad (34)$$

To simplify the calculation of the expectation values in the r.h.s. of Eqs. (33) and (34) it is essential to note that at $\xi = 1$ the operators $R(h, \xi)$ and $Q(h, \xi)$ coincide with the four-dimensional Lagrangian

$$R(h, 1) = Q(h, 1) = L(h). \quad (35)$$

Therefore we have

$$\left\langle \int d^4x \frac{\partial R(h,\xi)}{\partial \xi} \right\rangle_{\xi=1} = -i \frac{d}{d\xi} \left\{ \ln \int [dh] e^{i\int d^4x\, R(h,\xi)} \right\}_{\xi=1}, \quad (36)$$

$$\left\langle \int d^4x \frac{\partial Q(h,\xi)}{\partial \xi} \right\rangle_{\xi=1} = -i \frac{d}{d\xi} \left\{ \ln \int [dh] e^{i\int d^4x\, Q(h,\xi)} \right\}_{\xi=1}. \quad (37)$$

To carry out the calculation of the path integrals in the r.h.s. of Eqs. (36) and (37) we must avoid the first order terms in the expressions (26) and (27) of the operators $R(h,\xi)$ and $Q(h,\xi)$ by rewritting

$$R(h,\xi) = \Omega_7 \frac{1}{\varkappa^2 a^2} \frac{504}{(7+2\xi^2)[30\xi f(\xi)-\alpha]} + \tilde{R}^{(2)}(h,\xi), \tag{38}$$

$$Q(h,\xi) = \Omega_7 \frac{1}{\varkappa^2 a^2} \frac{504}{(2+7\xi^2)[30\varphi(\xi)-\alpha]} + \tilde{Q}^{(2)}(h,\xi), \tag{39}$$

where $\tilde{R}^{(2)}(h,\xi)$ or $\tilde{Q}^{(2)}(h,\xi)$ is obtained from the r.h.s. of Eq. (28) or Eq. (29) by means of the substitution

$$h^{(0)}_{ii} \longrightarrow h^{(0)}_{ii} - \sqrt{\Omega_7}\, \frac{168}{\varkappa(7+2\xi^2)[30\xi f(\xi)-\alpha]} \tag{40}$$

(for $\tilde{R}^{(2)}$), or

$$h^{(0)}_{ii} \longrightarrow h^{(0)}_{ii} - \sqrt{\Omega_7}\, \frac{168}{\varkappa(2+7\xi^2)[30\varphi(\xi)-\alpha]} \tag{41}$$

(for $\tilde{Q}^{(2)}$), resp. Then instead of Eqs. (33) and (34) we have

$$\int d^{11}x \sqrt{|\det g|}\, \langle g^{\alpha\beta} M_{\alpha\beta}(g,h)\rangle = \int d^{11}x \sqrt{|\det g|} \left\{ -\frac{dV(\xi)}{d\xi}\bigg|_{\xi=1} - \frac{1}{\varkappa^2 a^2}\frac{28(18-\alpha)}{110-3\alpha} - \frac{1}{\varkappa^2 a^2}\frac{112(285-2\alpha)}{(110-3\alpha)^2} \right\}, \tag{42}$$

$$\int d^{11}x\sqrt{|\det g|}\left\langle g^{ab}M_{ab}(g,h)\right\rangle=\int d^{11}x\sqrt{|\det g|}\left\{-\frac{dU(\xi)}{d\xi}\Big|_{\xi=1}\right.$$
$$\left.+\frac{1}{\varkappa^2a^2}\frac{28(42-\alpha)}{110-3\alpha}-\frac{1}{\varkappa^2a^2}\frac{784\,(30-\alpha)}{(110-3\alpha)^2}\right\}, \qquad (43)$$

where $V(\xi)$ and $U(\xi)$ may be called the extended effective potentials and are determined in the following manner

$$\int d^{11}x\sqrt{|\det g|}\,V(\xi)=i\ln\int[dh]\,e^{i\int d^4x\,\tilde{R}^{(2)}(h,\xi)}, \qquad (44)$$

$$\int d^{11}x\sqrt{|\det g|}\,U(\xi)=i\ln\int[dh]\,e^{i\int d^4x\,\tilde{Q}^{(2)}(h,\xi)}. \qquad (45)$$

