BR85/1343

IFT-P.18/84

"FROM GCM ENERGY KERNELS TO WEYL-WIGNER HAMILTONIANS"
A PARTICULAR MAPPING*

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*WORK SUPPORTED BY FINEP (UNDER CONTRACT 43/84/0083/00)
AND CNPq - BRASIL

ABSTRACT

A particular mapping is established which directly connects GCM energy kernels to Weyl-Wigner Hamiltonians, under the assumption of gaussian overlap kernel. As an application of this mapping scheme we derive the collective Hamiltonians for some giant resonances.

I - INTRODUCTION

The Weyl transform of an operator $\hat{A}$ (Leaf 1968,de Groot and Suttorp 1972), defined as

$$A_W(p,q) = \int_{-\infty}^{\infty} e^{\frac{ipv}{\hbar}} \langle q-\frac{v}{2} | \hat{A} | q+\frac{v}{2} \rangle dv$$

is an extremely convenient representation of operators acting on states of a Hilbert space since it allows us to treat quantum mechanical problems in a phase space-like fashion.In particular, this approach may be even applied to the description of a specific nuclear collective motion if we know how to construct the corresponding collective subspace of the full many-body Hilbert space. In this connection the Generator Coordinate Method (GCM) (Hill and Wheeler 1953, Griffin and Wheeler 1957) has proved to constitute a good starting point from which such a collective subspace can be found. In fact it has been put forth in the past years a series of papers presenting a scheme by which it is possible to write the projection operator of the subspace of interest(Piza et al 1977, Piza and Passos 1978), the crucial point of this approach being the diagonalization of the GCM overlap kernel; if this step of the procedure can be performed in a analytic way we are able to explicitly write a non-local Hamiltonian kernel with which the specific collective spectrum can be calculated. A further improvement of this method is achieved just by finding the Weyl transform of that non-local collective Hamiltonian kernel (Galetti and Piza 1981) since it is then possible to extract a Weyl-Wigner phase space version of that Hamiltonian which clearly exhibits a collective potential and an effective mass, i.e., we have at our disposal the explicit forms of those objects which can reveal us the fundamental collective properties of that particular degree of freedom of the nuclear system. It is clear that those expressions, appearing as global results coming from microscopic calculations, manifest the nucleon-nucleon interaction peculiarities in all their complexity. The task of separating a semiclassical contribution from those intricate expressions, however, is here again greatly facilitated by the very use of the Weyl scheme and has already been sucessfully applied to the ^{16}O monopole giant resonance calculation (Garcia 1983).

In the previous treatments (Galetti and Piza 1981, Garcia 1983) however, a power series expansion for the GCM reduced energy kernel was needed due to operational restrictions, thus implying in approximated final expressions. In this paper we present an approach - starting again from the GCM reduced energy kernel calculated from one real GC only - through which it is possible to exactly write the Weyl-Wigner collective Hamiltonian. We show how to establish a mapping between that GCM kernel, K (γ,η), and the Weyl-Wigner version of the Hamiltonian by means of an integral kernel $M(\gamma,\eta;$ $q,p)$ which bears all the kinematical contents of the transformation

$$\mathcal{H}(q,p) = \int_{-\infty}^{\infty} K(\gamma,\eta)\, M(\gamma,\eta;q,p)\, d\gamma\, d\eta$$

This mapping kernel is explicitly written for the case of gaussian GCM overlap kernel which, although particular, is of wide range of applications. As a result, the study of semi classical limits of nuclear collective motions is greatly simplified by virtue of the particularly simple form of the mapping kernel when compared to the previous treatments. Furthermore this scheme is developed in such a way to permit a closer contact with the Time Dependent Hartree Fock approach (TDHF) and its adiabatic approximation (ATDHF) (Villars 1975).

In section II a brief review of the collective space construction technique is presented and we further show how is possible to write the Weyl-Wigner version of the collective Hamiltonian of interest by the introduction of a mapping kernel. Giant resonances of ^{16}O are studied as applications of the method and the results are given in section III. Finally in section IV we present the final considerations and conclusions.

II - THE MAPPING KERNEL

It has already been shown (Piza and Passos 1978) that the Griffin-Wheeler equation (Griffin and Wheeler 1957) for one real GC

$$\int_{-\infty}^{\infty}\left[K(\alpha,\alpha') - E\,N(\alpha,\alpha')\right] f(\alpha')\,d\alpha' = 0\,, \qquad \text{II.1}$$

can be projected onto a collective subspace of the full many body Hilbert space given rise to a Schroedinger - like equa tion with a non-local energy kernel

$$\int_{-\infty}^{\infty} H(\tilde{k},\tilde{k}')\, g(\tilde{k}')\, d\tilde{k}' = E\, g(\tilde{k}) \qquad \text{II.2}$$

From a merely operational point of view all one has to do is to diagonalize the GCM overlap kernel

$$\int_{-\infty}^{\infty} U^{+}_{\tilde{k}}(\alpha)\; N(\alpha,\alpha')\, U_{\tilde{k}'}(\alpha')\, d\alpha\, d\alpha' = \Lambda(\tilde{k})\, \delta(\tilde{k}-\tilde{k}'),$$

where $\Lambda(\tilde{k})$ is the spectrum of $N(\alpha,\alpha')$, and construct the projection operator

$$\Pi(\tilde{k},\alpha) = \frac{U_{\tilde{k}}(\alpha)}{\Lambda^{1/2}(\tilde{k})} \;;$$

equation (II.2) then immediately follows from (II.1). Further, a collective potential and an effective mass can be extracted from $H(\tilde{k},\tilde{k}')$ just by considering the zeroth and second moments of that kernel respectively, followed by a Weyl transformation (Galetti and Piza 1981). This gives an approximated Weyl-Wigner version of the desired quantum mechanical collective hamiltonian.

We now want to describe the above previous procedure by means of the [illegible] Weyl-Wigner Hamiltonian formalism, [illegible]

$$\mathcal{H}(q,p) = \int_{-\infty}^{\infty} F(\alpha,\alpha')\, M(\alpha,\alpha';q,p)\, d\alpha\, d\alpha', \qquad \text{II.3}$$

where $F(\alpha,\alpha')$ is the reduced GCM energy kernel

$$F(\alpha,\alpha') = K(\alpha,\alpha')/N(\alpha,\alpha') \ ,$$

and $M(\alpha,\alpha';q,p)$ is a mapping kernel. The main feature of $M(\alpha,\alpha';q,p)$ is that it must only carry kinematical information; it depends on the particular form of the GCM overlap kernel of the problem at hand and on the Weyl transformation ingredients. In what follows we will study this mapping kernel in the particular case when the GCM overlap kernel is gaussian

$$N(\alpha,\alpha') = N(\alpha-\alpha') = e^{-c(\alpha-\alpha')^2} \ .$$

In this situation, as is well known (Galetti and Piza 1981), we have

$$\Lambda(\tilde{k}) = \left(\frac{\pi}{c}\right)^{1/2} e^{-\frac{\tilde{k}^2}{4c}} \ ,$$

$U_{\tilde{k}}(\alpha)$ is just a Fourier transformation and (II.3) can be easily written as

$$\mathcal{H}(q,p) = \frac{c^{1/2}}{2\pi^{3/2}} \int_{-\infty}^{\infty} F(\tilde{\gamma},\eta)\exp\left[\frac{ip\eta}{\hbar} + i\tilde{k}(\tilde{\gamma}-q) + \frac{p^2}{4\hbar^2 c} + \frac{\tilde{k}^2}{16c} - c\eta^2\right] d\tilde{\gamma}\, d\eta\, d\tilde{k} \qquad \text{II.4}$$

where $\tilde{\gamma} = \frac{\alpha+\alpha'}{2}$, $\eta = \alpha-\alpha'$ and $\tilde{k} = \tilde{k} - \tilde{k}'$.

Now, if we express $F(\tilde{\gamma},\eta)$ as a power series in $\tilde{\gamma}$ and η

$$F(\tilde{\gamma},\eta)=\sum_{n,m=0}^{\infty} A_{mn}\,\eta^{n}\tilde{\gamma}^{m},$$

then, if we first perform the $\tilde{\gamma}$ integration in (II.4), we are led to

$$\mathcal{H}(q,p)=\sum_{n,m=0}^{\infty} A_{mn}(-)^{n} i^{n+m}\hbar^{n}\frac{d^{m}}{d\tilde{k}^{m}}\,e^{\frac{\tilde{k}^{2}}{16c}-i\tilde{k}q}\Bigg|_{\tilde{k}=0}\,e^{\frac{p^{2}}{4c\hbar^{2}}}\frac{d^{n}}{dp^{n}}\,e^{-\frac{p^{2}}{4c\hbar^{2}}}.$$

By identifying the generating functions of the usual Hermite polynomials, this expression can be put in the form

$$\mathcal{H}(q,p)=\sum_{n,m=0}^{\infty} A_{mn}\left(\frac{i}{2c^{1/2}}\right)^{n}\frac{\partial^{n+m}}{\partial\xi^{n}\partial\nu^{m}}\,T(\xi,\nu;q,p)\Big|_{\xi=\nu=0},$$

where

$$T(\xi,\nu;q,p)=\exp\left(-\xi^{2}+\frac{\xi}{\hbar c^{1/2}}p-\frac{\nu^{2}}{16c}+\nu q\right),$$

or in a more convenient form as

$$\mathcal{H}(q,p)=\sum_{n,m=0}^{\infty} A_{mn}\frac{i^{n}}{2^{n+2m}\,c^{\frac{n+m}{2}}}\,H_{m}(2c^{1/2}q)\,H_{n}\left(\frac{p}{2\hbar c^{1/2}}\right). \qquad \text{II.5}$$

It is important to note that in this case all information connected with the nuclear interaction is embodied in the A_{mn} coefficients while the Hermite polynomials give, for each even (odd) value of m and n, all even (odd) powers of q and p down to zero (one); this fact permits us to isolate the terms corresponding to the fluctuation energy in a simple way, although it may not be ease to sum the final entangled series.