4. Existence of solutions

The extended effective potentials $V(\xi)$ and $U(\xi)$ may be calculated by means of the dimensional regularization method, as this was done for the scalar fields in many previous works[9-12]. In ν dimensions we have

$$\begin{aligned}V(\xi)=-\frac{1}{2}\frac{\Gamma(-\nu/2)}{(4\pi)^{\nu/2}}\frac{1}{a^\nu}\Big\{&3\sum_{n=0}^{\infty}D_n^{(0)}\Big(\xi\big[n(n+6)+42\big]-\alpha\Big)^{\nu/2}\\&+\sum_{n=0}^{\infty}D_n^{(0)}\Big(\xi\big[n(n+6)+30f(\xi)\big]-\alpha\Big)^{\nu/2}\\&+\sum_{n=2}^{\infty}D_n^{(0)}\Big(\xi\big[n(n+6)+18\big]-\alpha\Big)^{\nu/2}\\&+2\sum_{n=1}^{\infty}D_n^{(0)}\Big(\xi\big[n(n+6)+30\big]-\alpha\Big)^{\nu/2}\\&+2\sum_{n=1}^{\infty}D_n^{(1)}\Big(\xi\big[n(n+6)+35\big]-\alpha\Big)^{\nu/2}\end{aligned} \qquad (46)$$

$$+ \sum_{n=2}^{\infty} D_n^{(1)} \left(\xi \left[n(n+6) + 23 \right] - \alpha \right)^{\nu/2}$$

$$+ \sum_{n=2}^{\infty} D_n^{(2)} \left(\xi \left[n(n+6) + 30 \right] - \alpha \right)^{\nu/2} \Bigg\} , \tag{46}$$

$$\mathcal{U}(\xi) = -\frac{1}{2} \frac{\Gamma(-\nu/2)}{(4\pi)^{\nu/2}} \frac{1}{a^{\nu}} \Bigg\{ 3 \sum_{n=0}^{\infty} D_n^{(0)} \left[\frac{n(n+6)+42-\alpha}{\xi} \right]^{\nu/2}$$

$$+ \sum_{n=0}^{\infty} D_n^{(0)} \left[\frac{n(n+6)+30\varphi(\xi)-\alpha}{\xi} \right]^{\nu/2}$$

$$+ \sum_{n=2}^{\infty} D_n^{(0)} \left[\frac{n(n+6)+18-\alpha}{\xi} \right]^{\nu/2}$$

$$+ 2\sum_{n=1}^{\infty} D_n^{(0)} \left[\frac{n(n+6)+30-\alpha}{\xi} \right]^{\nu/2}$$

$$+ 2\sum_{n=1}^{\infty} D_n^{(1)} \left[\frac{n(n+6)+35-\alpha}{\xi} \right]^{\nu/2} \tag{47}$$

$$+ \sum_{n=2}^{\infty} D_n^{(1)} \left[\frac{n(n+6)+23-\alpha}{\xi} \right]^{\nu/2}$$

$$+ \sum_{n=2}^{\infty} D_n^{(2)} \left[\frac{n(n+6)+30-\alpha}{\xi} \right]^{\nu/2} \Bigg\} .$$