As shown, in this approach all integrations in (II.4)can be performed if a restriction is imposed: one must first integrate the $\tilde{\gamma}$ variable. This fact reveals the ultradistribution nature of our expansion, i.e., we are dealing with expressions involving derivatives of Dirac's δ distribution. Thus we are not allowed to first integrate the $\tilde{k}$ variable in (II.4) so as to obtain the mapping kernel in the form proposed in (II.3). This drawback however can be circumvented by the introduction of an integral representation of the Hermite polynomial(Gradshteyn and Ryzhik 1980)

$$H_n(\beta) = \frac{(2i)^n}{\pi^{1/2}} \int_{-\infty}^{\infty} x^n \exp\left[-(x-i\beta)^2\right] dx \,,$$

since then, assuming convergence, it is possible to write (II.5) as

$$\mathcal{H}(q,p) = \frac{2c}{\pi} \int_{-\infty}^{\infty} d\gamma\, d\eta \sum_{n,m=0}^{\infty} A_{mn} (-i\gamma)^m \eta^n \exp\left[-c\left(\eta - \frac{ip}{2c\hbar}\right)^2 - 4c(\gamma - iq)^2\right],$$

which can be finally expressed as

$$\mathcal{H}(q,p) = \frac{2c}{\pi} \int_{-\infty}^{\infty} F(-i\gamma,\eta) \exp\left[-c\left(\eta - \frac{ip}{2c\hbar}\right)^2 - 4c(\gamma - iq)^2\right]. \quad \text{II.6}$$

Two points must be stressed here. First, the mapping (II.6) is very simple of handling because the new mapping kernel is gaussian

$$M(\gamma,\eta;q,p) = \frac{2c}{\pi} \exp\left[-c\left(\eta - \frac{ip}{2c\hbar}\right)^2 - 4c(\gamma - iq)^2\right]$$

and, if desired, we need not to work with series. Second, under the convergence assumption, (II.6) is seen to be completely equivalent to (II.5); this fact assures we are preserving the whole kinematical content of the mapping when we pass from (II.5) to (II.6), but to accomplish this we had to introduce a new reduced energy kernel F $(-i\gamma,\eta)$; this new kernel however is obtained simply by replacing the old variable $\tilde{\gamma}$ by the new one $-i\gamma$.

Now, following the Weyl procedure (de Groot and Suttorp 1972) we can look for the collective Hamiltonian operator associated to $\mathcal{H}(q,p)$. It can be simply expressed as

$$H(\hat{Q},\hat{P})=\int_{-\infty}^{\infty} F(-i\gamma,\eta)\,\delta(q-\hat{Q})\,\delta(p-\hat{P})\exp\left[\frac{\hbar}{2i}\frac{\partial}{\partial p}\frac{\partial}{\partial q}\right]M(\gamma,\eta;q,p)\,d\gamma\, d\eta\, dp\, dq \,.$$

Noticing that

$$\exp\left(\frac{\hbar}{2i}\frac{\partial}{\partial p}\frac{\partial}{\partial q}\right)M(\gamma,\eta;q,p)=\exp\left(\frac{i}{4c}\frac{\partial}{\partial\eta}\frac{\partial}{\partial\gamma}\right)M(\gamma,\eta;q,p)$$

we can finally write, after a partial integration,

$$H(\hat{Q},\hat{P})=\int_{-\infty}^{\infty} M(\gamma,\eta;\hat{Q},\hat{P})\exp\left[\frac{i}{4c}\frac{\partial}{\partial\eta}\frac{\partial}{\partial\gamma}\right]F(-i\gamma,\eta)\,d\gamma\, d\eta \,.$$

Here all the coordinate operators must be placed to the left of the momentum operators so as to establish a unique correspondence. This simple expression allows us to write the collective Hamiltonian operator in a direct analytic way if we have the auxiliar energy kernel $F(-i\gamma,\eta)$. Moreover it can set the basis for a treatment of the connection between Time

Dependent Hartree Fock (TDHF) and the present approach.

III - APPLICATIONS

In what follows we will apply this mapping technique to the GCM calculations of giant resonances of ^{16}O, performed with harmonic oscillator wave functions and Skyrme interaction SIII. As usual the starting point is the obtention of the GCM kernels, however here we only quote the final results as the calculations can be found in (Flocard and Vautherin 1976).

A) Isovector dipole

In this case the generator coordinate is chosen as the half distance between the centers of the proton and neutron distributions, and the overlap kernel is exactly gaussian,

$$N(\alpha-\alpha') = N(\eta) = e^{-\frac{\eta^2}{b^2}} ,$$

where b is the harmonic oscillator parameter. The energy kernel is written as

$$K(\tilde{\gamma},\eta) = N(\eta)\Big[140.14+\frac{t_0}{2}(2+x_0)\,D_0(\tilde{\gamma})+\frac{t_3}{2}\,D_1(\tilde{\gamma})+\frac{3t_1-t_2}{8}\,D_2(\tilde{\gamma})$$

$$+\frac{t_1+t_2}{4}\,D_3(\tilde{\gamma})-\frac{\hbar^2}{2mb^4}\,\eta^2-\frac{t_1+t_2}{16b^4}\,D_0(\tilde{\gamma})\,\eta^2\Big], \qquad \text{III}.1$$

where t_0, t_1, t_2, t_3 and x_0 are the parameters of the Skyrme interaction and the D's are integrals written in terms of the density $\rho(\tilde{\gamma},\mathbf{r})$, and kinetic energy density $\tau(\tilde{\gamma},\mathbf{r})$

$$D_0(\tilde{\gamma}) = \int \rho(\tilde{\gamma}, \underline{r})\, d^3r$$

$$D_1(\tilde{\gamma}) = \int \rho^2(\tilde{\gamma}, \underline{r})\, \rho(-\tilde{\gamma}, \underline{r})\, d^3r$$

$$D_2(\tilde{\gamma}) = \int \bar{\nabla}\rho(\tilde{\gamma}, \underline{r}) \cdot \bar{\nabla}\rho(-\tilde{\gamma}, \underline{r})\, d^3r$$

$$D_3(\tilde{\gamma}) = \int \left[\rho(\tilde{\gamma}, \underline{r})\, T(-\tilde{\gamma}, \underline{r}) + \rho(-\tilde{\gamma}, \underline{r})\, T(\tilde{\gamma}, \underline{r})\right] d^3r .$$

By a mere substitution of $\tilde{\gamma}$ by $-i\gamma$ in (III.1) we get the auxiliar reduced energy kernel, and after a straightforward calculation we obtain the mapped Hamiltonian which exactly breaks into two contributions. The first part is independent of p and corresponds to the collective potential together with its fluctuation energy corrections

$$V(q) = 140.14 + 158.45 \exp\left(-\frac{4}{5}\frac{q^2}{b^2}\right) - 446.22 \exp\left(-\frac{4}{7}\frac{q^2}{b^2}\right) + \left[57.12 \exp\left(-\frac{4}{5}\frac{q^2}{b^2}\right) - 113.97 \exp\left(-\frac{4}{7}\frac{q^2}{b^2}\right)\right]\frac{q^2}{b^2} + \left[20.99 \exp\left(-\frac{4}{5}\frac{q^2}{b^2}\right) - 21.83 \exp\left(-\frac{4}{7}\frac{q^2}{b^2}\right)\right]\frac{q^4}{b^4} + \left[1.68 \exp\left(-\frac{4}{5}\frac{q^2}{b^2}\right) - 1.10 \exp\left(-\frac{4}{7}\frac{q^2}{b^2}\right)\right]\frac{q^6}{b^6} .$$

This collective potential is depicted in figure 1.With this expression for the collective potential we can now calculate the dipole incompressibility modulus

$$K = \frac{1}{A} \frac{d^2}{dq^2} V(q)\Big|_{q=0} .$$

The value $K = 2.8\ \mathrm{MeV.fm^{-2}}$ thus obtained is greater than that coming from a pure GCM energy kernel calculation $K = 2.64\ \mathrm{MeV\,fm^{-2}}$ (Flocard and Vautherin 1976). As can be seen, the difference comes from the fluctuation energy corrections which make the collective potential steeper than the diagonal part of the GCM energy kernel.

The second part of the Hamiltonian is proportional to p^2 and corresponds to the kinetic energy and its fluctuation energy corrections

$$T(p,q) = \frac{p^2}{2m}\left[\frac{1}{4} + 0.0053\left(15.102 + 3.3586\frac{q^2}{b^2} + 0.853\frac{q^4}{b^4}\right)\cdot \exp\left(-\frac{4}{7}\frac{q^2}{b^2}\right)\right] - \frac{\hbar^2}{4mb^4} - 0.0705\left(15.102 + 3.3586\frac{q^2}{b^2} + 0.853\frac{q^4}{b^4}\right) \exp\left(-\frac{4}{7}\frac{q^2}{b^2}\right)$$

The effective mass calculated from this expression is depicted in figure 2. As expected its asymptotic value goes to the reduced mass of a system constituted of two clusters, one composed of eight protons and the other eight neutrons respectively. It is worth to mention that similar results were obtained for the ^{4}He dipole giant resonance through a different approach some years ago (Passos and Cruz 1981).

B) Isoscalar quadrupole

For the description of this collective motion we adopt the harmonic oscillator parameters b_z and $b_\perp$, where $b_\perp$ is the parameter of the xy oscillator, as our generator coordinates. As usual (Griffin 1957), if we make the transformation $b_z = \ell^{\alpha}$ under the "volume conservation" condition, $b_\perp^2 b_z = b_0^3$, we find the overlap kernel

$$N(\eta) = \operatorname{sech}^{12}(2\eta)\operatorname{sech}^{24}(\eta). \qquad \text{III.2}$$

This particular form of the overlap kernel does not allow us to directly apply our mapping tecnique; however it is well known that we can, to a good degree of accuracy, consider the gaussian approximation to this overlap

$$N(\eta) \simeq e^{-C\eta^2} ,$$

where $C=36$ in this case; consequently we can treat, although in a approximated form, this collective motion.

Keeping these approximations in mind we will only quote the final expression for the GCM energy kernel

$$K(\tilde{\gamma},\eta) = N(\eta)\Big[G_1(\operatorname{sech} 2\eta\, e^{-4\tilde{\gamma}} + 2\operatorname{sech}\eta\, e^{2\tilde{\gamma}}) + G_2 \cosh\eta \cosh^{1/2} 2\eta + G_3(\cosh\eta \cosh^{3/2} 2\eta\, e^{-4\tilde{\gamma}} + 2\cosh^2\eta \cosh^{1/2} 2\eta\, e^{2\tilde{\gamma}}) + G_4 \cosh 2\eta \cosh^2\eta\Big], \quad \text{III.3}$$

where

$$G_1 = \frac{6\hbar^2}{m b_c^2} , \quad G_2 = \frac{93\, t_0}{2 b_c^3 (2\pi)^{3/2}} , \quad G_3 = \frac{105 t_1 + 80 t_2}{4 b_c^5 (2\pi)^{3/2}}$$