Here $D_n^{(0)}$, $D_n^{(1)}$ and $D_n^{(2)}$ denote the degrees of the degeneracy of the corresponding scalar, transverse vector and second rank transverse traceles symmetric tensor spherical harmonics in S_7. They were given in Ref.[10] Chodos and Myers[10] have shown also that all the sums over n in the r.h.s. of Eqs. (46) and (47) may be substituted by the corresponding sums taken from the common lowest value $n = -1$ without changing the whole expressions of the extended effective potentials $V(\xi)$ and $U(\xi)$. Applying the method of Candelas and Weinberg we transform each sum (from n =-1) in the r.h.s. of the modified expressions (46) and (47) into an integral which can be calculated with the help of the residue theorem at the unphysical values of ν such that this integral is convergent. After the analytical continuation to the physical value $\nu = 4$ we obtain

$$V'(1) = \frac{1}{a^4} A^{(7)}(\alpha) \;, \quad U'(1) = \frac{1}{a^4} A^{(4)}(\alpha) \;, \tag{48}$$

where

$$A^{(7)}(\alpha) = -\frac{1}{2^6 \pi} \left\{ \sum_{i=1}^{7} \varphi_i(\alpha) - \alpha \sum_{i=1}^{7} \psi_i(\alpha) - \frac{280}{27} \psi_2(\alpha) \right\} , \tag{49}$$

$$A^{(4)}(\alpha) = \frac{1}{2^6 \pi} \left\{ \sum_{i=1}^{7} \varphi_i(\alpha) + \frac{280}{27} \psi_2(\alpha) \right\} , \tag{50}$$

the functions $\varphi_i(\alpha)$ and $\psi_i(\alpha)$, $i = 1, 2, \dots 7$ are given in the Appendix.

The constant Z in the r.h.s. of Eq. (23) may be calculated by means of the ζ-function regularization method. In ν dimensions we obtain

$$Z = \lim_{s \to 0} \frac{1}{\Gamma(s)} \cdot \frac{\Gamma(s - \nu/2)}{\Gamma(-\nu/2)} U(1) .$$

As soon as $U(1) = V(1)$ is finite, we have $Z = 0$.

From the definitions (13), (14) and the relations (15), (17), (22), (42) and (48) we can derive the expressions of the last terms in the l.h.s. of Eqs. (11), (12) and then rewrite these relations in the form

$$B^{(7)}(\alpha)=\frac{\varkappa^2}{a^2}A^{(7)}(\alpha)\,,\quad B^{(4)}(\alpha)=\frac{\varkappa^2}{a^2}A^{(4)}(\alpha)\,, \tag{51}$$

where

$$B^{(7)}(\alpha)=\frac{7}{2}(\alpha-30)-\frac{7}{2}\cdot\frac{168}{110-3\alpha}-\frac{28(18-\alpha)}{110-3\alpha}-\frac{112(285-2\alpha)}{(110-3\alpha)^2}\,, \tag{52}$$

$$B^{(4)}(\alpha)=2(\alpha-42)-2\cdot\frac{168}{110-3\alpha}+\frac{28(42-\alpha)}{110-3\alpha}-\frac{784(30-\alpha)}{(110-3\alpha)^2}\,. \tag{53}$$

In particular, the parameter α determining the cosmological constant

$$\lambda=\frac{\alpha}{a^2}$$

for the given value of the radius a must satisfy following equation

$$\frac{B^{(7)}(\alpha)}{B^{(4)}(\alpha)}=\frac{A^{(7)}(\alpha)}{A^{(4)}(\alpha)}\,. \tag{54}$$

Using the expressions in the Appendix and the relations (49), (50), (52), (53) we have proved that the algebraic equation (54) has many roots satisfying the condition $\alpha\leqslant 34$. For each from these roots the ratio $\frac{\varkappa}{a}$ must take the corresponding value determined by Eq. (51).

5. Conclusion

We have shown that it is possible to construct a self-consistent theory of the pure gravity in the eleven-dimensional space spontaneously compactified into the direct product $M_4 \times S_7$ of the Minkowski space-time M_4 and the seven sphere S_7. The product α of the cosmological constant λ and the square of the radius a of S_7 can be chosen in such a way that the generalized Einstein equations (including the contribution of the quantum fluctuations) are satisfied on the one hand, and there are no tachyons on the other hand. For each admissible value of the product α the radius a of S_7 is completely determined if the constant $\varkappa$ is given.