$$G_4 = \frac{464\, t_3}{9 b_c^6 (\pi\sqrt{3})^3} .$$

By virtue of the particular dependence in $\tilde{\gamma}$ and η we will not try to separate from (III.3) a pure kinetic term plus a potential term. In fact (III.3) gives rise to all orders of the collective momentum p. However we can, following a procedure developed some years ago (Galetti and Piza 1981), construct a momentum independent contribution, identified with the collective potential, just by taking the η zeroth moment of the GCM energy kernel. In our present formalism this collective potential is then written as

$$V(q) = \int F(-i\gamma,\eta)\mathcal{M}(\gamma,\eta;q,p)\,\delta(p)\,d\gamma\, d\eta\, dp\ ,$$

the analytic expression of which being

$$V(q) = G_1\left(I_1 e^{-4q} + 2I_2 e^{2q}\right) + G_2 I_3 + G_3\left(I_4 e^{-4q} + 2I_5 e^{2q}\right) + G_4 I_5\ .$$

The new constants I's appearing in this potential are the summed effects of the corresponding fluctuaction energy terms and the η integration. Although this expression has a strong resemblance to the diagonal part of the GCM energy kernel (III.3), it is precisely the I's that cause a departu re from it; on the other hand it is also expected that for heavy nuclei these constants tend to unity. Figure 3 shows this collective potential, in which the confinement effect intro duced by the volume conservation constraint is remarkable, i.e., the potential grows exponentially as q goes to infi nity, contrary to the constant asymptotic behaviour expected when the constraint is not imposed.

Exactly in the some way as presented above, if we take the second moment of the GCM energy kernel we find an expression associated with the effective collective inertia. Thus

$$B(q) = \frac{1}{2}\int F(-i\gamma,\eta)\mathcal{M}(\gamma,\eta;q,p)\left(\frac{1}{2c}-\eta^2\right)\delta(p)\,d\gamma\, d\eta\, dp$$

permits us to write

$$B(q) = \frac{V(q)}{144} - \frac{1}{2}\left[G_1\left(J_1 e^{-4q} + 2J_2 e^{2q}\right) + G_2 J_3 + G_3\left(J_4 e^{-4q} + 2J_5 e^{2q}\right) + G_4 J_6\right],$$

where, as before, the J's are constants coming from the γ and η integrations; from the expression

$$\text{Kinetic energy} = -\frac{B(q)}{\hbar^2}\,p^2$$

we calculate the effective mass depicted in figure 4. The most conspicuous feature in this figure is the presence of two points of divergence giving rise to two regions in which the mass presents negative values. These singularities appear in connection with the volume conservation constraint and are also related to the relative strenght of the attractive and repulsive part of the nuclear interaction. A similar effect has been already presented and discussed by Giraud (Giraud and Grammaticos 1975) in the context of ^{16}O giant monopole resonance calculations with Skyrme interaction. Fortunately, for pratical purposes, i.e., low energy excitations, these negative value regions need not be considered ; however we must always keep in mind that a complete discussion of the collective dynamics requires no approximation, and, in this case, the physical interpretation of this inertia function is not obvious.

Finally, although we have performed an adiabatic approximation to this problem, a general expression for the complete collective Hamiltonian can be easily obtained from (II.3) through a direct mapping.

C) Isoscalar monopole

This giant resonance has been already treated by the power expansion approach, as presented in section II (Galetti and Piza 1981, Garcia 1983), and will be not discussed here; however we would like to point some interesting aspects of the mapped Hamiltonian concerning its semiclassical limit. First, it can be shown that the semiclassical limit of the monopole mapped Hamiltonian then found exhibits the excitation energy in complete agreement with the result obtained from a small amplitude calculation for a quantized liquid drop (Garcia 1983). Second, it is a trivial matter to establish a link between our collective potential and the corresponding ATDHF one as presented by Engel (Engel et al 1975) ; in fact it suffices to neglect all fluctuation energy terms coming from the mapping of the auxiliar energy kernel. Finally, the mass parameter, in the semiclassical limit, exactly coincides with that of the ATDHF approach.

IV - CONCLUSIONS

In this paper we have shown how to construct a particular kernel which maps directly an auxiliar reduced energy kernel from GCM to a Weyl-Wigner phase space Hamiltonian. The constructive procedure for this kernel is based on the hypothesis of gaussian GCM overlap. In this case the gaussian form of the mapping kernel allows one to easily establish the correspondence without the restrictions of expansion techniques, thus giving analytic expressions for the collective Hamiltonians. As far as the form of the mapping kernel is concerned one may be tempted to directly relate it to the well known

Bargmann kernel (Bargmann 1961). A closer inspection however shows that some differences arise. The way of establishing a connection between our result and that of Bargmann, as well as the wave packets mapping recently proposed by (Mizrahi 1983) is under study and will appear later.

Low lying energy levels for heavy nuclei can be easily calculated from this formalism although no great improvement over other results must be expected, since for low excitation energy the quadratic approximation dominates, and in this region it is not surprising that sufficiently well adapted theories give results in mutual agreement. This, in fact, has been verified by Arickx (Arickx and Broeckhove 1983) in the particular case of monopole giant resonance. On the other hand, as a final remark we would like to observe that the main purposes of obtaining this Weyl-Wigner phase space version of a collective Hamiltonian are to study its semiclassical limit, and its connection with other theories like TDHF or its adiabatic limit ATDHF. For the isoscalar monopole giant resonance this program has been partially achieved since it has been found a semiclassical Hamiltonian(Garcia 1983) which presents a hydrodynamical behaviour and is in complete agreement with the ATDHF version (Engel el al 1975); the related problem of finding a hydrodynamical limit, whether possible, for general mapped Hamiltonians is now under investigation.