In the eleven-dimensional supergravity beside of the metric *tensor* G_{AB} (or the corresponding viel-bein) there are other matter fields. The quantum fluctuations of the laters must contribute also to the curvatures and change the admissible values of the parameter α. The spontaneous compactification of the eleven-dimensional sypergravity will be studied in a subsequent work.

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Appendix

Setting

$$F_{i\sigma}^{(\pm)}(\beta) = \frac{1}{2(2\pi)^{3/2}} \left[\int_0^\infty \frac{X_i(x)}{\left(2\,\mathrm{sh}\,\frac{x}{2}\right)^8} \left(\beta x\right)^{\frac{\nu}{2} \pm \frac{1}{2}} Y_{\nu\sigma}^{(\pm)}\left(\beta x\right) x^{-\nu \mp 1} dx \right]_{\nu \to 4}$$

$$i = 1, 2, 3, \quad \sigma = \pm \quad ,$$

$$X_1(x) = sh\, x \quad , \quad X_2(x) = 8\, sh\, x - sh\, 2x ,$$

$$X_3(x) = 36\, sh\, x - 8\, sh\, 2x ,$$

$$Y^{(\pm)}_{\nu +}(\beta x) = J_{-\frac{\nu}{2} \mp \frac{1}{2}}(\beta x) \quad , \quad Y^{(\pm)}_{\nu -}(\beta x) = I_{-\frac{\nu}{2} \mp \frac{1}{2}}(\beta x) ,$$

where $J_\mu(x)$ and $I_\mu(x)$ are the Bessel functions, we have following relations

$$\varphi_1(\alpha) = 3\left[\theta(33-\alpha)\, F^{(+)}_{1+}\left(\sqrt{33-\alpha}\right) + \theta(\alpha-33)\, F^{(+)}_{1-}\left(\sqrt{\alpha-33}\right)\right] ,$$

$$\varphi_2(\alpha) = \theta\left(\frac{83}{3}-\alpha\right) F^{(+)}_{1+}\left(\sqrt{\frac{83}{3}-\alpha}\right) + \theta\left(\alpha-\frac{83}{3}\right) F^{(+)}_{1-}\left(\sqrt{\alpha-\frac{83}{3}}\right) ,$$

$$\varphi_3(\alpha) = \theta(9-\alpha)\, F^{(+)}_{1+}\left(\sqrt{9-\alpha}\right) + \theta(\alpha-9)\, F^{(+)}_{1-}\left(\sqrt{\alpha-9}\right) ,$$

$$\varphi_4(\alpha) = 2\left[\theta(23-\alpha)\, F^{(+)}_{1+}\left(\sqrt{23-\alpha}\right) + \theta(\alpha-23)\, F^{(+)}_{1-}\left(\sqrt{\alpha-23}\right)\right] ,$$

$$\varphi_5(\alpha) = 2\left[\theta(26-\alpha)\, F^{(+)}_{2+}\left(\sqrt{26-\alpha}\right) + \theta(\alpha-26)\, F^{(+)}_{2-}\left(\sqrt{\alpha-26}\right)\right] ,$$

$$\varphi_6(\alpha) = \theta(14-\alpha)\, F^{(+)}_{2+}\left(\sqrt{14-\alpha}\right) + \theta(\alpha-14)\, F^{(+)}_{2-}\left(\sqrt{\alpha-14}\right) ,$$

$$\varphi_7(\alpha) = \theta(23-\alpha)\, F^{(+)}_{3+}\left(\sqrt{23-\alpha}\right) + \theta(\alpha-23)\, F^{(+)}_{3-}\left(\sqrt{\alpha-23}\right) ,$$

and the similar expressions of $\psi_i(\alpha)$, $i = 1, 2, \dots 7$ in terms of six functions $F^{(\pm)}_{i\sigma}(\beta)$, $i = 1, 2, 3$, $\sigma = \pm$.
Residue calculations give

$$F^{(+)}_{1\pm}(x) = \frac{1}{6!}\Big\{ 8x^4\left(x^4 \pm 5x^2 + 4\right)\zeta_3^{(\pm)}(x)$$

$$\pm 12x^3\left(7x^4 \pm 25x^2 + 12\right)\zeta_4^{(\pm)}(x) \pm 6x^2\left(85x^4 \pm 205x^2 + 52\right)\zeta_5^{(\pm)}(x)$$

$$+ 30x\left(73x^4 \pm 110x^2 + 12\right)\zeta_6^{(\pm)}(x) + 45\left(155x^4 \pm 130x^2 + 4\right)\zeta_7^{(\pm)}(x)$$

$$1260x\left(\pm 13x^2 + 5\right)\zeta_8^{(\pm)}(x) + 630\left(\pm 43x^2 + 5\right)\zeta_9^{(\pm)}(x)$$

$$+ 28.350x\,\zeta_{10}^{(\pm)}(x) + 14.175\,\zeta_{11}^{(\pm)}(x) \Big\},$$

$$F^{(-)}_{1\pm}(x) = \frac{1}{6!}\Big\{ \pm 4x^3\left(x^4 \pm 5x^2 + 4\right)\zeta_2^{(\pm)}(x)$$

$$\pm 2x^2\left(13x^4 \pm 45x^2 + 20\right)\zeta_3^{(\pm)}(x) + 12x\left(9x^4 \pm 20x^2 + 4\right)\zeta_4^{(\pm)}(x)$$

$$+ 6\left(55x^4 \pm 70x^2 + 4\right)\zeta_5^{(\pm)}(x) + 150x\left(\pm 5x^2 + 1\right)\zeta_6^{(\pm)}(x)$$

$$+ 45\left(\pm 27x^2 + 5\right)\zeta_7^{(\pm)}(x) + 1260x\,\zeta_8^{(\pm)}(x) + 630\,\zeta_9^{(\pm)}(x)\Big\},$$

$$F^{(+)}_{2\pm}(x) = \frac{1}{5!}\Big\{ 8x^4\left(x^4 \pm 10x^2 + 9\right)\zeta_3^{(\pm)}(x)$$

$$\pm 12x^3\left(7x^4 \pm 50x^2 + 27\right)\zeta_4^{(\pm)}(x) \pm 6x^2\left(85x^4 \pm 410x^2 + 117\right)\zeta_5^{(\pm)}(x)$$

$$+ 30x\left(73x^4 \pm 220x^2 + 27\right)\zeta_6^{(\pm)}(x) + 45\left(155x^4 \pm 260x^2 + 9\right)\zeta_7^{(\pm)}(x)$$

$$+ 1260x\left(\pm 13x^2 + 10\right)\zeta_8^{(\pm)}(x) + 630\left(\pm 43x^2 + 10\right)\zeta_9^{(\pm)}(x)$$

$$+ 28.350x\,\zeta_{10}^{(\pm)}(x) + 14.175\,\zeta_{11}^{(\pm)}(x)\Big\},$$

$$F^{(-)}_{2\pm}(x) = \frac{1}{5!}\Big\{ \pm 4x^3\left(x^4 \pm 10x^2 + 9\right)\zeta_2^{(\pm)}(x)$$

$$\pm 2x^2\left(13x^4 \pm 90x^2 + 45\right)\zeta_3^{(\pm)}(x) + 12x\left(9x^4 \pm 40x^2 + 9\right)\zeta_4^{(\pm)}(x)$$

$$+6\left(55x^4 \pm 140x^2+9\right)\zeta_5^{(\pm)}(x)+300x\left(\pm 25x^2+3\right)\zeta_6^{(\pm)}(x)$$