Acknowledgment.

The author is greatly indebted to Dr. Salomon M.Mizrahi for valuable discussions and a critical reading of the manuscripts.

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Figure Captions

Fig. 1

The collective potential for the ^{16}O isovector dipole giant resonance as a function of the half distance between the neutron and proton distributions.

Fig. 2

The inertia function for the ^{16}O isovector dipole giant resonance as a function of the half distance between the neutron and proton distributions in units of a nucleon mass.

Fig. 3

The collective potential for the ^{16}O isoscalar quadrupole giant resonance as a function of the z-harmonic oscillator parameter.

Fig. 4

The inertia function for the ^{16}O isoscalar quadrupole giant resonance as a function of the z-harmonic oscillator parameter in units of a nucleon mass.

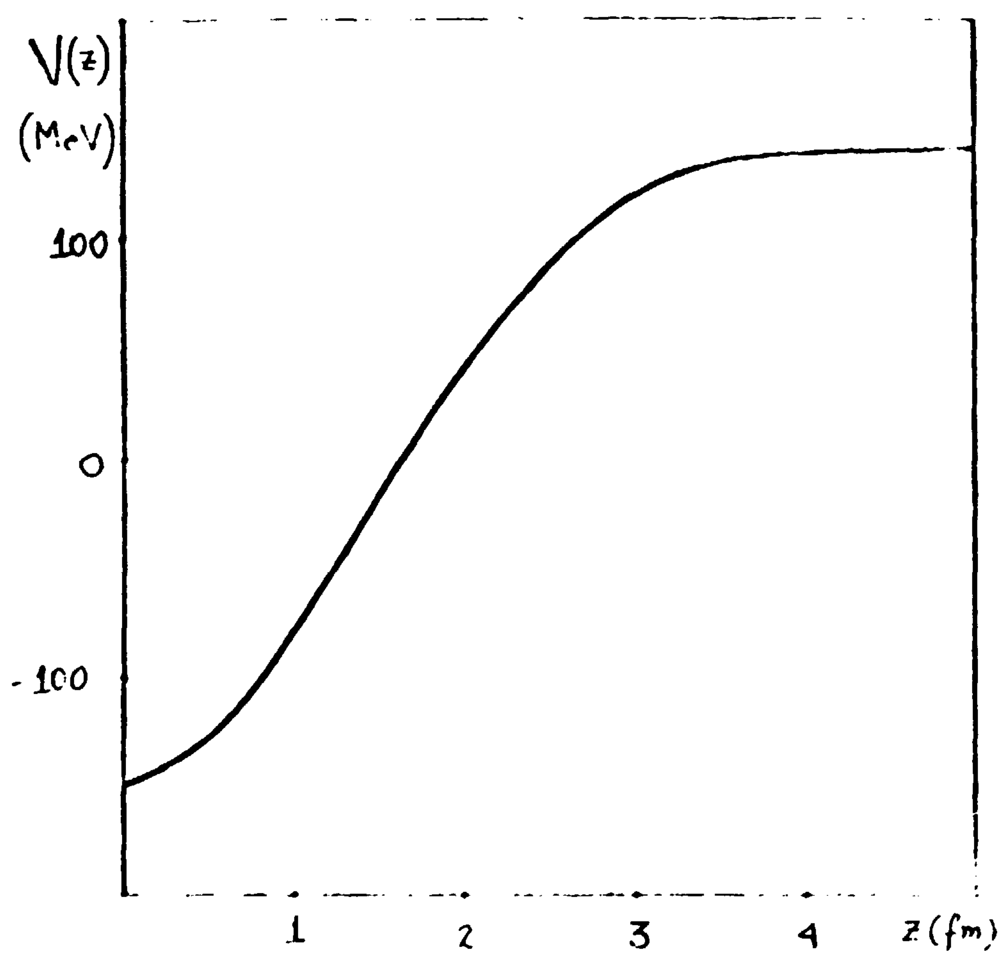


Fig. 1

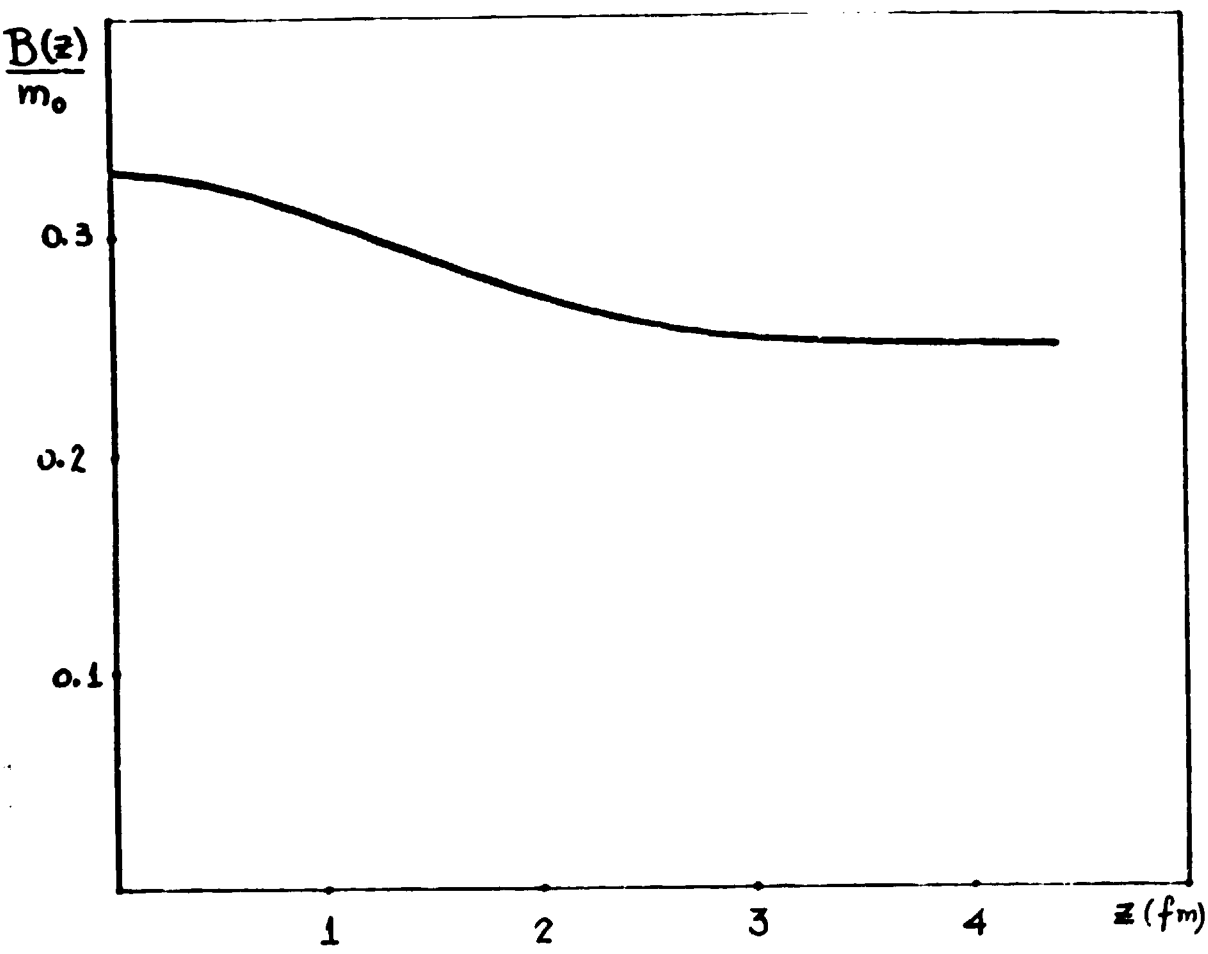


Fig. 2

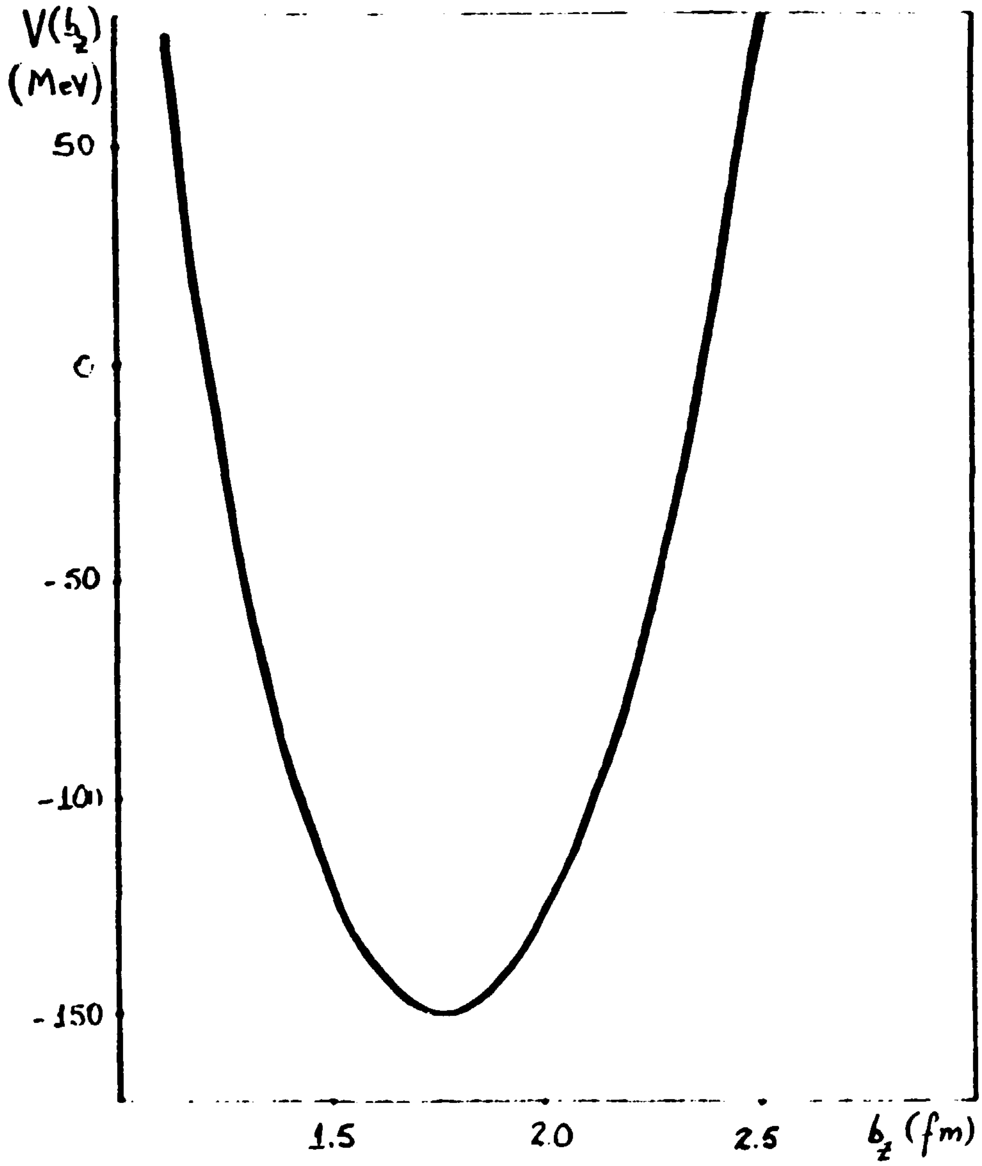
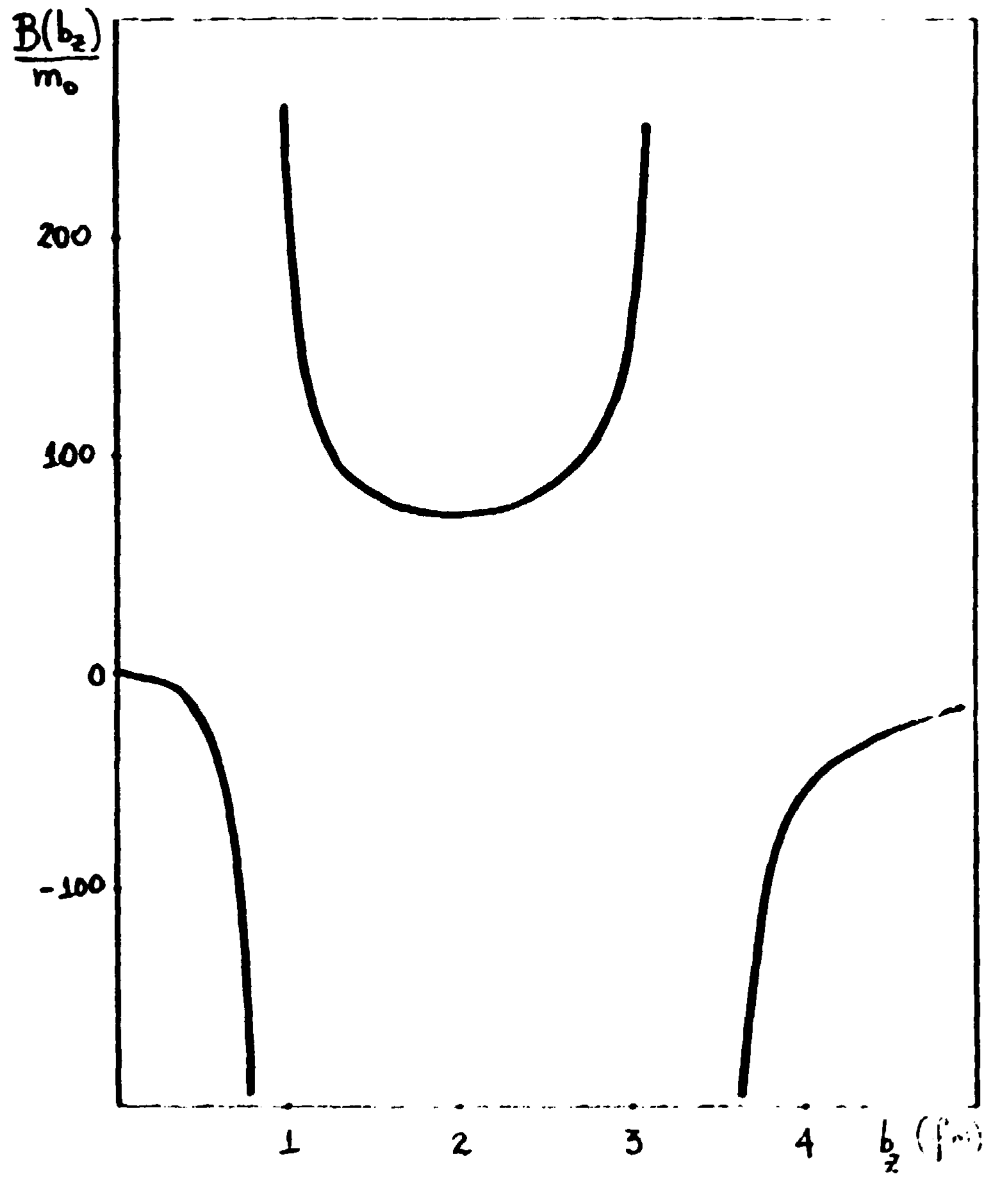


Fig. 3



B(b_z) / m_0
200
100
0
-100
1
2
3
4
b_z (fm)