$$+45\left(\pm 27x^2+10\right)\zeta_7^{(\pm)}(x)+1260x\zeta_8^{(\pm)}(x)+630\zeta_9^{(\pm)}(x)\Big\},$$

$$F_{3\pm}^{(+)}(x)=\frac{1}{36}\Big\{8x^4\left(x^4\pm 17x^2+10\right)\zeta_3^{(\pm)}(x)$$

$$\pm 12x^3\left(7x^4\pm 85x^2+48\right)\zeta_4^{(\pm)}(x)\pm 6x^2\left(85x^4\pm 697x^2+208\right)\zeta_5^{(\pm)}(x)$$

$$+30x\left(73x^4\pm 374x^2+48\right)\zeta_6^{(\pm)}(x)+45\left(155x^4\pm 442x^2+16\right)\zeta_7^{(\pm)}(x)$$

$$+1260x\left(\pm 13x^2+17\right)\zeta_8^{(\pm)}(x)+630\left(\pm 43x^2+17\right)\zeta_9^{(\pm)}(x)$$

$$+28.350x\zeta_{10}^{(\pm)}(x)+14.175\zeta_{11}^{(\pm)}(x)\Big\},$$

$$F_{3\pm}^{(-)}(x)=\frac{1}{36}\Big\{\pm 4x^3\left(x^4\pm 17x^2+16\right)\zeta_2^{(\pm)}(x)$$

$$\pm 2x^2\left(13x^4\pm 153x^2+80\right)\zeta_3^{(\pm)}(x)+12x\left(9x^4\pm 68x^2+16\right)\zeta_4^{(\pm)}(x)$$

$$+6\left(55x^4\pm 238x^2+16\right)\zeta_5^{(\pm)}(x)+30x\left(\pm 25x^2+51\right)\zeta_6^{(\pm)}(x)$$

$$+45\left(\pm 27x^2+17\right)\zeta_7^{(\pm)}(x)+1260x\zeta_8^{(\pm)}(x)+630\zeta_9^{(\pm)}(x)\Big\},$$

$$\zeta_k^{(+)}(x)=\sum_{n=1}^{\infty}\frac{e^{-2\pi n x}}{(n\pi)^k}, \quad k=2,3,\dots 11,$$

$$\zeta_k^{(-)}(x)=\begin{cases}\displaystyle\sum_{n=1}^{\infty}\frac{\sin 2\pi n x}{(n\pi)^k}, & k=2,4,6,8,10,\\ \displaystyle\sum_{n=1}^{\infty}\frac{\cos 2\pi n x}{(n\pi)^k}, & k=3,5,7,9,11.\end{cases}$$

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Нгуен Ван Хьеу E2-85-134

Квантовые флуктуации и спонтанная компактификация 11-мерной гравитации

Рассматривается редукция теории гравитации в 11-мерном пространстве к теории поля в четырехмерном пространстве-времени Минковского посредством спонтанной компактификации семи лишних измерений. Вычислены вклады квантовых флуктуаций симметричного тензорного поля в 11-мерном пространстве в кривизне пространства-времени и компактифицированного пространства внутренней симметрии. Показано, что существуют значения космологической константы, для которых отсутствуют тахионы. Получается самосогласованная квантовая теория полей в спонтанно компактифицированном пространстве $M_4 \times S_7$, где M_4- пространство-время Минковского, а S_7 - семимерная сфера.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

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Nguyen Van Hieu E2-85-134

Quantum Fluctuations and Spontaneous Compactification of Eleven-Dimensional Gravity

The reduction of the eleven-dimensional pure gravity to a field theory in the four-dimensional Minkowski space-time by means of the spontaneous compactification of the extra dimensions is investigated. The contribution of the quantum fluctuations of the eleven-dimensional second rank symmetric tensor field to the curvatures of the space-time and the compactified space of the extra dimensions are calculated in the one-loop approximation. It is shown that there exist the values of the cosmological constant such that the resulting four-dimensional field theory is self-consistent.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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